Status and Prospects of Theoretical Predictions for Weak Gauge Boson Production Processes at Lepton and Hadron Colliders

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(Dated: November 6, 2001)

For the envisioned precision measurement of the W-boson mass at present and future lepton and hadron colliders it is crucial that the theoretical predictions for the underlying production processes are well under control. We briefly describe the status of the predictions for the W-pair-production processes at e^+e^- colliders, $e^+e^- \to W^+W^- \to 4f$, and for W- and Z-boson production at pp and $p\bar{p}$ colliders, $p\overset{(-)}{p} \to W^\pm \to \ell^\pm\nu_\ell$ and $p\overset{(-)}{p} \to Z,\gamma \to \ell^+\ell^-$ ($\ell = e, \mu$). We also discuss the theoretical improvements needed to meet the experimental accuracies one hopes to achieve in future experiments.

I. INTRODUCTION

One of the main tasks at present and future lepton and hadron colliders is a precise measurement of the W-boson mass, M_W. The present value of the W-boson mass, $M_W = 80.451 \pm 0.033$ GeV [1], has been obtained from combining measurements carried out at LEP2, the CERN $\bar{p}p$ collider, and in Run I of the Tevatron. The W-boson mass plays a major role in the indirect determination of the Standard Model Higgs-boson mass, M_H, from a global fit to the electroweak precision observables. A more precise knowledge of M_W will greatly improve the indirect bounds on M_H as discussed in detail, for example, in Ref. [2].

In future experiments at the Tevatron, one expects to measure M_W with a precision of 20–40 MeV [3]. At the LHC one hopes to achieve an accuracy of 15 MeV [4]. At a future linear e^+e^- collider (LC) a precision of about 6 MeV [5, 6] from a dedicated threshold scan of the W-pair production cross section and of the order of 15 MeV [7] or even better [8] at $\sqrt{s} = 500$ GeV may be achievable. In order to measure the W-boson mass with such high precision it is necessary to fully understand and control QCD and electroweak corrections to single-W production at hadron colliders and to W-pair production at lepton colliders.

At a linear collider, one expects that about 10^6 W-boson pairs are produced per year, compared to a yield of $\mathcal{O}(10^4)$ W-boson pairs at LEP2. This implies that the $e^+e^- \to W^+W^- \to 4f$ cross section can be measured with a precision of the order of a few per mille. The measurement of the W-pair differential cross section provides a tool for a precise measurement of the (charged) triple gauge-boson couplings (TGCs) [9]. Already at LEP2 the precision achieved for the TGCs requires the inclusion of electroweak $\mathcal{O}(\alpha)$ corrections in the calculation of

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the cross section. They affect the shape of the angular distribution from which the TGCs are extracted and, in case of the C and P conserving TGCs, may introduce theoretical uncertainties as large as the experimental uncertainties [10, 11]. Thus, for the precision envisioned at a LC [6, 12] the inclusion of electroweak 1-loop corrections is indispensable.

Here we give a status report of radiative corrections to $e^+e^- \to W^+W^- \to 4f$ and to weak gauge-boson production at hadron colliders. In Section II we briefly describe the presently available calculations of electroweak corrections to W-pair production at LEP2 and LC center-of-mass (CM) energies, and discuss their uncertainties and the theoretical challenges for the envisioned precision of M_W at a future LC. In Section III we describe the status of calculations of radiative corrections to W- and Z-boson production at hadron colliders. QCD corrections only indirectly influence the W-mass determination [3] and their effect on the p_T distribution of the W boson is well described by calculations [13–15] which resum soft-gluon emission terms. We therefore focus entirely on electroweak radiative corrections in our discussion. In Section IV we, finally, briefly discuss the field-theoretical definition of the W-boson mass. The precise definition of M_W is important at CM energies much larger than the gauge-boson masses, and needs to be addressed for precision measurements of M_W at future e^+e^- and hadron colliders.

II. ELECTROWEAK RADIATIVE CORRECTIONS TO $e^+e^- \to WW \to 4f$

A complete calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to WW \to 4f$ is technically very difficult. Moreover, the requirement to include finite W-width effects poses severe problems with gauge invariance. Although there is ongoing work in this direction [16, 17], a full calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to WW \to 4f$ is currently not available. A suitable approach to include $\mathcal{O}(\alpha)$ corrections to W-pair production is provided by the double-pole approximation (DPA): electroweak $\mathcal{O}(\alpha)$ corrections are only considered for the terms that are enhanced by two resonant W bosons. The intrinsic DPA uncertainty is estimated to be of the order of $\alpha\Gamma_W/(\pi M_W)$, i.e. $\lesssim$0.5%, whenever the cross section is dominated by doubly-resonant contributions. This is the case at LEP2 energies sufficiently above threshold and up to about 500 GeV. The DPA is not a valid approximation close to the W-pair production threshold. At higher energies contributions from single resonant (single W production) and non-resonant diagrams become sizeable, and appropriate cuts have to be imposed to extract the WW signal. All present calculations of $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to WW \to 4f$ rely on the DPA [18–22]. However, the technical details of the DPA implementation differ somewhat and lead to small differences in the predicted cross sections within the expected uncertainties. These differences can be studied in detail with the state-of-the-art Monte Carlo (MC) generators `RacoonWW` [19, 23, 24] and `YFSWW3` [20, 21, 25], and have to be treated as systematic theoretical uncertainties in the extraction of M_W and TGCs. In the following, we briefly discuss the theoretical uncertainties associated with those MC generators, and the theoretical precision required in order to meet the goal of $\delta M_W = 6$ MeV in a threshold scan at a LC. A detailed comparison of all presently available calculations for $e^+e^- \to 4f$ can be found in Ref. [26].

A. Theoretical Uncertainties of Current Calculations of $e^+e^- \to WW \to 4f$

A tuned numerical comparison between the state-of-the-art MC generators `RacoonWW` and `YFSWW3`, supported by a comparison with a semi-analytical calculation [18] and a study of the intrinsic DPA ambiguity with `RacoonWW` [19, 26] and `YFSWW3` [26], shows that the current theoretical uncertainty for the total W-pair production cross section is about 0.5% for CM energies between 170 GeV and 500 GeV [26]. This is in agreement with the expected intrinsic DPA uncertainty of $\lesssim$0.5% for these energies. A tuned comparison has also been performed for `RacoonWW` and `YFSWW3` predictions for the W invariant-mass distribution, the distribution of the W production angle, as well as several photon observables at $\sqrt{s} = 200$ GeV [26–28] and $\sqrt{s} = 500$ GeV [27–29]. Taking the observed differences [26] between the `RacoonWW` and `YFSWW3` predictions as a guideline, a theoretical uncertainty of about 1% can be assigned to the distribution of the W production angle and the W invariant-mass distribution in the W resonance region.

The theoretical uncertainties of the $e^+e^- \to WW \to 4f$ cross section translate into uncertainties of the W mass and the TGCs extracted from data. A recent study [30] based on the MC generators `KoralW` and `YFSWW3` finds a theoretical uncertainty of $\delta M_W = 5$ MeV due to unknown electroweak corrections at LEP2 energies. This is consistent with a qualitative 'naive' estimate for the shift in M_W derived from the observed difference [26] between the `RacoonWW` and `YFSWW3` predictions for the W invariant-mass distribution. Using the MC generator `YFSWW3`, the ALEPH collaboration [10] has derived (preliminary) results for the shifts in the extracted values for the TGCs due to the inclusion of electroweak corrections. A study of the theoretical uncertainty induced by missing electroweak radiative corrections for the TGC parameter $\lambda = \lambda_\gamma = \lambda_Z$, extracted from the W angular distribution at LEP2, is in progress [31].

A major difference between `RacoonWW` and `YFSWW3` lies in their treatment of visible photons. In `RacoonWW` real photon radiation is based on the full $4f+\gamma$ matrix element and collinear leading higher-order initial-state radiation up to $\mathcal{O}(\alpha^3)$. In `YFSWW3`, on the other hand, multi-photon radiation to W-pair production is combined with $\mathcal{O}(\alpha^2)$ leading-logarithmic photon radiation in W decays by using `PHOTOS` [32]. A comparison [27] of `RacoonWW` and `YFSWW3` predictions for photon observables finds relative differences of less than 5% at $\sqrt{s} =$ 200 GeV and about 10% at $\sqrt{s} = 500$ GeV between the two generators. Precise knowledge of the photon observables will be needed for the measurement of photonic quartic gauge-boson couplings in $4f+\gamma$ production at a future LC. Even for the LEP2 measurement of these couplings [9], an improvement in the current theoretical prediction for the photon-energy spectrum is desirable.

B. M_W Measurement at a Future Linear Collider

The W-boson mass can be measured in W-pair production at a LC either in a dedicated threshold scan operating the machine at $\sqrt{s} \approx 161$ GeV, or via direct reconstruction of the W bosons at intermediate and high energies $\sqrt{s} = 170$–1500 GeV. Both strategies have been used with success at LEP2.

In the threshold region, the total W-pair production cross section, σ_{WW}, is very sensitive to the W-boson mass. A rough estimate of the statistical power of a W-mass measurement from σ_{WW} in this energy region at a LC can be obtained from the corresponding LEP2 studies. As discussed in Refs. [33, 34] the sensitivity for M_W is largest in the region around $\sqrt{s} = 161$ GeV at which point the statistical uncertainty can be estimated to be

$$\delta M_W^{\rm stat} \approx 90 \text{ MeV} \left[\frac{\varepsilon \int \mathcal{L} dt}{100 \text{ pb}^{-1}}\right]^{-1/2} . \tag{1}$$

Here, ε is the efficiency for detecting W bosons. For $\varepsilon = 0.67$ and an integrated luminosity of 100 fb^{-1}, one finds from Eq. (1)

$$\delta M_W^{\rm stat} \approx 3.5 \text{ MeV}. \tag{2}$$

The systematic uncertainty due to an overall multiplicative factor C to the cross section, such as efficiency and luminosity, can be parametrized in the form [33, 34],

$$\delta M_W^{\rm syst} = 17 \text{ MeV} \left[\frac{\Delta C}{C} \times 100\right] , \tag{3}$$

where $\Delta C/C$ is the relative error on the quantity C. Assuming that the efficiency and the integrated luminosity can be determined with a precision of $\Delta\varepsilon/\varepsilon = 0.25\%$ and $\Delta\mathcal{L}/\mathcal{L} = 0.1\%$, one finds that M_W can be measured with an uncertainty of

$$\delta M_W \approx 6 \text{ MeV}, \tag{4}$$

provided that the theoretical uncertainty is under control in the energy region of interest. This estimate has been confirmed by a more recent study [5] of the precision of a M_W measurement from a threshold scan at a LC using simulated data and taking into account experimental systematic uncertainties. However, as emphasized in Ref. [5], for such a precise M_W measurement not to be jeopardized by the theoretical uncertainty (i.e. to keep the theoretical uncertainty below 1–2 MeV) the calculations of the relevant observables in the energy region of interest need to be of a precision of $\mathcal{O}(0.1\%)$.

Presently, the total W-pair production cross section in the threshold region is known with an accuracy of about 1–2% [35, 36], since predictions are based on an improved-Born approximation which neglects non-universal electroweak corrections. As mentioned earlier, the DPA is not a valid approximation in the threshold region, as the $e^+e^- \to 4f$ cross section in this region is not dominated by W-pair production. At present there is no study available on how the requirement that the theoretical uncertainty of M_W extracted from a threshold scan has to be less than 1–2 MeV translates into a constraint on the theoretical precision for σ_{WW} in the threshold region. One could argue that only the shape of the total W-pair production cross section in the threshold region needs to be known with high precision but not the overall normalization. However, it is expected that the non-universal $\mathcal{O}(\alpha)$ corrections modify the shape of the cross section curve in this energy region. If one (pessimistically) assumes that the theoretical uncertainty of the cross section will not improve and that the shape of σ_{WW} in the narrow region of $\sqrt{s} \approx 161$ GeV considered in Ref. [5] is not predicted with sufficient accuracy by the improved-Born approximation, the uncertainty of the W mass obtained from a threshold scan is completely dominated by the theoretical error, and the precision of the W mass is limited to [34]

$$\delta M_W \approx \delta M_W^{\text{theor}} \;\approx\; 17 \text{ MeV} \left[\frac{\Delta\sigma}{\sigma} \times 100\right] \approx 20\text{–}30 \text{ MeV}. \tag{5}$$

Thus, in order to understand and reduce the theoretical uncertainty of the M_W measurement from a threshold scan to the desired level, the full $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to 4f$ in the threshold region are needed.

The full treatment of the processes $e^+e^- \to 4f$ at the one-loop level is of enormous complexity. While the real Bremsstrahlung contribution, $e^+e^- \to 4f + \gamma$, is known exactly for all final states [37–40] there are severe theoretical problems with the virtual $\mathcal{O}(\alpha)$ corrections. A full calculation of the virtual contribution to $e^+e^- \to 4f$ can lead to $\mathcal{O}(10^4)$ complicated one-loop diagrams involving 5- and 6-point functions which exhibit potential numerical instabilities. Besides these technical challenges, there are serious theoretical problems with gauge invariance in connection with the instability of the W and Z bosons, as discussed in Section IV.

Using direct reconstruction of W bosons and assuming an integrated luminosity of 500 fb^{-1} at $\sqrt{s} = 500$ GeV, one expects a statistical error of $\delta M_W^{\text{stat}} \approx 3.5$ MeV [8]. Systematic errors are dominated by jet resolution effects. Using $Z\gamma$ events where the Z decays into 2 jets and the photon is lost in the beam pipe for calibration, a systematic error $\delta M_W^{\text{syst}} < 10$–$15$ MeV is expected. The resulting overall precision of the W-boson mass from direct W reconstruction at a LC operating at an energy well above the W-pair threshold is [6, 8]

$$\delta M_W \approx 10\text{–}15 \text{ MeV}. \tag{6}$$

For theoretical predictions in the high-energy range, i.e. $\sqrt{s} > 500$ GeV, the same calculations as for the LEP2-energy range can be used as a starting point. However, in view of a 10–15 MeV precision, the study of the theoretical uncertainty of M_W due to electroweak corrections of Ref. [30] should be repeated at 500 GeV. Moreover, large electroweak logarithms of Sudakov-type become increasingly important and the theoretical uncertainty is expected to become worse due to missing higher-order corrections. At $\sqrt{s} = 1$ TeV the typical size of the corrections from $\mathcal{O}(\alpha^2)$ Sudakov logarithms to cross sections for single-W and W-pair production processes can be estimated to be of the order of one and several per cent, respectively. These double logarithmic contributions are expected to exponentiate, which has been confirmed by explicit two-loop calculations [41–43]. Subleading contributions are also under study for specific processes, since they are expected to have large coefficients at one loop. For the relevant literature and a review of the present understanding of electroweak Sudakov logarithms we refer to Refs. [44, 45]. The effect of these corrections on the W mass extracted from data has not been studied yet.

III. ELECTROWEAK RADIATIVE CORRECTIONS TO WEAK BOSON PRODUCTION IN HADRONIC COLLISIONS

The determination of the W-boson mass in a hadron collider environment requires a simultaneous precision measurement of the Z-boson mass, M_Z, and width, Γ_Z. When compared to the value measured at LEP1, the two quantities help to accurately calibrate detector components [46–49]. It is therefore necessary to understand electroweak corrections for both W- and Z-boson production.

QED corrections are known to produce a considerable shift in the measured W- and Z-boson masses [46–49]. The shift in M_W is significantly larger than the uncertainty expected in future experiments. In the calculation [50] which was used in the analysis of the Tevatron Run I data, only the final-state photonic corrections were correctly included. The sum of the soft and virtual parts was indirectly estimated from the inclusive $\mathcal{O}(\alpha^2)$ $Z \to \ell^+\ell^-(\gamma)$ and $W \to \ell\nu_\ell(\gamma)$ width and the hard photon bremsstrahlung contribution. Initial-state, interference, and weak contributions to the $\mathcal{O}(\alpha)$ corrections were ignored altogether. The ignored parts of the $\mathcal{O}(\alpha)$ electroweak radiative corrections, combined with effects of multiple photon emission (higher-order corrections), have been estimated to contribute a systematic uncertainty of $\delta M_W = 15$–20 MeV to the measurement of the W-boson mass [46–49]. Given the expected accuracy for M_W in Run II of the Tevatron and the LHC, improved calculations of the electroweak corrections to weak-boson production in hadronic collisions are needed.

The full electroweak $\mathcal{O}(\alpha)$ corrections to resonant W-boson production in a general four-fermion process were computed in Ref. [51] with special emphasis on obtaining a gauge-invariant decomposition into a photonic and non-photonic part. The results were used in Ref. [52] to calculate the $\mathcal{O}(\alpha)$ electroweak corrections to $p\overset{(-)}{p} \to W \to \ell\nu_\ell$ in the pole approximation. The cross section for W-boson production via the Drell–Yan mechanism at parton level, $q_i\bar{q}_{i'} \to f\bar{f}'(\gamma)$, in the pole approximation can be written in the form

$$d\hat{\sigma}^{(0+1)} = d\hat{\sigma}^{(0)} \left\{1 + 2\mathcal{R}e\left[\tilde{F}^{\rm initial}_{\rm weak}(\hat{s} = M_W^2) + \tilde{F}^{\rm final}_{\rm weak}(\hat{s} = M_W^2)\right]\right\} + \sum_{\substack{\rm a=initial,final,\\ \rm interf.}} \left[d\hat{\sigma}^{(0)}\, F^{\rm a}_{\rm QED}(\hat{s},\hat{t}) + d\hat{\sigma}^{\rm a}_{2\to 3}\right], \tag{7}$$

where the Born-cross section, $d\hat{\sigma}^{(0)}$, is of Breit–Wigner form, and $\hat{s}$ and $\hat{t}$ are the usual Mandelstam variables in the parton CM frame. The (modified) weak corrections and the virtual and soft-photon emission from the initial and final-state fermions (as well as their interference) are described by the form factors $\tilde{F}^{\rm a}_{\rm weak}$ and $F^{\rm a}_{\rm QED}$, respectively. The IR-finite contribution $d\hat{\sigma}^{\rm a}_{2\to 3}$ describes real photon radiation away from soft singularities. Mass singularities of the form $\ln(\hat{s}/m_f^2)$ arise when the photon is emitted collinear to a charged fermion and the resulting singularity is regularized by retaining a finite fermion mass (m_f). $F^{\rm initial}_{\rm QED}$ and $d\hat{\sigma}^{\rm initial}_{2\to 3}$ still include quark-mass singularities which need to be extracted and absorbed into the parton distribution functions (PDFs). The absorption of the quark-mass singularities into the PDFs can be done in complete analogy to gluon emission in QCD, thereby introducing a QED factorization scheme dependence. The latter results in a modified scale dependence of the PDFs, which is expected to have a negligible effect on the observable cross sections [4, 53, 54]. Comparing the W-mass shifts obtained using the calculations of Refs. [50] and [52], one finds that the proper treatment of virtual and soft corrections and the inclusion of weak corrections induces an additional shift of $\mathcal{O}(10$ MeV) in the extracted W-boson mass.

The calculation of the electroweak $\mathcal{O}(\alpha)$ corrections to W production described in Ref. [52] was carried out in the pole approximation, i.e. corrections which are very small at the W pole, such as the WZ box diagrams, were ignored and the form factors $\tilde{F}^{\rm a}_{\rm weak}$ were evaluated at $\hat{s} = M_W^2$. A calculation of the full $\mathcal{O}(\alpha^3)$ matrix elements for $p\overset{(-)}{p} \to W \to \ell\nu_\ell$ has recently appeared in Refs. [55, 56]. An independent calculation which will also discuss the impact of the electroweak radiative corrections on the M_W mass extracted from various observables [3] is in progress [57]. While the corrections ignored in Ref. [52] change the differential cross section in the W-pole region by less than 1% [55], they become large at high $\ell\nu_\ell$ invariant masses $m(\ell\nu_\ell)$ due to Sudakov-like logarithms of the form $\ln^2[m(\ell\nu_\ell)/M_W]$. They significantly affect the transverse mass distribution above the W peak, which serves as tool for a direct measurement of the W width, Γ_W. Taking these corrections into account in future measurements of Γ_W will be important.

A calculation of the $\mathcal{O}(\alpha)$ QED corrections to $p\overset{(-)}{p} \to \gamma, Z \to \ell^+\ell^-$ based on the full set of contributing one-loop Feynman diagrams was carried out in Ref. [58]. Purely weak corrections were ignored in this first step towards a complete calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $p\overset{(-)}{p} \to \gamma, Z \to \ell^+\ell^-$. The difference in the extracted Z-boson mass when comparing the approximate calculation of Ref. [50] with the full calculation of the $\mathcal{O}(\alpha)$ QED corrections was found to be of $\mathcal{O}(10$ MeV). However, in order to properly calibrate the Z-boson mass and width using the available LEP data, it is desirable to use exactly the same theoretical input that has been used to extract M_Z and Γ_Z at LEP, i.e. to include the purely weak corrections to $p\overset{(-)}{p} \to \gamma, Z \to \ell^+\ell^-$ and the $\mathcal{O}(g^4m_t^2/M_W^2)$ corrections to the effective leptonic weak mixing parameter, $\sin^2\theta^\ell_{\rm eff}$, and the W-boson mass [59], in the calculation. A calculation of the complete $\mathcal{O}(\alpha)$ corrections to $p\overset{(-)}{p} \to \gamma, Z \to \ell^+\ell^-$ which also takes into account the $\mathcal{O}(g^4m_t^2/M_W^2)$ corrections has recently been completed [60]. The additional corrections taken into account in Ref. [60] enhance the differential cross section in the Z-peak region by up to 1.2%. Since they are not uniform, they are expected to shift the Z-boson mass extracted from data upward by several MeV. Detailed simulations of this effect, however, have not been carried out yet.

For the analysis of Run IIA data [3] an $\mathcal{O}(\alpha^3)$ calculation of W and Z production will likely be sufficient. This is not the case for the precision of M_W expected with Run IIB data ($\delta M_W \leq 20$ MeV). The main contribution to the shift in M_W originates from final-state photon radiation. An explicit calculation of real two-photon radiation in W- and Z-boson production [61] indicates that, in order to measure the W mass with a precision of less than 20 MeV in a hadron collider environment as foreseen in Run IIB and at the LHC, it will be necessary to take into account multi-photon radiation effects.

IV. THEORETICAL ISSUES AT HIGH LUMINOSITY AND ENERGY

For precision measurements of M_W with values of δM_W smaller than about 40 MeV, the precise definition of the W mass and width become important when these quantities are extracted. For stable particles the location of the pole in the particle's propagator provides a proper definition of the mass parameter, which is known as the pole mass. For unstable particles, such as the W and Z bosons, the pole position becomes a complex quantity, so that the actual definition of an associated real mass parameter involves some convention. The imaginary part of the pole position is related to the decay width of the unstable particle. In LEP1 precision physics, usually the convention, called the on-shell mass scheme, was adopted of identifying the Z-boson mass with the zero in the real part of the inverse propagator, which coincides with the real part of the complex pole position in one-loop order, but differs at two loops and beyond. Field-theoretical studies [62, 63] showed that the complex pole is related to the bare mass of the unrenormalized field theory in a gauge-invariant way, while the on-shell definition involves gauge dependences starting at two-loop accuracy.

In practice the issue of the mass definition enters when the Breit–Wigner resonance shape is parametrized by the mass and the width. In the following we consider a single W-boson resonance, as it appears, for instance, in $q\bar{q}' \to W \to \ell\nu_\ell$. In a field-theoretical description, the resonance propagator results from a Dyson summation of the imaginary part of the W vacuum polarization, and the matrix element $\mathcal{M}$ behaves as

$$\mathcal{M} \sim \frac{R}{k^2 - M_W^2 + iM_W\Gamma_W(k^2)} \tag{8}$$

in the vicinity of the resonance ($k^2 \sim M_W^2$), where k is the momentum transfer in the propagator. This leads to an energy-dependent width $\Gamma_W(k^2)$, which is given by $\Gamma_W(k^2) = \Gamma_W \times (k^2/M_W^2)$ if the decay fermions are taken to be massless. In Eq. (8) we did not yet specify the precise definition of the mass and width parameters. In the on-shell mass definition the form (8) naturally arises in any loop order, thus we identify M_W and Γ_W with the on-shell quantities in the following. The relation to the complex pole position, denoted by $\overline{M}_W^2 - i\overline{M}_W\overline{\Gamma}_W$, is easily obtained by inserting the explicit form of $\Gamma_W(k^2)$ into Eq. (8), and identifying terms. This leads to

$$\mathcal{M} \sim \frac{\overline{R}}{k^2 - \overline{M}_W^2 + i\overline{M}_W\overline{\Gamma}_W} \tag{9}$$

with [51, 64, 65]

$$\begin{aligned}
\overline{M}_W &= M_W/\sqrt{1+\gamma^2} = M_W - \frac{\Gamma_W^2}{2M_W} + \ldots, \\
\overline{\Gamma}_W &= \Gamma_W/\sqrt{1+\gamma^2} = \Gamma_W - \frac{\Gamma_W^3}{2M_W^2} + \ldots, \\
\overline{R} &= R/(1+i\gamma) = R\left(1 - i\frac{\Gamma_W}{M_W} + \ldots\right),
\end{aligned} \tag{10}$$

where $\gamma = \Gamma_W/M_W$. Thus, if the measured W resonance is fitted to a propagator that is parametrized with a constant width, the measured mass parameter will be $\overline{M}_W$, which is about 27 MeV smaller than the on-shell value M_W. This procedure ignores distortion effects of the resonance shape, in particular those induced by radiation. In Ref. [55] it has been verified numerically that the reparametrization (10) works extremely well if electroweak $\mathcal{O}(\alpha)$ corrections are taken into account.

In the past, an energy-dependent W width has been used in measurements of the W mass at the Tevatron [46–49]. The Monte Carlo programs available for the W-mass analysis at LEP2 (see Ref. [26] for an overview) in contrast use a constant W width. Since the difference between the W mass obtained using a constant and an energy-dependent width is of the same size as the current uncertainties from LEP2 and Tevatron data and significantly larger than those expected from future collider experiments, it is important to correct for this difference.

Unfortunately, the Dyson summation of the resonance propagators, as described above, carries the risk of breaking gauge invariance. Gauge invariance works order by order in perturbation theory. By resumming the self-energy corrections, one only takes into account part of the higher-order corrections. Apart from being theoretically questionable, breaking gauge invariance may result in large numerical errors in cross section calculations. This has been illustrated by specific examples in Refs. [38, 65, 66].

In order to restore gauge invariance, one can adopt the strategy of finding the minimal set of Feynman diagrams that is necessary for compensating the terms caused by an energy-dependent width that violate gauge invariance. In Refs. [65–69] this approach was systematically carried out by a gauge-invariant Dyson summation of the fermion-loop corrections which lead to the gauge-boson decay widths in the propagators. This procedure provides a consistent treatment of lowest-order matrix elements, improved by running couplings. For lowest-order predictions there are also other methods leading to gauge-invariant results, such as the complex-mass scheme [38] or the use of effective Lagrangians [70]. However, a practical general way of combining Dyson-summed propagators with full $\mathcal{O}(\alpha)$ corrections (including bosonic loops) is still not available. To this end, the background-field quantization seems to be a promising framework, since Dyson summation preserves all Ward identities induced by gauge invariance [71] in contrast to conventional quantization.

For the resonance process $q\bar{q}' \to W \to \ell\nu_\ell$ the gauge-invariant inclusion [51, 55] of the $\mathcal{O}(\alpha)$ correction was possible in a simple way, but this is due to the occurrence of a single s-channel resonance. While the naive introduction of fixed gauge-boson widths might be sufficient for the full calculation of the $\mathcal{O}(\alpha)$ corrections to the four-fermion processes in the LEP2-energy range, the calculation of these corrections at high energies certainly requires some progress towards a solution of the gauge-invariance problem.

V. CONCLUSIONS

While the full electroweak $\mathcal{O}(\alpha)$ corrections to the Drell–Yan-like production of W and Z bosons are known, all calculations of $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to WW \to 4f$ are based on the leading term of an expansion about the double resonance pole, an approach that is known as double-pole approximation. For the W-mass measurements at LEP2 and in Run IIA (2 fb^{-1}) of the Tevatron, the accuracy of the corresponding predictions is sufficient.

In order to measure the W-boson mass with a precision of less than 20 MeV in Run IIB (15 fb^{-1}) of the Tevatron and at the LHC, radiative corrections beyond $\mathcal{O}(\alpha)$ have to be controlled. In particular, multi-photon radiation effects [3, 61] have to be taken into account.

To fully exploit the potential of a future LC for a precision M_W measurement in a threshold scan, the full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4f$ are needed. On the other hand, for a determination of M_W by reconstruction at intermediate and high energies ($\sqrt{s} = 170-1500$ GeV), which is comparable in precision with the measurements at Run IIB and the LHC, the existing calculations for radiative corrections to $e^+e^- \to WW \to 4f$ could be sufficient, but this expectation needs to be confirmed by explicit calculations, or proper estimates, of the missing corrections.

Acknowledgments

One of us (U.B.) would like to thank the Phenomenology Institute of the University of Wisconsin, Madison, and the Fermilab Theory Group, where part of this work was carried out, for their splendid hospitality. The work of U.B. is supported by NSF grant PHY-9970703. The work of A.D. is supported in part by the Swiss Bundesamt für Bildung und Wissenschaft contract 99.0043 and the work of A.D., S.D., M.K., and M.R. by the European Union under contract HPRN-CT-2000-00149. S.D. is supported by a Heisenberg fellowship. The work of D.W. is supported by the U.S. Department of Energy under grant DE-FG02-91ER40685.

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