Theoretical and Experimental Status of the Indirect Higgs Boson Mass Determination in the Standard Model

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The impact of theoretical and experimental uncertainties on the indirect determination of the Higgs boson mass, M_H, in the Standard Model (SM) is discussed. Special emphasis is put on the electroweak precision observables M_W (the W boson mass) and $\sin^2\theta_{\rm eff}$ (the effective leptonic mixing angle). The current uncertainties of the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$ due to missing higher order corrections are conservatively estimated to $\delta M_W \approx 7$ MeV and $\delta\sin^2\theta_{\rm eff} \approx 7\times 10^{-5}$. Expectations and necessary theoretical improvements for future colliders are explored. Results for the indirect M_H determination are presented based on the present experimental and theoretical precisions as well as on improvements corresponding to the prospective situation at future colliders. The treatment of the different future colliders is done in a uniform way in order to allow for a direct comparison of the accuracies that can be reached. Taking all experimental, theoretical, and parametric uncertainties into account, a current upper bound on M_H of ~ 200 GeV is obtained. Furthermore we find in a conservative approach that a Linear Collider with GigaZ capabilities can achieve a relative precision of about 8% (or better) in the indirect determination of M_H.

I. INTRODUCTION

In this contribution we address the status and possible future developments in the measurements of and the theoretical predictions for the most important electroweak precision observables. We estimate their precision from upcoming and proposed accelerator experiments. In all cases we quote uncertainties which we believe to be realistically achievable, not excluding even greater precisions. As a result of imposing similar standards in all cases, our quoted uncertainties should be directly comparable. Similarly, we attempt to anticipate which improvements can be expected in the theoretical predictions for the observables. Again, we believe that our estimates can be realistically achieved with a dedicated effort and allow some leeway for even more precise calculations.

Within the SM, the mass of the Higgs boson, M_H, can be constrained indirectly with the help of electroweak precision observables (EWPO). As a result of a global analysis, Fig. 1 [1] shows $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min}$ as an approximately quadratic function of $\log M_H$. Therefore, the 95% CL upper limit can be approximated by $\Delta\chi^2 = 2.71$, corresponding to a 95% CL upper bound of $M_H < 196$ GeV at present.

Among the experimental measurements of EWPO which are used in global fits, the W boson mass, M_W, and the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$, have the largest impact on the extracted value of M_H. Although the current relative precision of M_W is better by a factor of 1.8 compared to $\sin^2\theta_{\rm eff}$, the latter is the most relevant parameter for the indirect M_H determination due to its more pronounced dependence on the Higgs mass. For equal relative experimental precisions, it yields a 3.1 times higher sensitivity (for M_H around 115 GeV). Other observables include the leptonic Z boson width, Γ_l; the mass and width of the Z boson, M_Z and Γ_Z; the peak hadronic cross section of the Z boson, $\sigma^0_{\rm had}$; EWPO from deep inelastic neutrino scattering; and others. Furthermore the top quark mass, m_t, enters in the global fit; its value and its error have a strong impact on the extracted M_H value.

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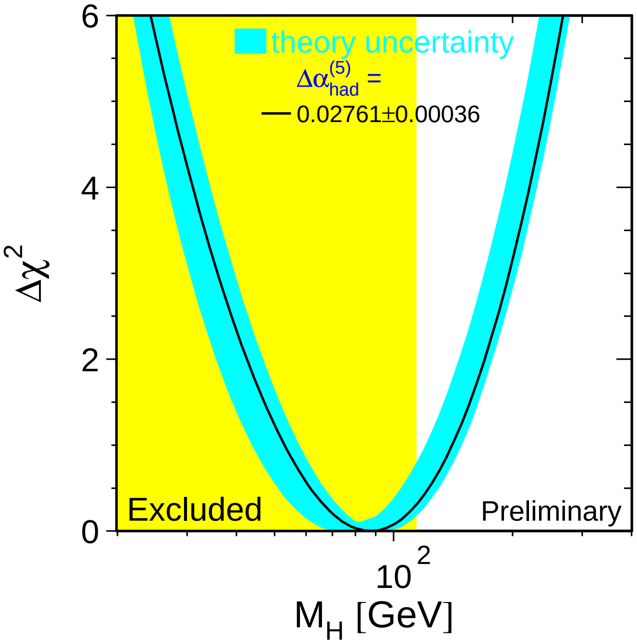

FIG. 1: $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ from a global fit to all available data [1] as a function of the SM Higgs boson mass, M_H. The width of the "Blue Band" indicates the effect of "intrinsic" uncertainties from unknown higher order corrections (see text). The yellow region is excluded by direct Higgs searches at LEP2 [2].

The precision of the fit results depends on the experimental uncertainties of the measured values of the EWPO and the theoretical uncertainties of their predictions. When discussing these uncertainties, one has to take into account that most of the EWPO, for example M_Z, M_W and $\sin^2\theta_{\rm eff}$, are not directly measurable quantities, but are related to measured cross-sections and asymmetries by a deconvolution or unfolding procedure. They are therefore often called "pseudo-observables", in order to distinguish them from directly measured "primordial" observables. The unfolding procedure is in general affected by theoretical uncertainties (and a certain degree of model dependence), which enter the systematic experimental error of the pseudo-observables. We will refer to this kind of theoretical uncertainties as *primordial* theoretical uncertainties in the following. A second kind of theoretical uncertainty arises in the prediction for pseudo-observables, e.g. M_W and $\sin^2\theta_{\rm eff}$, in terms of the chosen input parameters within a certain model, e.g. the Standard Model or the Minimal Supersymmetric Standard Model. We use the phrase *intrinsic* uncertainties for the ones arising from unknown higher-order corrections in the perturbative expansion, as well as for other uncertainties arising from computational limitations. Finally, *parametric* errors originate from the limited experimental precision on the input parameters. The effect of the intrinsic uncertainties is indicated by the width of the "Blue Band" in Fig. 1.

The SM predictions for the EWPO are calculated in terms of a small set of input parameters: $M_Z, G_\mu, \alpha(M_Z), m_\ell, m_q, m_t, M_H$, and $\alpha_s(M_Z)$. The fine structure constant, $\alpha(0)$, the Z boson mass, M_Z, the lepton masses, m_ℓ, and the Fermi constant, G_μ, are currently the most precisely measured input parameters [3], and their errors have negligible effects on the fit results [4–6]. The dominant uncertainties presently arise from the experimental error on the top quark mass, $m_t = 174.3 \pm 5.1$ GeV [3], the hadronic contribution to the fine structure constant, $\Delta\alpha_{\rm had}$ [7, 8] (the value used in Fig. 1 is from Ref. [9]), as well as M_H. $\alpha_s(M_Z)$ is constrained mainly by Γ_Z, R_l, and $\sigma^0_{\rm had}$, with little theoretical uncertainty as long as one ignores the possibility of large new physics effects.

In practice, both EWPO and input parameters are used as constraints in the fits subject to their experimental uncertainties (which, as explained above, contain the primordial theoretical uncertainties related to extraction of the EWPO). The only distinction is that the input parameters are treated as fit parameters, and the EWPO are computed in terms of these. For example, m_t which appears only in loops is chosen as input. Moreover, one usually prefers to compute less precise quantities in terms of more precise ones. The fit results are insensitive to these choices.

Table I summarizes the current status of the experimental uncertainties and the precision one expects to achieve at future colliders for the most relevant EWPO, M_W and $\sin^2\theta_{\rm eff}$, and the top quark mass, together with the expected experimental error on M_H, assuming the SM Higgs boson has been discovered with $M_H \approx 115$ GeV. The entries in the table attempt to represent the combined results of all detectors and channels at a given collider, taking into account correlated systematic uncertainties.

TABLE I: The expected experimental uncertainties (including theory errors for the experimental extraction, i.e. the primordial uncertainties, see text) at various colliders are summarized for $\sin^2\theta_{\rm eff}$, M_W, m_t, and M_H (the latter assuming $M_H = 115$ GeV). Each column represents the combined results of all detectors and channels at a given collider, taking into account correlated systematic uncertainties.
Run IIA refers to an integrated luminosity of 2 fb^{-1} (per detector) collected at the Tevatron with the Main Injector, while Run IIB (IIB*) assumes the accumulation of 15 (30) fb^{-1}. The numbers for $\sin^2\theta_{\rm eff}$ are obtained by scaling (see Ref. [10]) the uncertainties of Run I [11] to the quoted integrated luminosities. A detailed analysis [12] has shown that the uncertainties for $\sin^2\theta_{\rm eff}$ approximately scale with $1/\sqrt{\mathcal{L}}$. Earlier estimates [12] were based on the approximation of a linear relationship between the forward-backward asymmetry, $A_{\rm FB}$, and $\sin^2\theta_{\rm eff}$. The numbers given here have additionally been corrected to reflect the full tree level relation between $A_{\rm FB}$ and $\sin^2\theta_{\rm eff}$. The values for M_W are taken from Ref. [10], while the M_H uncertainty is from Ref. [13].
The upper end of the $\delta\sin^2\theta_{\rm eff}$ range (used for the fits in Table III) at the LHC corresponds to the statistical uncertainty which can be obtained in one year of running at high luminosity (100 fb^{-1}) after combining the e and μ channels and the two experiments [14]. Systematic uncertainties and cross correlations have been ignored in this estimate. However, one can gain considerable leeway by accumulating data over several years. Moreover, one may be able to increase the pseudorapidity range (see text) potentially allowing even greater precision. This is indicated in the range. The uncertainty of 15 MeV quoted for M_W at the LHC is challenging but should be feasible due to the enormous statistics [14]. For the Higgs boson mass uncertainties at the LHC, see Ref. [15].
LC denotes a linear collider operating at $\sqrt{s} = 500$ GeV. The uncertainty quoted for M_W is based on an integrated luminosity of 500 fb^{-1} [16]. (The entry in parentheses assumes a fixed target polarized Møller scattering experiment using the e^- beam [17, 18], thus corresponding to an effective mixing angle at a scale of $\mathcal{O}(0.5$ GeV). It is not used in the fits.)
GigaZ collectively denotes an LC operating at $\sqrt{s} = M_Z$ or $\sqrt{s} \approx 2M_W$ with a luminosity of $\mathcal{L} \approx 5\times10^{33}\,{\rm cm}^{-2}\,{\rm s}^{-1}$. The GigaZ error for M_W combines the 5.2 MeV experimental error [19] (requiring about one year of running) with beam energy and theory uncertainties (see text) which for definiteness we assume close to 3 MeV each (which is challenging). The determination of $\sin^2\theta_{\rm eff}$ with the quoted precision at GigaZ can be performed in 50-100 days of running, see Ref. [20] for details. δM_H at the LC/GigaZ is discussed in detail in Refs. [21–23].
δm_t from the Tevatron [10, 12] and the LHC [24] is the uncertainty in the top pole mass. We included an irreducible uncertainty of order $\Lambda_{\rm QCD} \sim 0.5$ GeV from non-perturbative and renormalon ambiguities. The precision listed for GigaZ and the LC is for the $\overline{\rm MS}$ top mass, see Refs. [25, 26]. The relatively smaller uncertainty at GigaZ compared to the LC is due to the higher precision in α_s (from other GigaZ observables) which affects the extraction of m_t.

	now	Tev. Run IIA	Run IIB	Run IIB*	LHC	LC	GigaZ
$\delta\sin^2\theta_{\rm eff}(\times10^5)$	17	78	29	20	14–20	(6)	1.3
δM_W [MeV]	33	27	16	12	15	10	7
δm_t [GeV]	5.1	2.7	1.4	1.3	1.0	0.2	0.13
δM_H [MeV]	—	—	$\mathcal{O}(2000)$		100	50	50

The current measurement of $\sin^2\theta_{\rm eff}$ is dominated by the left-right asymmetry from SLD and the b-quark forward-backward asymmetry from LEP1. In the future, constraints are expected to come from precise measurements of the forward-backward asymmetry in $p\overline{p}(pp) \to Z + X \to \ell^+\ell^- + X$ at the Tevatron and the LHC, and from a possible linear collider (LC). At the LC, $\sin^2\theta_{\rm eff}$ can be determined from left-right asymmetries when operating at the Z peak (GigaZ). Another effective mixing angle (at a much lower energy scale) can also be measured in fixed-target Møller scattering. We do not consider this measurement, which would provide additional input for precision tests of the SM, in more detail here. In both cases, polarized beams are needed.

The current precision of M_W is dominated by the direct mass reconstruction of W-pair events at LEP2. Transverse-mass fits from Run I at the Tevatron and data from threshold scans at LEP2 also contribute significantly but carry less statistical weight. Future LHC and Tevatron estimates are based on fits to the transverse mass distribution, the lepton transverse momentum distribution, and the W/Z transverse mass ratio [12]. The LC estimate in the continuum is based on the direct mass reconstruction of W-pair events, similar to the LEP2 analysis [16]. The GigaZ projection assumes a dedicated threshold scan, which requires that the knowledge of the absolute beam energy can be controlled better than 2.5 MeV [19].

The determination of the top quark mass at the Tevatron (present and future) and at the LHC is based on kinematic reconstruction, and thus represents a measurement of the pole mass [10, 12, 24]. At the LC,

m_t can be determined from an energy scan near the $t\bar{t}$ production threshold. The extracted value can be chosen to correspond to a suitably defined threshold mass or another short distance mass such as the $\overline{\rm MS}$ mass [25, 26]. The threshold analysis gives correlated measurements of α_s and m_t, and the last entry in Table I represents the combination of the threshold scan with the precise α_s determination from GigaZ [27, 28]. A recent calculation [29] may allow for an even better determination of m_t up to ~ 50 MeV.

In Section II we summarize the current status of the intrinsic and parametric uncertainties in the predictions for the most relevant EWPO and analyze their impact on the current prediction for M_H within the SM. In Section III we discuss necessary improvements of primordial theoretical uncertainties which are required for the extraction of M_W, $\sin^2\theta_{\rm eff}$, and m_t at future colliders with a precision as envisaged in Table I. We also analyze the necessary improvements in the predictions of the EWPO in order to match the experimental precisions. Based on estimates of prospective improvements of the experimental and theoretical uncertainties, we study the accuracy which can be achieved in the indirect determination of M_H at future colliders.

II. CURRENT THEORETICAL UNCERTAINTIES OF EWPO AND THE PREDICTION OF M_H

The dominant parametric uncertainty of the EWPO presently arises from the experimental error of the top quark mass, $\delta m_t = 5.1$ GeV. This error induces a parametric uncertainty of 32 MeV and 16×10^{-5} in the W mass and the leptonic effective mixing angle, respectively. The corresponding errors from the uncertainty in $\Delta\alpha_{\rm had}$, $\delta\Delta\alpha_{\rm had} = 0.0002$, are 3.7 MeV and 7×10^{-5}. Furthermore, the imperfect knowledge of the strong coupling constant, $\delta\alpha_s(M_Z) = 0.0028$, introduces uncertainties of 2 MeV and 3.5×10^{-5} and also an uncertainty in $\Delta\alpha_{\rm had}$ of about $\delta\Delta\alpha_{\rm had} = 0.0001$. While the uncertainty induced by the top quark mass is about as large as the present experimental error of M_W and $\sin^2\theta_{\rm eff}$, the parametric uncertainties induced by the errors of $\Delta\alpha_{\rm had}$ and $\alpha_s(M_Z)$ are already smaller than the prospective experimental errors on M_W and $\sin^2\theta_{\rm eff}$ at the Tevatron and the LHC (see Table I). On the other hand, the accuracies reachable at GigaZ will clearly require a significantly improved experimental precision not only of m_t (see Table I), but also of $\Delta\alpha_{\rm had}$ and $\alpha_s(M_Z)$. An improved determination of $\alpha_s(M_Z)$ with little theoretical uncertainty is, in fact, expected from GigaZ itself [27, 28].

Concerning the intrinsic uncertainties of the EWPO from unknown higher orders, recent progress has been made for the prediction of M_W by the inclusion of the full fermionic two-loop corrections [30], superseding the previous expansions in m_t^2/M_W^2. Since this expansion yielded similar values (with the same sign) for the m_t^4/M_W^4 and the m_t^2/M_W^2 terms (casting some doubt on the convergence), the full fermionic two-loop corrections constitute an important step towards a very precise M_W prediction. The difference between the expansion calculation and the full result can reach up to about 4 MeV, depending on M_H. The only missing two-loop corrections to M_W are the pure bosonic contributions. The M_H dependence of the bosonic two-loop contributions to M_W has recently been evaluated [31], indicating corrections of $\mathcal{O}(1$ MeV). For $\sin^2\theta_{\rm eff}$ the situation is slightly worse, since a result for the full fermionic two-loop corrections is not yet available, and one has to rely on the expansion in powers of m_t^2/M_W^2 [32]. Beyond two-loop order, the results for the pure fermion-loop contributions (incorporating in particular the leading terms in $\Delta\alpha$ and $\Delta\rho$) are known up to the four-loop order [33]. Furthermore, the QCD corrections of $\mathcal{O}(\alpha\alpha_s^2)$ are known [34, 35]. More recently, also the leading three-loop terms of $\mathcal{O}(G_F^3 m_t^6)$ and $\mathcal{O}(G_F^2\alpha_s m_t^4)$, which enter via the quantity $\Delta\rho$, have been calculated in the limit of vanishing Higgs boson mass. The results have been found to be quite small, which is familiar from the $M_H = 0$ limit of the $\mathcal{O}(G_F^2 m_t^4)$ result [36]. In the latter case, the extension to finite values of M_H and the inclusion of subleading terms led to an increase in the numerical result by a factor of up to 20.

In order to quantify the remaining intrinsic uncertainties of the EWPO, one has to perform estimates of the possible size of uncalculated higher-order corrections. The results of calculations based on different renormalization schemes or on different prescriptions for incorporating non-leading contributions in resummed or expanded form differ from each other by higher-order corrections. One way of estimating the size of unknown higher-order corrections is thus to compare the results for the prediction of the EWPO from different codes in which the same corrections are organized in a somewhat different way. A detailed description of different "options" used in this comparison can be found in Ref. [37] and an update in Ref. [38]. This prescription may lead to an underestimate of the theoretical uncertainty if at an uncalculated order a new source of potentially large corrections (e.g. a certain enhancement factor) sets in. In general, it is not easy to quantify how large the variety of different codes and different "options" should be in order to obtain a reasonable estimate of the theoretical uncertainty.

The "Blue Band" in Fig. 1 is obtained according to the prescription described above, using the codes `ZFITTER` [39] and `TOPAZO` [40]. At present, the theoretical uncertainty represented by the width of the "Blue Band" mainly arises from the intrinsic uncertainties in the prediction for $\sin^2\theta_{\rm eff}$, while the intrinsic uncertainty in the prediction for M_W, being significantly smaller than the experimental error, is less important. A shift in the prediction for M_W, on the other hand, induces a shift in $\sin^2\theta_{\rm eff}$ according to

$$\sin^2\theta_{\rm eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right)\kappa(M_W^2), \tag{1}$$

where κ is a calculable function in the SM. While the shift in M_W induced by going from the result of the expansion in powers of m_t^2/M_W^2 to the result of the full fermionic two-loop corrections is known, the corresponding result for $\kappa(M_W^2)$ is still missing. The effect of inserting the new result for M_W in Eq. (1), which amounts to an upward shift of about 8×10^{-5} in $\sin^2\theta_{\rm eff}$ (for $M_H \approx 115$ GeV), has been (conservatively) treated as a theoretical uncertainty in the "Blue Band" of Fig. 1.

Other (related) methods to estimate the size of missing higher order corrections are to vary the renormalization scales and schemes. While these methods usually give an order of magnitude estimate and a lower bound on the uncertainty, they can lead to underestimates whenever there are sizeable but scheme- and scale-invariant contributions. For example, the lowest order flavor singlet contribution to Z decay, a separately gauge invariant and finite set of corrections, cannot be estimated by scale variations of the non-singlet contribution or by using different "options" for resumming non-leading contributions in computer codes.

In the following we use a simple minded, but rather robust and, in the past, quite successful method for estimating the uncertainties from unknown higher orders [41]. The idea is to collect all relevant enhancement and suppression factors and setting the remaining coefficient (from the actual loop integrals) to unity. If, in a given order, terms with different group theory factors contribute, one can often choose the largest one as an estimate for the uncertainty. Our results are summarized in Table II. They are in good agreement with the estimates of the current uncertainties of M_W and $\sin^2\theta_{\rm eff}$ performed in Refs. [31, 42–44].

TABLE II: Theoretical uncertainties from unknown higher-order corrections to $\sin^2\theta_{\rm eff}$ and M_W. $\hat{s}$ denotes the $\overline{\rm MS}$ mixing angle, $N = 12$ is the number of fermion doublets in the SM, $C_F = 4/3$ and $C_A = 3$ are QCD factors, and $N_C = 3$ is the number of colors. The corrections in the upper part of the table are assumed to enter the predictions in the same way as $\Delta\alpha$ (only the leading top quark correction of $\mathcal{O}(\alpha\alpha_s^2)$ enters via $\Delta\rho$), while the ones in the lower part are assumed to enter via $\Delta\rho$. The fermionic contributions of $\mathcal{O}(\alpha^2)$ refer to the non-leading terms beyond the next-to-leading term of the expansion in powers of m_t^2/M_W^2. The uncertainty in $\sin^2\theta_{\rm eff}$ has been estimated from the known correction to M_W using Eq. (1) (see text). The $\mathcal{O}(\alpha\alpha_s^2)$ corrections, which are completely known both for M_W and $\sin^2\theta_{\rm eff}$, are included in the table for completeness. However, the light fermion corrections are not yet included in all codes currently used for performing electroweak fits (and have not been published yet as an independent explicit formula); our error estimate for M_W and $\sin^2\theta_{\rm eff}$ corresponds to ± 1.7 MeV and $\pm 3.3 \times 10^{-5}$, respectively. In order to estimate effects of finite M_H and subleading terms in the lower part of the table, we have taken the average of the individual coefficients of the result in the limit $M_H = 0$ [45] (which in this limit conspire to yield a small answer), resulting in the numerical prefactors there.

order	sector	estimate	size ($\times 10^5$)	M_W[MeV]	$\sin^2\theta_{\rm eff}$ ($\times 10^5$)
α^2	fermionic	$N(\alpha/4\pi\hat{s}^2)^2$	8.7	complete [30]	4.1
α^2	bosonic	$(\alpha/\pi\hat{s}^2)^2$	11.6	2.1	4.1
$\alpha\alpha_s^2$	top-bottom doublet	$N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	4.7	complete [34]	complete [34]
$\alpha\alpha_s^2$	light doublets	$2\ N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	9.4	complete [35]	complete [35]
$\alpha^3 m_t^6$	heavy top	$5.3\ N_C^2(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	7.0	4.1	2.3
$\alpha^3 m_t^6$	heavy top	$3.3\ N_C(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	1.5	0.9	0.5
$\alpha^2\alpha_s m_t^4$	heavy top	$3.9\ N_C C_F \alpha^2\alpha_s m_t^4/16\pi^3\hat{s}^4 M_W^4$	7.8	4.5	2.5
$\alpha\alpha_s^3 m_t^2$	heavy top	$N_C C_F C_A^2 \alpha\alpha_s^3 m_t^2/4\pi^4\hat{s}^2 M_W^2$	2.3	1.3	0.8
	total			7	7

We have performed a global fit to all data in the Standard Model based on the present experimental and parametric uncertainties and using the estimates of Table II for the intrinsic theoretical uncertainties from unknown higher-order corrections. For the theoretical predictions the program `GAPP` [41] has been used. In contrast to the fit in Fig. 1, where the theory uncertainties are represented by the width of the blue band, we have added theoretical and experimental errors in quadrature. As a result we find

$$M_H = 97^{+53}_{-36}\ \text{GeV}, \tag{2}$$

and a 95% CL upper bound of $M_H < 194$ GeV. These numbers are very close to the result of the fit in Fig. 1 [1].

Concerning the interpretation of the fit result, it should be kept in mind that it is based on the assumption that the Standard Model provides the correct description of the experimental measurements. This means, in particular, that the resulting bound on M_H does not reflect the quality of the fit, i.e. it does not contain information about how well the SM actually describes the data.

III. FUTURE INDIRECT DETERMINATIONS OF M_H

For the analysis in this section, we anticipate that in the future the currently missing corrections indicated in the upper part of Table II will become available, and that the uncertainties listed in the lower part will be reduced by a factor of two.

In the following we will discuss the anticipated future experimental precisions of the EWPO reachable at the next generation of colliders as given in Table I in view of necessary improvements of the primordial theoretical uncertainties. In each case we also investigate whether the prospective parametric and intrinsic theoretical uncertainties of the EWPO will be sufficiently under control in order to match the projected experimental precision.

- Tevatron Run IIA (2 fb^{-1}/experiment):
  In order to measure the W mass with the precision anticipated for Run IIA, it is necessary to take into account QCD and electroweak radiative corrections to W production. In particular, the understanding of QED radiative corrections which shift the W mass extracted from data by 50 – 150 MeV [46–49] is crucial for a precision W mass measurement. The determination of the W mass in a hadron collider environment requires a simultaneous precision measurement of the Z boson mass, M_Z, and width, Γ_Z. When compared to the value measured at LEP1, the two quantities help to accurately calibrate detector components. It is therefore also necessary to understand the EW corrections to Z boson production in hadronic collisions. In order to properly calibrate the Z boson mass and width using the available LEP1 data, it is desirable to obtain the predictions for the Z observables in hadronic collisions with an accuracy which is comparable with that of the theoretical input which has been used to extract M_Z and Γ_Z at LEP1. During the last three years, results for the full $\mathcal{O}(\alpha)$ corrections to W [50, 51] and Z boson production [52, 53] became available. The remaining uncertainties from unknown higher order corrections have been estimated to be of $\mathcal{O}(5\text{ MeV})$ [12].

  QCD corrections only indirectly influence the W mass determination via the angular distribution of the decay lepton [12]. However, in order to correctly reconstruct the transverse momentum of the neutrino, it is crucial to accurately predict the W transverse momentum distribution. This is achieved using the observed Z boson p_T distribution together with calculations [54–56] which resum the QCD corrections to the W and Z p_T distributions to all orders, and a parameterization of non-perturbative effects at small p_T [57]. The systematic uncertainties due to the knowledge of the p_T^W distribution are estimated to be $\delta M_W \approx 5$ MeV in Run IIA. Incomplete knowledge of the parton distribution functions (PDFs) will contribute an uncertainty of similar size [12].

  The effective leptonic mixing angle is expected to be measured from the precise determination of the forward-backward asymmetry in $p\bar{p} \rightarrow Z + X \rightarrow l^+l^- + X$ at the Z peak. The main theoretical uncertainty originates from the incomplete knowledge of the PDFs [12].

  In summary, for the precision M_W and $\sin^2\theta_{\text{eff}}$ measurements envisioned at Run IIA, no further improvements in the primordial uncertainties of the theoretical predictions for W and Z observables are needed.

  As in Run I, the top quark mass measurement is mainly based on the direct kinematic reconstruction of the $t\bar{t}$ events in the lepton+jets channel, $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow l^+\nu q\bar{q}'b\bar{b}$. This channel provides a large and clean sample for mass reconstruction, resulting in a measurement of the top pole mass, and thus does not require additional theoretical input. In Run II the uncertainty in the lepton+jets channel will be dominated by systematic effects, which are largely dominated by the uncertainty on the jet energy scale and the modeling of QCD radiation in top events.

  The improvement in the experimental determination of m_t at Run IIA will reduce the parametric theoretical uncertainties of M_W and $\sin^2\theta_{\text{eff}}$ to values below the experimental errors of these observables. Similarly, the present values of the intrinsic theoretical uncertainties of M_W and $\sin^2\theta_{\text{eff}}$ (see Table II) are smaller than the envisaged experimental errors (see Table I). On the other hand, an improvement of the theoretical prediction of $\sin^2\theta_{\text{eff}}$, in particular the inclusion of the missing corrections of $\mathcal{O}(\alpha^2)$, would lead to a significant reduction of the width of the "Blue Band" shown in Fig. 1. It is not obvious, however, that full two-loop results for M_W and $\sin^2\theta_{\rm eff}$ will become available already at the time scale of Run IIA, which is expected to be completed within the next two to three years.

- Tevatron Run IIB (15 fb^{-1}/experiment):
  The main contribution to the shift in M_W induced by the QED corrections originates from final state photon radiation. An explicit calculation of real two photon radiation in W and Z boson production [58] indicates that, in order to measure the W mass with a precision of less than 20 MeV in a hadron collider environment as foreseen in Run IIB and at the LHC, it will be necessary to take into account multi-photon radiation effects. Moreover, an improved understanding of the uncertainty due to PDFs is needed. At the Tevatron the PDFs can be constrained by a measurement of the W charge asymmetry. The estimated uncertainty on the W mass due to PDF uncertainties in Run I was 15 MeV, which is expected to improve in Run II.

  Given the estimated time scale of about 6–8 years until Run IIB will be completed, it seems reasonable to hope for a considerable improvement of the intrinsic theoretical uncertainties of the EWPO, which would arise from full results at the two-loop level and improved predictions for the dominant higher-order corrections. The parametric uncertainty induced by the experimental error of m_t will be further reduced at Run IIB, but will still play an important role in the indirect determination of the Higgs boson mass.

- Tevatron Run IIB* (30 fb^{-1}/experiment):
  If the current Fermilab booster is replaced by a high intensity proton driver, it is conceivable that an integrated luminosity of 30 fb^{-1} can be achieved with the Tevatron by 2008–9 [59]. For integrated luminosities larger than 15 fb^{-1}, the uncertainty on the W mass extracted using the traditional transverse mass method is dominated by systematic uncertainties associated with the production and decay model [10]. This uncertainty can be reduced significantly by using the W/Z transverse mass ratio [12] to measure M_W. Extrapolating from the present uncertainties of $\delta M_W(\text{stat}) = 211$ MeV and $\delta M_W(\text{sys}) = 50$ MeV obtained using the W/Z transverse mass ratio method (see Ref. [12] and references therein), one finds that an overall uncertainty of $\delta M_W = 10 - 15$ MeV might be achievable for an integrated luminosity of 30 fb^{-1}.

- LHC:
  M_W and m_t will be measured using techniques similar to those employed at the Tevatron. In order to improve the experimental uncertainty of $\sin^2\theta_{\rm eff}$ at the LHC, it will be necessary to detect one of the leptons originating from $Z \to l^+l^-$ over the entire pseudorapidity range of $|\eta| < 5$ [14]. This requires an electron jet rejection factor of < 0.01 in the forward region ($2.5 < |\eta| < 5$) of the electromagnetic calorimeter. The relevance of a more precise determination of PDFs in this respect remains to be investigated.

  The improvement in the measurement of M_W at the LHC is due to the large statistics which is expected to result in very small statistical errors and good control of many systematic uncertainties. However, as in Run IIB, theoretical improvements are needed, e.g. for radiative W decays, the modeling of the p_T^W distribution, and for constraining PDFs. In Ref. [14] it has been argued that it should be possible to obtain an uncertainty on the W mass due to PDFs smaller than 10 MeV.

- LC (without GigaZ option):
  As for the M_W measurement at LEP2, the determination of the W mass at the LC at center of mass (CM) energies above the W^+W^- production threshold will be based on direct reconstruction of W-pair events in 4-fermion production processes. The small experimental uncertainty at LEP2 and the LC requires the inclusion of electroweak radiative corrections to the predictions for the underlying production processes, $e^+e^- \to WW \to 4f$. The full treatment of the processes $e^+e^- \to 4f$ at the one-loop level is of enormous complexity. Nevertheless, there is ongoing work in this direction [60, 61]. While the real Bremsstrahlung contribution is known exactly, there are severe theoretical problems with the virtual $O(\alpha)$ corrections. A detailed description of the status of predictions for $e^+e^- \to 4f$ processes can be found in [62]. A suitable approach to include $O(\alpha)$ corrections to gauge-boson pair production is a double-pole approximation (DPA): electroweak $\mathcal{O}(\alpha)$ corrections are only considered for the terms that are enhanced by two resonant gauge bosons. All present calculations of $O(\alpha)$ corrections to $e^+e^- \to WW \to 4f$ rely on a DPA [63–67], and different versions of a DPA have been implemented in the state-of-the-art Monte Carlo (MC) generators `RacoonWW` [64, 68, 69] and `YFSWW3` [65, 66, 70]. The intrinsic DPA error is estimated to be of the order of $\alpha\Gamma_W/(\pi M_W)$, i.e. $\lesssim$ 0.5%, whenever the cross section is dominated by doubly-resonant contributions. This is the case at LEP2 for energies sufficiently above threshold. The DPA is not a valid approximation close to the W-pair production threshold. At higher energies, the contributions of single resonant and non-resonant diagrams become sizeable, and appropriate cuts may need to be imposed to extract the WW signal.

An estimate of the theoretical uncertainty of the M_W measurement at LEP2 due to electroweak corrections when using the state-of-the-art MC programs has been given in [71] by exploiting numerical results obtained at 200 GeV with `KORALW` and `YFSWW3`. Using idealized event selections and a simple fitting procedure, the theoretical uncertainty on M_W is estimated to be about 5 MeV. In view of an envisioned 10 MeV measurement at the LC in the continuum this analysis should be repeated using realistic LEP2 event selection criteria, and for the LC CM energy of 500 GeV.

At the LC, the top quark mass can either be extracted from a $t\bar{t}$ threshold scan that would determine a suitably defined threshold mass [25, 26], or in the continuum by direct kinematical reconstruction of $e^+e^- \to \bar{t}t \to W^+W^-b\bar{b} \to l^+\nu l^-\bar{\nu}b\bar{b}$ events [72] which determines the pole mass. The remaining theoretical uncertainties are sufficiently small to allow a measurement of the threshold mass with a precision of $\mathcal{O}(50\ \text{MeV})$ [25, 26, 29]. The measurement of the pole mass at higher energies with an accuracy of 200 MeV or better may be possible [72], but is limited in precision by QCD renormalon effects which are of $\mathcal{O}(\Lambda_{QCD})$.

The precise measurement of m_t at the LC will eliminate the main source of parametric uncertainties of the EWPO. The uncertainties induced in M_W and $\sin^2\theta_{\text{eff}}$ by the experimental error of m_t will be reduced by the LC measurement to the level of 1 MeV and 0.5×10^{-5}, respectively, i.e. far below the uncertainty corresponding to the present error of $\delta\Delta\alpha_{\text{had}}$.

- GigaZ:
A determination of M_W with the GigaZ option is based on a dedicated threshold scan. Presently, the predictions for the W^+W^- cross section in the threshold region are based on an improved-Born approximation [73, 74] which neglects non-universal electroweak corrections. Thus, the total W-pair cross section in the threshold region is only known with an accuracy of about 1.4% [73]. This translates into a theoretical uncertainty on the W mass of about 24 MeV [12, 75, 76]. Since the extracted value for M_W may be more sensitive to the shape of the cross section than its normalization, it has been suggested that this estimate is too pessimistic, neglecting possible cancellations in cross section differences. However, as discussed in more detail in Ref. [77], it is expected that the non-universal corrections noticeably affect the shape in the threshold region. Thus, in order to achieve the target precision of $\delta M_W = 7$ MeV, a full $\mathcal{O}(\alpha)$ calculation of the process $e^+e^- \to WW \to 4f(+\gamma)$ in the threshold region is needed. This is a very difficult task, in particular since currently no practicable solution of the gauge invariance problem associated with finite W-width effects in loop calculations exists. Aiming at an accuracy of $\delta M_W \approx 7$ MeV will clearly require a considerable effort from the theory side. Besides an estimated future primordial theoretical uncertainty of $\sim 2-3$ MeV, the experimental error for M_W also includes the uncertainty arising from the beam energy. It has to be controlled at the level of ~ 2.5 MeV, which, although it is of higher precision as currently foreseen for TESLA or NLC, might be achievable with some additional effort [19].

At GigaZ one hopes to improve the current precision of $\sin^2\theta_{\text{eff}}$ by more than an order of magnitude. This is envisaged by a precise measurement of A_{LR} [20, 78] using the Blondel scheme [79]. A_{LR} is then given as a function of polarized cross sections, where both beams have different combinations of polarizations. Due to the anticipated drastic improvement in the accuracy, a reanalysis of the effect of primordial uncertainties in the determination of $\sin^2\theta_{\text{eff}}$ might become necessary. This determination of A_{LR} requires that both beams can be polarized independently and that the polarizations of the colliding e^+ and e^- bunches with opposite helicity states are equal (or that their difference is precisely determined; see Ref. [20] for details). A precision of $\delta A_{\text{LR}} \approx 8 \times 10^{-5}$ seems to be feasible [20, 21], resulting in $\delta\sin^2\theta_{\text{eff}} \approx 10^{-5}$.

The $t\bar{t}$ threshold analysis at the LC will result in correlated measurements of α_s and m_t. Since an independent and more precise determination of $\alpha_s(M_Z)$ would be possible with GigaZ (to ± 0.0010, from other GigaZ observables: the Z width with 1 MeV uncertainty, and R_l with 0.05% uncertainty [27, 28]), an improved value for m_t can be expected as well.

In view of the increased precision of $\sin^2\theta_{\text{eff}}$ at GigaZ and the largely reduced error of m_t at the LC, it will be very important to reduce the uncertainty of $\delta\Delta\alpha_{\text{had}}$ at least to the level of $\delta\Delta\alpha(M_Z) = 7 \times 10^{-5}$ [7], corresponding to parametric uncertainties of M_W and $\sin^2\theta_{\text{eff}}$ of 1.5 MeV and 2.5×10^{-5}, respectively. This will require improved measurements of $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ below about $\sqrt{s} \leq 5$ GeV. In case the uncertainty of $\Delta\alpha(M_Z)$ could even be improved by another factor of two (and taking also into account the expected improvement in the $\alpha_s(M_Z)$ determination at GigaZ), the limiting factor in the parametric uncertainty of $\sin^2\theta_{\text{eff}}$ would arise from the experimental error of M_Z ($\delta M_Z = 2.1$ MeV induces an uncertainty of 1.4×10^{-5} in $\sin^2\theta_{\text{eff}}$), which is not expected to improve in the foreseeable future.

With the prospective future improvements of higher order corrections to the EWPO discussed above (i.e. complete electroweak two-loop results and a reduction of the uncertainties in the lower part of Table II by a factor of two), the intrinsic theoretical uncertainties of the EWPO will be comparable to or smaller than the parametric uncertainties and the experimental errors at GigaZ (see also Ref. [80].)

In summary, the projected experimental accuracies at GigaZ require on the theory side a considerable effort to reduce primordial theoretical uncertainties. In addition, improvements of the intrinsic and parametric uncertainties of the EWPO are needed. These tasks appear challenging, but, in view of the time scale of at least a decade, not unrealistic.

Based on the uncertainties expected at the next generation of colliders and our estimates of present and future theoretical uncertainties, we list in Table III the (cumulative) precision of M_H at different colliders which one hopes to achieve from EWPOs. Results are given for $\delta M_H/M_H$ obtained from M_W alone, from $\sin^2\theta_{\rm eff}$ alone, and from all precision data, taking into account the intrinsic and the parametric theoretical uncertainties and their correlated effects.

TABLE III: The expected *cumulative* precision, $\delta M_H/M_H$, from future collider data, given the error projections in Tables I and II. Intrinsic theoretical and parametric uncertainties and their correlated effects on M_W and $\sin^2\theta_{\rm eff}$ are taken into account. In the first row, our estimate for the current intrinsic uncertainties in M_W and $\sin^2\theta_{\rm eff}$ from unknown higher order corrections as given in Table II is used. In the other rows we assume that complete two-loop results for the most relevant EWPO are available, and that the uncertainties in the lower part of Table II have been reduced by a factor of two. This corresponds to future intrinsic theoretical uncertainties in M_W and $\sin^2\theta_{\rm eff}$ of 3 MeV and 1.7×10^{-5}, respectively[a]. As in Eq. (2) we have added the theoretical and experimental errors in quadrature. We also assume $\delta\Delta\alpha(M_Z) = 7\times10^{-5}$ [7]. (Using the very optimistic value of 5×10^{-5} would improve the δM_H uncertainty at GigaZ to 7%.) The last row also assumes a determination of $\alpha_s(M_Z)$ with an uncertainty of ±0.0010 from other GigaZ observables.

$\delta M_H/M_H$ *from:*	M_W	$\sin^2\theta_{\rm eff}$	all
now	106 %	60 %	58 %
Tevatron Run IIA	72 %	39 %	35 %
Tevatron Run IIB	37 %	33 %	25 %
Tevatron Run IIB*	30 %	29 %	23 %
LHC	22 %	25 %	18 %
LC	15 %	24 %	14 %
GigaZ	12 %	8 %	8 %

If the SM is the correct low energy theory, the Higgs boson will be discovered in Run II at the Tevatron or at the LHC. In this case, the indirect determination of M_H from precision electroweak measurements will constitute an important internal consistency check of the SM. Possible new scales beyond the SM could manifest themselves in a disagreement of the directly and indirectly determined M_H value [28, 81].

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