Update of Golden measurements at a Neutrino Factory

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In this talk, we present an update and a refinement of the neutrino factory sensitivity on the mixing angle θ_{13} and the leptonic CP violating phase δ. We will concentrate exclusively on the LMA-MSW scenario. Compared to previous works, the expected uncertainties on the solar and atmospheric oscillation parameters and in the average Earth matter density along the neutrino path are added. Moreover, by exploring the full range for θ_{13} and δ, one discovers that, at fixed neutrino energy, E_ν, and baseline, L, there exists the possibility of degenerate solutions when measuring simultaneously (θ_{13}, δ). Although the spectral analysis helps in disentangling fake from true solutions, a leftover degeneracy remains when using a magnetized iron detector with realistic energy resolution. Including all these new considerations, one still reaches the conclusion that an intermediate baseline of $\mathcal{O}(3000)$ km is the best option to tackle simultaneously θ_{13} and δ, although a combination of two baselines turns out to be very important in resolving the degeneracies.

1. Introduction

In the next 5 – 10 years one will expect significative improvements in the knowledge of the parameters that determine neutrino oscillations. Planned solar, atmospheric and long baselines neutrino experiments will be able to define almost completely the pattern of neutrino oscillations. In particular in the next years it will be:

- determined if the LMA-SMW solution for the solar oscillation is the real solution chosen by Nature (by Kamland and Borexino experiments);
- measured the atmospheric mixing angle and mass difference with a precision of the 10% level (for the actual SuperK central value—Minos);
- confirmed weather or not the LSND signal reveals the existence of light sterile neutrino(s) (MiniBooNe and then BooNe if the signal is confirmed).

Nevertheless, there is a strong case for going further in the fundamental quest of the neutrino masses and mixing angles, as a necessary step to unravel the scale(s) of the underlying fundamental theory behind neutrino oscillations. In fact, in ten years from now there may be no significant improvement in the knowledge of:

1. the sign of Δm^2_{23}, which determines wheater the three-family neutrino spectrum is "hierarchical" or "degenerate" type (i.e one heavy state and two almost degenerate light ones or the reverse);
2. the mixing angle θ_{13}, that in a certain sense define the mixing between the solar and the atmospheric realms, for which the present best bounds is given by CHOOZ, $\sin^2\theta_{13} \leq 5\times 10^{-2}$;
3. the presence (or not) of CP violation in the leptonic sector, through a phase, δ, analogous to the CP-violating phase in the quark-mixing matrix.

The best opportunities for studying these topics will be given by the so called neutrino factory [1], that allows the measure of the transition probabilities $\nu_e(\bar{\nu}_e) \to \nu_\mu(\bar{\nu}_\mu)$. These are precisely the "golden measurements" at the neutrino factory. The enormous physics reach of such a signal in the context of three-family neutrino mixing was first realized in [2], where the emphasis was put

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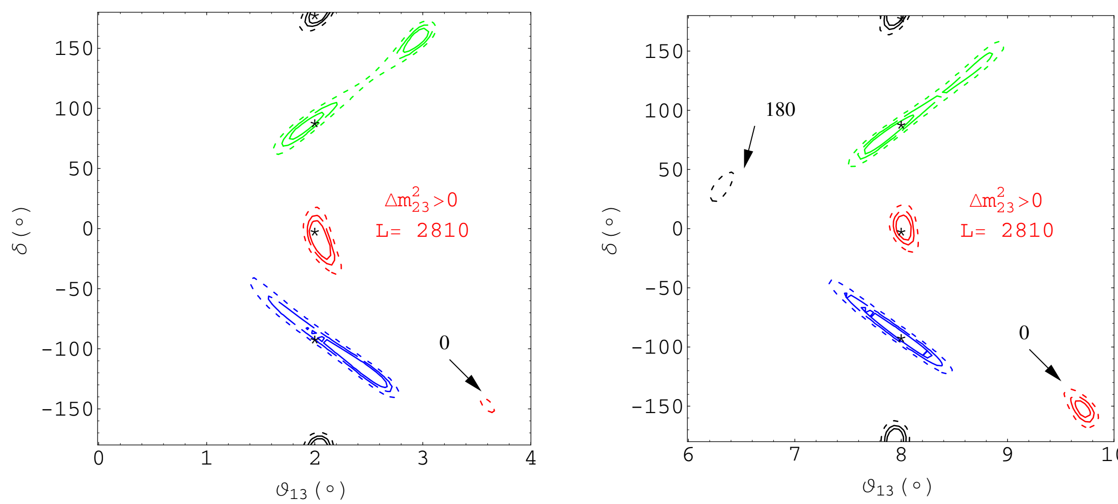

Figure 1: Simultaneous fits of δ and θ_{13} at $L = 2810$ km for different central values of $\bar{\delta}$ and for $\bar{\theta}_{13} = 2^\circ$ (left plot) and $\bar{\theta}_{13} = 8^\circ$ (right plot). The value of $\bar{\delta}$ for the degenerate solutions is also indicated.

on the separate measurements of θ_{13} and δ. But, as realized in [3], a strong correlation between θ_{13} and δ can seriously complicate the measurement especially in the case of small θ_{13} angle and large δ phase. In [4] the possibility of simultaneous determination of (θ_{13}, δ) was for the first time analyzed. A realistic experimental setup was considered with a total number of useful muons $n_\mu = 10^{21}$ in five years and a muon energy of $E_\mu = 50$ GeV. Moreover realistic efficiencies and backgrounds for a magnetized iron detector of 40 KTon of fiducial mass were included [5, 6]. In [4] it was realized that the best baselines for such a simultaneous determination is given by a baseline of $\mathcal{O}(3000)$ km. Shorter baselines are affected negatively by higher backgrounds and longer baselines are disfavored by a loss of statistic and an overwhelming matter effect that wipes out every possibility to detect CP violation signals. The analysis of [4] was done for $\theta_{13} \geq 1^\circ$ and $0 \leq \delta \leq \pi/2$. Moreover all the other oscillation parameters as well as the Earth matter density were assumed perfectly known. The motivation for [7] was to exceed these limitations. The results obtained will be presented shortly in this talk. Anyway, it is worth to mention from the beginning that the the main conclusions reached in [4] do not suffer any dramatic change.

2. Simultaneous determination of θ_{13} and δ

As argued in the introduction in ten years from now, our knowledge on the quantity θ_{13} and δ will remain practically unchanged. The physics scenarios will be completely different weather the solar solution will be the SMA-MSW, LOW and VO or the LMA-MSW solution [8, 9].

If the true solar solution will happen to be one of SMA-MSW, LOW or VO solution, then the discover of the leptonic CP violation in neutrino oscillation will be out of reach, due to the small mixing angle θ_{sun} and/or too small solar mass difference Δm^2_{sun}. Only θ_{13} will affect the oscillation probability and the neutrino factory could measured it with down to $\mathcal{O}(10^{-5})$ [2, 4]. The preferred baselines should be placed at a medium-large distance ($\geq$ 3000 km) for optimizing detector performances. An update analysis of this case, with the inclusion of all the parameters expected errors, can be found in [7].

In the case the LMA-MSW solution will appear to be the one chosen by Nature then CP violation can play a fundamental role and its discovery should be included as one of the most important goals for the neutrino factory. As actually the LMA-MSW solution seems preferred by SK measurement of a (small) day-night asymmetry, in the following of this talk we will analyze in more detail this second scenario. The main challenge of the neutrino factory (or any other experiment that may be planned in order to complete the measurements of the neutrino oscillations parameter space) will be to measure simultaneously the mixing angles θ_{13} and CP violating phase, δ. The best way to determine these two parameters is through the sub leading transitions $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ by searching for wrong-sign muons [1, 2] for both polarities of the beam, i.e. μ^+ and μ^- respectively. A convenient and precise approximation of the probability for this transition is obtained by expanding to second order in the two small parameters, θ_{13} and Δm^2_{12}, (small when compared with all the relevant energy scales at terrestrial distances). Defining $\Delta_{ij} \equiv \Delta m^2_{ij}/(2E)$, the result is [4]:

$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)} = s^2_{23}\sin^2 2\theta_{13}\left(\frac{\Delta_{13}}{\tilde{B}_\mp}\right)^2 \sin^2\left(\frac{\tilde{B}_\mp L}{2}\right) + c^2_{23}\sin^2 2\theta_{12}\left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\left(\frac{A L}{2}\right) + \tilde{J}\,\frac{\Delta_{12}}{A}\,\frac{\Delta_{13}}{\tilde{B}_\mp}\,\sin\left(\frac{AL}{2}\right)\sin\left(\frac{\tilde{B}_\mp L}{2}\right)\cos\left(\pm\delta - \frac{\Delta_{13} L}{2}\right), \quad (1)$$

where $\tilde{J} \equiv \cos\theta_{13}\ \sin 2\theta_{13}\ \sin 2\theta_{23}\ \sin 2\theta_{12}$, L is the baseline expressed in km and $\tilde{B}_\mp \equiv |A \mp \Delta_{13}|$. The matter parameter, A, is given in terms of the average electron number density, $n_e(L)$ taken from [10]. The probability in eq.(1) depends on three different terms. We can refers to them respectively as atmospheric, solar and interference term. The atmospheric term vanishes in the limit $\theta_{13} \to 0$, while the solar term vanish if $\Delta_{12} \to 0$. The interference term is vanishing in both previous limits. Note that all the CP information is carried by the interference term. We will refer to "*atmospheric regime*" ("*solar regime*") when the atmospheric (solar) term dominates in eq.(1). Roughly speaking, for the neutrino energy and baseline of relevance here, this happens for $\Delta m^2_{12} \approx 10^{-4}$ eV^2 and $\theta_{13} \geq (\leq) 1^\circ$.

The first question to understand is if by measuring $P_{\nu_e\nu_\mu}$ and $P_{\bar{\nu}_e\bar{\nu}_\mu}$, it is possible to determine unambiguously θ_{13} and δ at fixed neutrino energy, E_ν, and baseline, L. The answer is no. In fact it can be shown numerically and analytically (e.g. using the approximate expansion of Equation (1)) that for a fixed neutrino energy E_ν and baseline L there exists always (at least) another degenerate solution (θ_{13}, δ), which gives the same probabilities than the "true" values chosen by nature $(\bar{\theta}_{13}, \bar{\delta})$.

As the existence of "degenerate" solutions is an energy dependent condition, one could think that the inclusion of all the neutrino spectral information should remove the degeneracy. Simultaneous χ^2 fits of the parameters of $(\theta_{13}\,,\,\delta)$ for three reference baselines $L = 732$, 2810 and 7332 km (as well as for various combinations of them) has been performed. A muon beam of 50 GeV (five energy bins of 10 GeV) providing a total number of 10^{21} useful μ^+ and μ^- decays (which is the same working setup as in [4]) has been considered. Realistic efficiencies and backgrounds for a 40 Kton magnetized iron detector [5] have been included. All the results correspond to central values of the parameters in the LMA-MSW scenario: $\Delta m^2_{12} = 10^{-4}$ eV^2, $\Delta m^2_{23} = 3\times 10^{-3}$ eV^2 and $\theta_{12} = \theta_{23} = 45^\circ$ (except in Fig. 3, where the full range of Δm^2_{12} is considered). In Figures 1 the 68.5%, 90% and 99% contours resulting from the fits for $L = 2810$ km, for four central values of $\bar{\delta} = -90^\circ, 0^\circ, 90^\circ, 180^\circ$ and for $\bar{\theta}_{13} = 2^\circ$ (left) and $\bar{\theta}_{13} = 8^\circ$ (right) are shown (*atmospheric regime*). The true (input) solutions are depicted as a star. As can be seen the energy dependence of the signals is not significant enough (within our setup) to resolve the expected two-fold degeneracy. The "realistic" degeneracy shows up either as a new separate solution or as a broadening of the resolution of the true solution. Similar results are obtained also for shorter or longer baselines. The relevant limitation in our analysis is not, as one could suspect, the number of bins used. Degenerate solutions are, in fact, clearly visible also in fits that include several bins in energy (i.e 10). The stronger constraint, limiting the possibility of resolving the degeneracies comes from the very low (but unavoidable in our setup) detector efficiency for neutrino energies below 10 GeV. Degenerate solutions disappear increasing the statistics by a factor five at the intermediate baseline or if simultaneous fits for the combination of two baselines is performed. For example the two-fold degeneracy disappears completely in the combination of the intermediate baseline $\mathcal{O}(3000)$ km with the large baseline $\mathcal{O}(7000)$ km (but not with the short baseline $\mathcal{O}(700)$ km). If detectors with a lower detection threshold were used [11] one could probably obtain a "unique" solution with just one intermedaite baseline. The degeneracy is present also in the *solar regime*. Here, for any $\bar{\delta}$, a $\bar{\theta}_{13} = 0$ mimicking solution always appears. A more detailed discussion can be found in [7].

The second part of the talk is devoted to present an upgraded version of the analysis done in [4] taking into account all the errors in the other oscillation parameters as well as the error on the matter density parameter. Recent analysis of the expected uncertainty in the knowledge of the atmospheric parameters at the neutrino factory indicate a $\sim 1\%$ uncertainty in Δm^2_{23} and $\sin^2 2\theta_{23}$

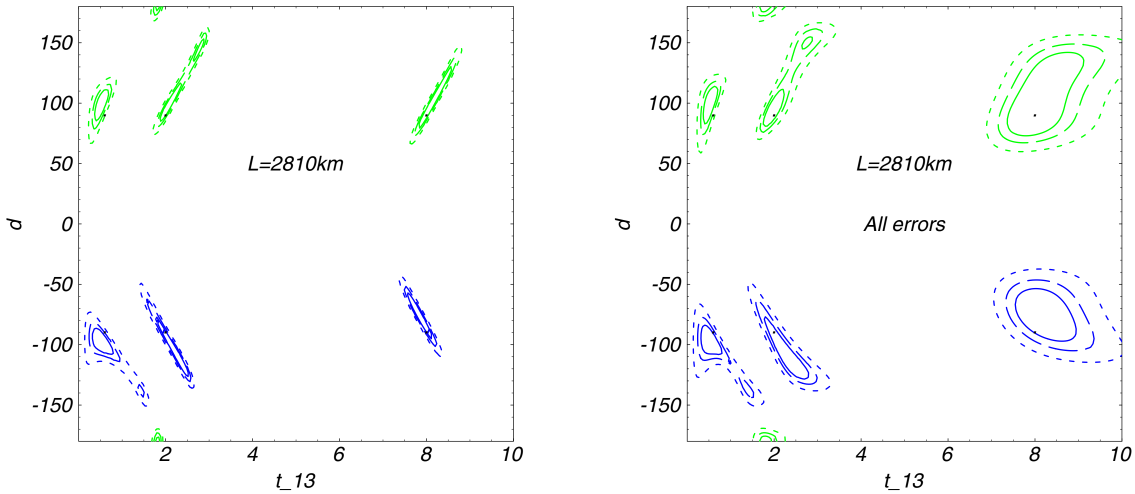

Figure 2: Fits of δ and θ_{13} at $L = 2810$ km including statistics, backgrounds and efficiencies (left plot) and adding also all the errors on the remaining parameters (right plot) with $\Delta A/A = 1\%$.

[12]. Although these analyses have been done for the SMA-MSW solution or assuming that the solar parameters are known, we will assume that in the LMA-MSW scenario the errors on the solar parameters or in the matter term do not change this result. For the solar parameters in the LMA-MSW regime we include the results of the analyses of the Kamland reach [13]: 2% error in Δm^2_{12} and ± 0.04 in $\sin^2 2\theta_{12}$, for (almost) maximal θ_{12}, both at 1σ. For the uncertainty on the matter parameter, A, we could not find any estimate in the literature. The dispersion of the different models of the Earth density profile [15] indicates an uncertainty of 1–2% for trajectories which do not cross the core. Analysis with $\Delta A/A = 10\%$ were also performed. The most important effects result from the uncertainty in θ_{23} and in the matter parameter A (once Δm^2_{12} and $\sin^2 2\theta_{12}$ are assumed to be known from Kamland as discussed above), with the former affecting mainly the measurement of θ_{13} and the latter the sensitivity to δ. In Figure 2 the comparison between the "old" and the "new" analysis is shown. On the left side plot all the other oscillation parameters (except the ones depicted on the axes) are assumed know. Instead, on the right side plot all the expected parameter uncertanities are included, with $\Delta A/A = 1\%$. As one can see the expected error on δ does not change significantly. Anyway even for $\Delta A/A = 10\%$, the effect on δ is still less important than the error induced by correlations between θ_{13} and δ. The precision on θ_{13} is slightly affected by the inclusion of all the parameters uncertainties, expecially for "large" values of θ_{13} (e.g 8°), being almost unaffected for smaller values (e.g. $\leq 1^\circ$). The error in θ_{13} is dominated by the uncertainty on θ_{23}.

Finally, it is interesting to understand how much of the LMA-MSW range can be covered in the discovery of CP violation. This is illustrated in Fig. 3 with a rough exclusion plot. For the hypothetical nature values $\bar{\delta} = 90^\circ$ and the best combination of baselines, $L = 2810 + 7332$ km, the line corresponds to the minimum value of Δm^2_{12} at which the 99%CL error on the phase reaches 90° degrees, and is thus indistinguishable from 0° or 180° (i.e. no CP violation). All errors on the parameters previously described have been included. Sensitivity to CP violation for $\theta_{13} >$ few tenths of degree and $\Delta m^2_{12} \geq 3 \times 10^{-4}$ eV^2 should be at reach.

3. Conclusion

At the hypothetical time of the neutrino factory, the parameters θ_{13} and δ may be still practically unknown and will have to be simultaneously measured. A relevant problem unearthed is the generic existence, at a given (anti)neutrino energy and fixed baseline, of a second "degenerate" solution (θ_{13}, δ) which gives the same oscillation probabilities for neutrinos and antineutrinos than the true values chosen by nature. These degeneracies can disappear if there exist a significant energy or baseline dependence. Anyway, also if in our configuration we are taking into

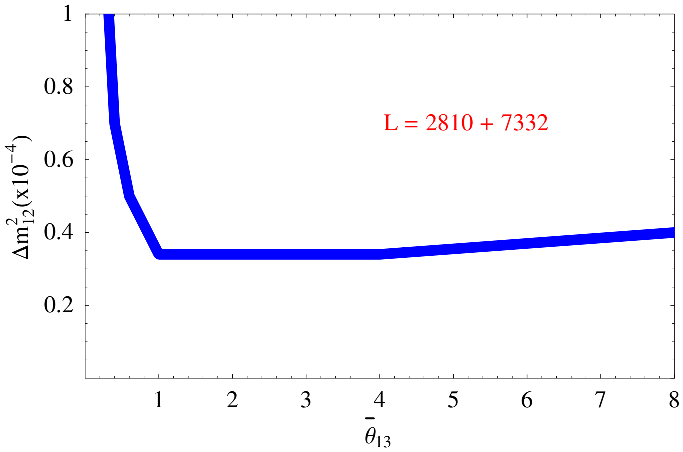

Figure 3: Sensitivity reach for CP violation as defined in the text on the plane $(\Delta m^2_{12}, \bar{\theta}_{13})$ for the combination of baselines $L = 2810$ and 7332 km. All errors are included.

account the energy spectrum, binning in energy, this is not enough to resolve completely the degeneracies at a single baseline. Simultaneous fits for the combination of any two baselines have been performed. The two-fold degeneracy disappears completely in the combination of the intermediate baseline $\mathcal{O}(3000)$ km with the large baseline $\mathcal{O}(7000)$ km but not with the short baseline $\mathcal{O}(700)$ km. The expected uncertainty on the knowledge of the rest of the oscillation parameters ($\sin^2\theta_{23}$, Δm^2_{23}, $\sin^2\theta_{12}$, Δm^2_{12}) and on the Earth electron density has been included in the analysis. The most important effects come from the error on θ_{23}, which affects mainly the uncertainty in θ_{13}, and from the uncertainty on the Earth matter profile, which affects mainly the extraction of δ.

After this analysis the overall conclusion remains substantially in agreement with previous works [4]. The optimal distance for studying simultaneously θ_{13} and CP-violation effects is still of $O(3000)$ km, although the combination of two baselines, one of which being preferably a very long one, is very important in resolving degeneracies.

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