Phenomenology of Generalized Higgs Boson Scenarios

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I outline some of the challenging issues that could arise in attempting to fully delineate various possible Higgs sectors, with focus on the minimal supersymmetric model, a general two-Higgs-doublet model, and extensions thereof.

In a broad sense the three basic topics of this review will be: (i) extended Standard Model Higgs sectors; (ii) perturbations of 'Standard' minimal supersymmetric model (MSSM) Higgs sector phenomenology; and (iii) Higgs phenomenology for SUSY models beyond the MSSM. Also interesting are Higgs-like particles and how their phenomenology would differ from or affect that for Higgs bosons. Such particles include: radions; top-condensates; and the pseudo-Nambu-Goldstone bosons of technicolor. However, I will not have space to discuss these latter objects.

1. Extended SM Higgs Sectors

Even within the SM context, one should consider extended Higgs sector possibilities.

· One can add one or more singlet Higgs fields. This leads to no particular theoretical problems (or benefits) but Higgs discovery can be much more challenging.

· One can consider more than one Higgs doublet field, the simplest case being the general two-Higgs-doublet model (2HDM). A negative point is that, in the general case, the charged Higgs mass-squared is not automatically positive (as required to avoid breaking electromagnetism). A more complete model context might, however, lead to a 2HDM Higgs sector having $m_{H^\pm}^2 > 0$ as part of an effective low-energy theory. A positive point is that CP violation can arise in a Higgs sector with more than one doublet and possibly be responsible for all CP-violating phenomena.

· One can include Higgs triplet fields. If there is a neutral member of the triplet representation and it has a non-zero vev, then $\rho = m_W / (m_Z \cos\theta_W)$ is no longer computable [1] (even if representations and vevs are chosen so that $\rho = 1$ at tree level); ρ becomes another input parameter to the theory. It is not clear how negatively one should regard this loss of predictability. Of course, if the neutral vev = 0, then there is no impact on EWSB and $\rho = 1$ remains natural.

· As regards higher representations, we should not forget that there are special choices of T and Y for the Higgs multiplet, the next simplest after $T = 1/2, |Y| = 1$ being $T = 3, |Y| = 4$, that yield $\rho = 1$ at tree level and finite loop corrections to ρ even if the neutral field has non-zero vev [2].

Coupling constant unification is also an important ingredient in evaluating the attractiveness of an extended SM Higgs sector. Let us denote by $N_{T,Y}$ the number of Higgs representations of weak-isospin T and hypercharge Y. It is easy to show [3] that certain choices of the $N_{T,Y}$'s can yield coupling constant unification for SM matter content (i.e. no SUSY), although not at as high a scale as the standard $M_U \sim 10^{16}$ GeV. For example, $N_{\frac{1}{2},1} = 2, N_{1,0} = 1$ yields $\alpha_s(m_Z) = 0.115$ and $M_U = 1.6 \times 10^{14}$ GeV. In the context of extra dimensions, it may be that unification should occur at scales not so far above a TeV. An example in the SM extension context with low unification scale is $N_{\frac{1}{2},1} = N_{\frac{1}{2},3} = N_{1,2} = N_{1,0} = 4, N_{3,4} = 3$, which leads to $\alpha_s(m_Z) = 0.112, M_U = 1000$ TeV

and $\alpha_U = 0.04$. Even lower values of M_U are possible in the case of MSSM matter content (*i.e.* including superpartners of the SM particles). For example, if $N_{\frac{1}{2},1} = N_{1,2} = N_{1,0} = 4, N_{3,4} = 4$ one obtains $\alpha_s(m_Z) = 0.114$, $M_U = 4$ TeV, and $\alpha_U = 0.07$. So, perhaps one should not discard complicated Higgs sectors out of hand.

Even the simplest extensions of the Higgs sector can lead to dramatic changes in our ability to detect Higgs bosons. Discovery prospects will often depend strongly on the type of collider, although given sufficient $\sqrt{s}$ it is usually the case that an e^+e^- collider is the best option.

Current data provide some important hints and constraints regarding the Higgs sector [4]. As is well known, the simplest interpretation of the precision electroweak data is the existence of a rather light SM-like Higgs boson (the preferred mass being below the LEP experimental lower limit of 114 GeV). The $1-CL$ plots as a function of SM-like Higgs boson mass show that it is also possible to interpret the LEP2 data as being due to a spread-out Higgs signal, *e.g.* several Higgs bosons in the < 114 GeV region, each with an appropriate fraction of the SM ZZ coupling. Such a situation was considered in [5] (see also [6] and references therein). The simplest Higgs sector for which this could occur is one obtained by adding a modest number of singlet Higgs fields to the minimal one-doublet SM Higgs sector. For an appropriate Higgs potential that mixes the many neutral fields, the physical Higgs bosons would be mixed states sharing the WW/ZZ coupling strength squared and having decays to a rather confused set of final states. If these Higgs bosons had masses spread out every $10-20$ GeV (*i.e.* smaller than the detector resolution in a typical decay channel), a broad/diffuse 'continuum' Higgs signal would be the result. Fortunately, the constraints outlined below imply that a future e^+e^- linear collider with $\sqrt{s} \sim 500$ GeV would be guaranteed to detect even such a signal for currently anticipated integrated luminosities. In particular, if we define $\langle M^2 \rangle \equiv \sum_i C_i^2 m_{h_i}^2$, where $C_i g m_W$ is the strength of the $h_i WW$ coupling ($\sum_i C_i^2 = 1$ being required by unitarity), precision electroweak data requires that $\langle M^2 \rangle \lesssim (200 - 250 \text{ GeV})^2$. (In the SUSY context, even allowing for the most general Higgs representations, RGE evolution starting with perturbative couplings at M_U implies this same result.) This sum rule implies [5] that enough of the resulting diffuse excess in the recoil M_X mass distribution for $e^+e^- \rightarrow ZX$ ($Z \rightarrow e^+e^-, \mu^+\mu^-$) would be confined to the $M_X \in [100, 200 \text{ GeV}]$ mass region that it could be detected with $L \gtrsim 200$ fb^{-1} accumulated luminosity at $\sqrt{s} = 500$ GeV, despite the significant background [7]. It is far from clear that such a no-lose theorem can be established for this scenario at the LHC. In particular, the $\gamma\gamma$ decay width is reduced (due to less W loop contribution) for each of the overlapping Higgs states. Meanwhile, the Wh and Zh production processes would be weak for each of the individual Higgs bosons, h_i, and these h_i signals would be spread out and overlapping in mass. The $t\bar{t}h_i$ signals might retain a roughly SM-like rate for each h_i, but again the signals would be spread out by experimental effects and overlapping, so that search techniques using bump hunting could not be employed.

A more popular SM Higgs sector extension is the general two-Higgs-doublet model (2HDM) with its five physical Higgs bosons ($h_{1,2,3}^0$, $H^\pm$, or in the CP-conserving (CPC) case $h^0, H^0, A^0, H^\pm$). An interesting question is whether or not we are guaranteed to find at least one of the Higgs bosons of a general 2HDM given current precision electroweak constraints. The answer is yes, but direct Higgs detection might only be possible at the LHC if the LC has $\sqrt{s} \lesssim 1$ TeV. One case in which this situation arises is [8] if the only light Higgs boson has no WW/ZZ couplings and all the other Higgs bosons have mass $\gtrsim 1$ TeV. As we describe shortly, these heavy Higgs bosons can be chosen to have mass splittings such that the S, T parameters fall within the current 90% precision electroweak ellipse.

At the LC, the relevant discovery processes for a h with no WW, ZZ couplings are: $e^+e^- \rightarrow t\bar{t}h$ and $e^+e^- \rightarrow b\bar{b}h$ [9]; $e^+e^- \rightarrow Z^* \rightarrow Zhh$ [10, 11]; $e^+e^- \rightarrow e^+e^- W^*W^* \rightarrow e^+e^- hh$ [11, 12]; $\gamma\gamma \rightarrow h$ [13]. That these processes might have reasonable rates follows from the couplings involved. It can be shown [9, 14] that for any h with no (or very small) WW/ZZ coupling (both $h = A^0$ and $h = h^0$ are possibilities in the CPC case) at least one of the couplings $t\bar{t}h$ or $b\bar{b}h$ must be substantial: $(\hat{S}_h^t)^2 + (\hat{P}_h^t)^2 = \cot^2\beta$, $(\hat{S}_h^b)^2 + (\hat{P}_h^b)^2 = \tan^2\beta$, where $\hat{S}$ and $\hat{P}$ are the 1 (*e.g.* $h = h^0$) and γ_5 (*e.g.* $h = A^0$) couplings defined relative to usual SM-type weight. The quartic couplings, $ZZhh$ and W^+W^-hh, arise from the gauge covariant structure $(D_\mu \Phi)^\dagger (D^\mu \Phi)$ and are of guaranteed magnitude. The $\gamma\gamma \rightarrow h$ coupling derives from fermion loops, and above we saw that not both the $b\bar{b}h$ and $t\bar{t}h$ coupling can be suppressed. Unfortunately, despite these guarantees for non-suppressed couplings, there is a wedge-like region of parameter space where discovery of a light decoupled h will be quite difficult.

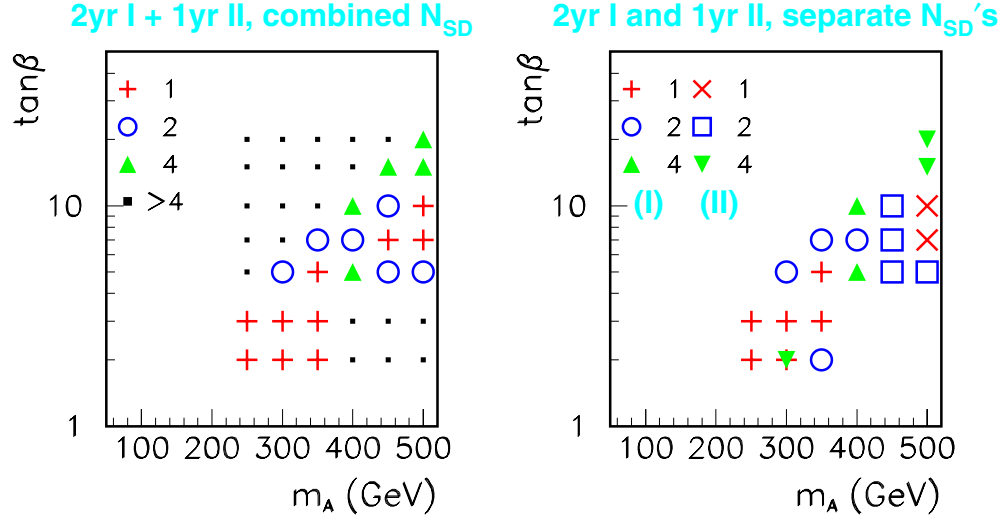## Luminosity Factor Required for 4σ Discovery of 2HDM A

Figure 1: Assuming a machine energy of $\sqrt{s} = 630$ GeV, we show the $[m_{A^0}, \tan\beta]$ points for which two 10^7 sec years of $\gamma\gamma$ collision operation using the type-I (broad) $E_{\gamma\gamma}$ spectrum and one 10^7 sec year using the type-II peaked spectrum configuration will yield $S/\sqrt{B} \geq 4$ for the A^0 of a general 2HDM, assuming all other 2HDM Higgs bosons have mass of 1 TeV. In the left-hand window we have combined results from the type-I and type-II running using $S/\sqrt{B} = \sqrt{S_I^2/B_I + S_{II}^2/B_{II}}$. In the right-hand window we show the separate results for $S_I/\sqrt{B_I}$ and $S_{II}/\sqrt{B_{II}}$. Also shown are the additional points for which a 4σ signal level is achieved if the total luminosity is doubled or quadrupled (the '2' and '4' symbol cases) relative to the 2+1-year luminosities we are employing. (In the LH window, the small black squares indicate the additional points sampled for which even a luminosity increase of a factor of 4 does not yield a 4σ signal.) Such luminosity increases could be achieved for some combination of longer running time and/or improved technical designs. For example, the factor of '2' results probably roughly apply to TESLA. Cuts and procedures are as described in [13].

Turning first to $t\bar{t}h$ and $b\bar{b}h$ production, the former (latter) always yields significant rates if $\tan\beta$ is small (large) enough (and the process is kinematically allowed), while $e^+e^- \rightarrow b\bar{b}h$ always works if $\tan\beta$ is large enough. But, even for high $\sqrt{s}$ and $L = 1000$ fb^{-1}, there remains a wedge of moderate $\tan\beta$ for which neither process provides adequate event rate. The wedge expands to lower and higher $\tan\beta$ values as m_h increases. The corresponding wedge at the LHC is even larger. For the lower values of m_h, double Higgs production via the quartic couplings will allow discovery at the LC even in the wedge region. For instance, the process $e^+e^- \rightarrow Z^* \rightarrow Zhh$ gives a decent event rate for $m_h \lesssim 150$ GeV ($m_h \lesssim 250$ GeV) for $\sqrt{s} = 500$ GeV ($\sqrt{s} = 800$ GeV), while $WW \rightarrow hh$ fusion production probes $m_h \lesssim 200$ ($m_h \lesssim 300$). A careful assessment of backgrounds is required to ascertain just what the mass reach of these processes actually is.

If the $\gamma\gamma$ collider option is implemented at the LC, $\gamma\gamma \rightarrow h$ will provide a signal for a decoupled h over a significant portion of the wedge region. The results from the quite realistic study of [13] are illustrated in Fig. 1, which focuses on the case of $h = A^0$ and $m_{A^0} \geq 250$ GeV. The crosses and pluses indicate 4σ discovery points after 3 years of appropriate running at the NLC. The higher TESLA luminosity for $\gamma\gamma$ collisions would allow 4σ discovery for the additional points indicated by the circles and squares.

Finally, although we don't present details here, a muon collider would probably be able to provide 4σ signals for any h in the $m_h < 500$ GeV wedge region after about 3 years of appropriately configured operation, assuming the nominal Higgs factory luminosities discussed during this workshop. For more details, see [15].

Is the type of scenario being considered (a light decoupled h and all other Higgs bosons heavy) consistent with precision electroweak constraints? In fact, it can be arranged [8]. For example, consider the case of $h = A^0$ and a SM-like h^0 with mass ~ 1 TeV. The heavy h^0 leads to large $\Delta S > 0$ and large $\Delta T < 0$ contributions, which on its own would place the S, T prediction of the
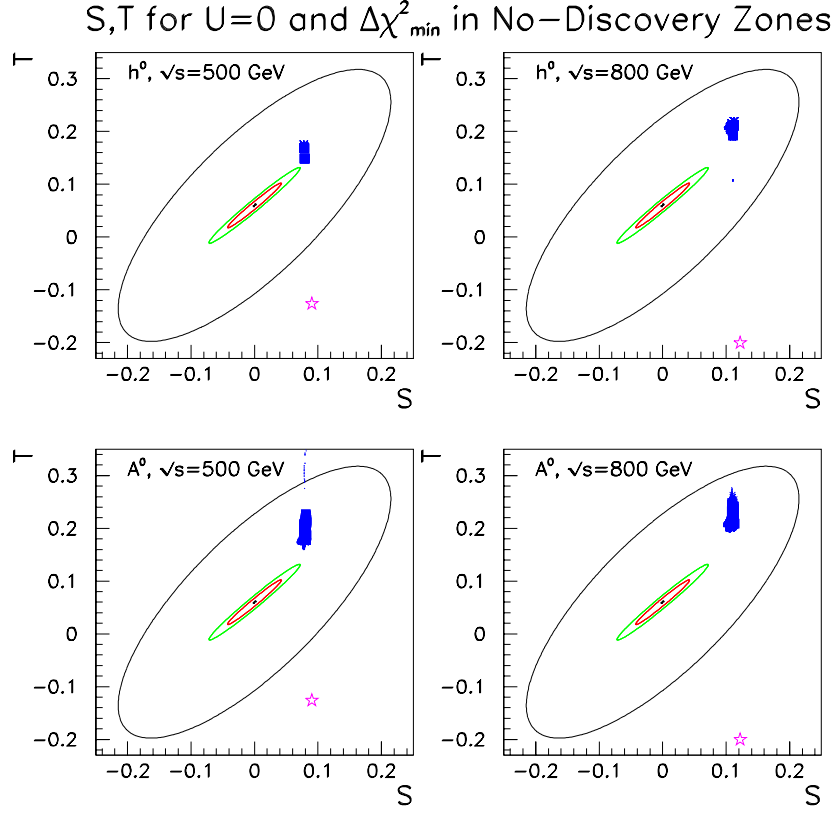
S,T for U=0 and $\Delta\chi^2_{min}$ in No-Discovery Zones

Figure 2: The outer ellipses show the 90% CL region from current precision electroweak data in the S,T plane for $U = 0$ relative to a central point defined by the SM prediction with $m_{h_{SM}} = 115$ GeV. The blobs of points show the S,T predictions for 2HDM models with a light h^0 or A^0 with no WW/ZZ coupling and with $\tan\beta$ such that the light h^0 or A^0 cannot be detected in $b\bar{b}$+Higgs or $t\bar{t}$+Higgs production at either the LC or the LHC; the mass of the SM-like Higgs boson of the model is set equal to $\sqrt{s} = 500$ GeV (left) or 800 GeV (right) and the heavier Higgs masses have been chosen to minimize the χ^2 of the full precision electroweak fit. The innermost (middle) ellipse shows the 90% (99.9%) CL region for $m_{h_{SM}} = 115$ GeV after Giga-Z LC operation *and* a $\Delta m_W \lesssim 6$ MeV threshold scan measurement. The stars to the bottom right show the S,T predictions in the case of the SM with $m_{h_{SM}} = 500$ GeV (left) or 800 GeV (right).

2HDM model well outside the current 90% CL ellipse — see the stars in Fig. 2. However, large $\Delta T < 0$ contribution from the SM-like h^0 can be compensated by a large $\Delta T > 0$ from a small mass non-degeneracy (weak isospin breaking) of the still heavier H^0 and $H^\pm$ Higgs bosons. In detail, for a light A^0 one finds

$$\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2} \left\{ \frac{c_W^2}{s_W^2} \frac{m_{H^\pm}^2 - m_{H^0}^2}{2} - 3m_W^2 \left[\log\frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2}\log\frac{m_W^2}{m_Z^2} \right] \right\} \tag{1}$$

from which we see that the first term can easily compensate the large negative contribution to $\Delta\rho$ from the $\log(m_{h^0}^2/m_W^2)$ term. The overall arrangement of this light A^0 case is further illustrated in the lower windows of Fig. 2. The blobs correspond to 2HDM parameter choices for which $m_{h^0} = \sqrt{s}$ (so that it cannot be observed at the LC of this $\sqrt{s}$) and $m_{H^\pm} - m_{H^0} \sim$ few GeV has been chosen (with both $m_{H^\pm}, m_{H^0} \sim 1$ TeV) so that the S,T prediction is well within the 90% CL ellipse, while at the same time m_{A^0} and $\tan\beta$ are precisely in the wedge of parameter space for which the LHC and e^+e^- LC operation would not allow discovery of the A^0. However, this scenario can only be pushed so far. In order to maintain perturbativity for all the Higgs self couplings, it is necessary that $m_{h^0} \lesssim 1$ TeV, implying that it would be detected at the LHC. Giga-Z operation and

a $\Delta m_W = 6$ MeV WW threshold scan at the LC (with the resulting ellipse sizes illustrated in Fig. 2) would be very important to confirm that the S, T values were indeed those corresponding to a S, T location like that of the blobs of Fig. 2. If no other new physics was detected at the LC or LHC that could cause the extra $\Delta T > 0$, searching for a possibly light decoupled A^0 would become a high priority.
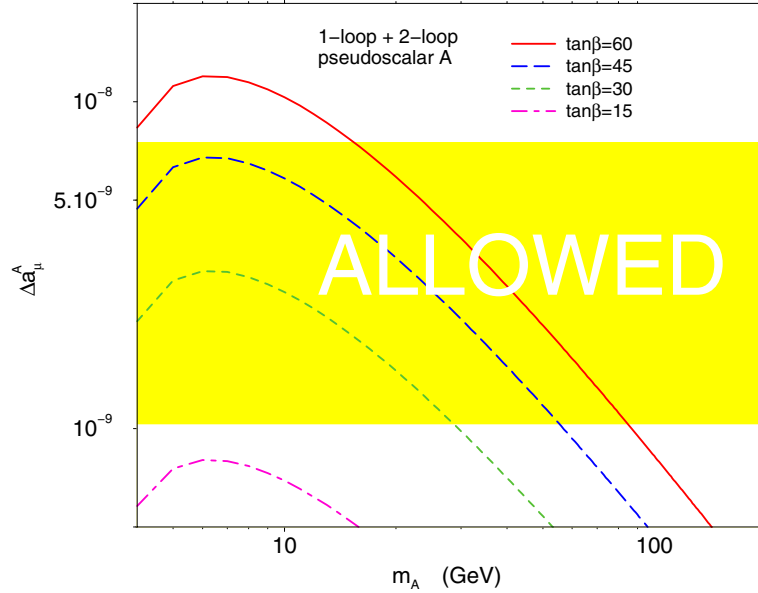

Figure 3: Explanation of new BNL a_μ value via a light 2HDM A^0, from [17].

Interestingly, the recent observation [16] of a discrepancy ($10.3 \times 10^{-10} < \Delta a_\mu < 74.9 \times 10^{-10}$ at 95% C.L. ($\pm 1.96\sigma$) for 'standard' $\sigma(e^+e^- \to \mathrm{hadrons})$ at low $\sqrt{s}$) with SM predictions for a_μ can be explained by the existence of a light A^0 [17]. A light A^0 (h^0) gives a positive (negative) contribution to a_μ, dominated by the two-loop Bar-Zee graph. As shown in Fig. 3, rather small values of m_{A^0} and large values of $\tan\beta$ are needed to explain the entire Δa_μ. In the indicated range of $\tan\beta > 17$, the A^0 will be found at the LC for sure and possibly also at the LHC. However, it seems possible that the Δa_μ discrepancy will turn out to be not quite so large as currently stated, either as statistics improve or because the forthcoming low-E $\sigma(e^+e^- \to \mathrm{hadrons})$ data alters the SM prediction. Smaller values for Δa_μ would be best explained by smaller $\tan\beta$ and higher m_{A^0} values that could lie inside the LC/LHC no-discovery wedge region.

Extra dimensions and related ideas can have a tremendous impact on Higgs phenomenology. There is only space for the most cursory of reviews. In the simplest model, SM particles live on a 'brane' (3+1 dimensions), and gravity resides in the bulk [18, 19]. The new physics scale, Λ, typically identified with the string scale, M_S, is possibly as small as 1 TeV. Since the quadratic divergence at 1-loop for $m_{h_{\mathrm{SM}}}^2$ is cutoff by the string physics at M_S, a light Higgs boson would be natural in the SM. Small fermionic couplings could arise if the brane is 'fat' and the fermion fields (other than the top) are localized within the brane so as to have little overlap with the Higgs field(s) [20]. Some important results of these ideas are the following.

- Extra contributions to precision electroweak parameters from effective operators proportional to $1/M_S$ can be substantial, and in a fashion somewhat analogous to the 2HDM discussion yield an extra positive ΔT contribution that would allow for the SM Higgs boson to be heavy [21, 22]. As in the 2HDM case, $m_{h_{\mathrm{SM}}} \lesssim 1$ TeV is required and other signals of the extra dimensional physics would emerge at energy scales near a TeV.

- The KK graviscalar excitations could provide the mechanism for electroweak symmetry breaking [23]. In the simple case studied, all SM particles live on the brane. One must minimize an effective potential consisting of $V(\phi) - \mathcal{L}_{\mathrm{mass}}(\phi_{KK}^{\vec{n}}) - \mathcal{L}_{\mathrm{mix}}(\phi_{KK}^{\vec{n}}, \phi)$, where ϕ is the usual Higgs field, $\mathcal{L}_{\mathrm{mass}}$ contains the quadratic mass terms for the KK graviscalar

fields $\phi_{KK}^{\vec{n}}$, and $\mathcal{L}_{\text{mix}} \propto \kappa \sum_{\vec{n}} \phi_{KK}^{\vec{n}} T_\mu^{\mu,\,\text{Higgs}} \propto \kappa \sum_{\vec{n}} \phi_{KK}^{\vec{n}} V(\phi)$ arises because gravity sees the energy-momentum tensor. Although $\kappa \propto 1/M_P$ is small, there are many KK modes. After integrating out KK modes, one finds $\overline{V}_{\text{tot}} = V(\phi) - \overline{D} V^2(\phi)$, where $\overline{D} \equiv \kappa^2 \frac{\delta-2}{\delta+2} \sum_{\text{all }\vec{n}} \frac{1}{m_{\vec{n}}^2}$ (δ =number of extra dimensions). For $\delta > 2$, the sum is divergent – after regulation by the string, $\overline{D} \sim M_S^{-4}$ with a coefficient whose sign depends upon the string regulation. It is possible that $\overline{D} < 0$. Note that even if $V(\phi) = \frac{1}{2}m^2\phi^2 + \Xi$ (*i.e.* no quartic self interactions), $\mathcal{L}_{\text{mix}}$ generates ϕ^4 interactions (of correct sign if $\overline{D} < 0$). If $\overline{D} < 0$, then $\overline{V}_{\text{tot}}$ has a minimum at $V(\phi) = \frac{1}{2\overline{D}}$, which determines values for ϕ and the ϕ_{KK} fields at the minimum. Expanding about the vev's, rescaling $\phi \to \hat{\phi}$ for canonical normalization, and diagonalizing the mass matrix, one finds: a Higgs boson s_{phys} with $m_{s_{\text{phys}}}^2 > 0$; standard WW/ZZ couplings for s_{phys} (with tiny corrections); no fermionic couplings of s_{phys} at tree level; and large decays of s_{phys} to states containing two graviscalar KK excited states (which are invisible decays).

· There is a possibility (for a normal EWSB minimum) of large mixing between graviscalar-KK excitations and the SM Higgs that could lead to an effectively invisible Higgs boson [24]. For this, one must introduce a $-\frac{\zeta}{2}R(g)\phi\phi^\dagger$ interaction, where R is the usual Ricci scalar. This interaction leads to an addition to T_μ^μ for the ϕ: in unitary gauge $\Delta T_\mu^\mu = -6\zeta v\, m_h^2 h$ (where h is the usual physical Higgs boson eigenstate in the absence of mixing) and the graviscalar KK modes $\phi_{KK}^{\vec{n}}$ couple to this: $\mathcal{L} \ni \frac{f(\delta)}{M_P} \sum_{\vec{n}} \phi_{KK}^{\vec{n}} T_\mu^\mu$. The resulting h–$\phi_{KK}^{\vec{n}}$ mixing must be removed by rediagonalization, and the physical Higgs ends up having some (invisible) KK-graviscalar excitation components and KK pair decay modes.

In fact, there are many models in which the SM Higgs decays invisibly. (Aside from extra dimension models discussed above, there are models with invisible Majoron decays and the like [25].) Thus, it is important to assess discovery prospects for an invisibly decaying Higgs boson. This has been studied for various colliders by many people. I give a very brief summary. At LEP2 or the LC, one simply looks for $e^+e^- \to ZX$. For any Higgs with ZZ coupling, the recoil M_X distribution will show a peak. The LEP2 limit on a single Higgs with SM-like coupling to ZZ derived by looking for excess $e^+e^- \to ZX$ events is $m_h \geq 114$ GeV [26], *i.e.* essentially at the kinematic limit, even after allowing for the most general mixture between normal and invisible decay modes. The LC discovery potential for an invisibly decaying h with SM-like ZZ coupling would presumably also approach the ZX kinematic threshold. What is possible for a Higgs with only fermionic couplings that decays invisibly has not been studied in the LC specific context. Presumably, $e^+e^- \to Zhh$. $t\bar{t}h$ and $b\bar{b}h$ (all of which provide an event trigger of visible plus missing energy) would all be useful. Discovery of a h with SM-like WW/ZZ couplings that decays invisibly is more difficult at hadron colliders than at an LC. One would employ Wh, Zh production [27, 28] or $WW \to h$ fusion (with jet tagging) [29]. At the Tevatron [30], it will take $L > 5$ fb^{-1} of integrated luminosity just to surpass the LEP2 limit. At the LHC with $L = 100$ fb^{-1}, Wh, Zh production will probe up to $m_h \sim 200$ GeV; in WW fusion, the estimated reach is $300 - 500$ GeV. For any h with SM-like $t\bar{t}$ coupling, $t\bar{t}h$ production will provide a good signal at the LHC for $m_h \lesssim 250 - 300$ GeV assuming $L = 100$ fb^{-1} [31]. Of course, this latter mode, which relies on the $t\bar{t}h$ coupling, is complementary to the Wh and Zh modes that rely on the VVh coupling. Further work on both is desirable.

There is no space to more than briefly mention Higgs triplet models. Higgs triplet representations with $|Y| = 2$ are an integral part of any left-right symmetric model (LRM) in which neutrino masses arise via the see-saw mechanism. Basic collider phenomenology for such models is studied in [32, 33, 34, 35]. The 2×2 notation for the $|Y| = 2$ Higgs triplet fields is $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$. The most important new aspect of a Higgs triplet model (HTM) is the lepton-number-violating coupling: $\mathcal{L}_Y = ih_{ij}\psi_i^T C\tau_2 \Delta \psi_j + \text{h.c.}, \quad i, j = e, \mu, \tau$, which, among other things, leads to $e^-e^- \to \Delta^{--}$ and $\mu^-\mu^- \to \Delta^{--}$ couplings. Limits on the h_{ij} by virtue of the $\Delta^{--} \to \ell^-\ell^-$ couplings are best expressed by writing $|h_{\ell\ell}^{\Delta^{--}}|^2 \equiv c_{\ell\ell} m_{\Delta^{--}}^2$ (GeV) . A pre-1999 summary of these limits can be found in [35, 36]. The strongest of these limits are (there are no limits on $c_{\tau\tau}$): $c_{ee} < 10^{-5}$ (Bhabbha); $c_{\mu\mu} < 5 \times 10^{-7}$ (($g - 2)_\mu$ – I have updated this limit to reflect the BNL a_μ data — the predicted contribution has the wrong sign); and $\sqrt{c_{ee}c_{\mu\mu}} < 10^{-7}$ (muonium-antimuonium). The most likely case (advocated in [36]) is that $\langle \Delta^0 \rangle = 0$, in which case $\rho = 1$ remains natural [1]. (In the LRM, the Δ^0 with zero vev would be the neutral member of the

'left' triplet. It would be the members of this 'left' triplet to which the ensuing discussion applies. The 'right' triplet has very different phenomenology.) For $\langle \Delta^0 \rangle = 0$, the total width $\Gamma^T_{\Delta^{--}}$ would be small and large s-channel e^-e^- and $\mu^-\mu^-$ production rates are predicted. The strategy would be as follows. One would first discover the Δ^{--} in $p\overline{p} \to \Delta^{--}\Delta^{++}$ with $\Delta^{--} \to \ell^-\ell^-, \Delta^{++} \to \ell^+\ell^+$ ($\ell = e, \mu, \tau$) at the upgraded Tevatron or at the LHC [37] if $m_{\Delta^{--}} \lesssim 1$ TeV. According to how it decays – $\Delta^{--} \to e^-e^-$, $\mu^-\mu^-$, or $\tau^-\tau^-$ – we would know which lepton violating couplings are most substantial. Then, if $m_{\Delta^{--}}$ is in a mass range accessible to the LC or a muon collider, it would be of paramount importance to build the relevant e^-e^- and/or $\mu^-\mu^-$ colliders and study s-channel production of the Δ^{--} in order to determine the actual size of these couplings. For $c_{\ell\ell}$ near current upper limits, event rates would be enormous [3, 36]; equivalently one can probe very small $c_{\ell\ell}$ — at least a factor of $10^8 - 10^9$ improvement over current limits would be possible. Most importantly, if the magnitudes of c_{ee} and $c_{\mu\mu}$ are such as to be relevant to neutrino mass generation, observation of Δ^{--} in s-channel e^-e^- and $\mu^-\mu^-$ production would be possible and would allow an actual measurement of these very fundamental couplings.

2. Higgs Bosons in Supersymmetry

Supersymmetry remains the most attractive solution to the naturalness and hierarchy problems. Further, the MSSM implies coupling constant unification at $M_U \sim$ few $\times\, 10^{16}$ GeV and generates EWSB automatically via RGE evolution from M_U beginning with universal soft-supersymmetry-breaking masses. These very attractive features argue strongly for the MSSM model or the simplest generalizations thereof that maintain its attractive features. Overall, it is clearly important to consider the discovery and study of Higgs bosons in the SUSY context [2].

The MSSM contains exactly two doublets ($Y = +1$ and $Y = -1$), as required to give masses to both up and down quarks. Two doublets are also required in order that the anomalies generated by the higgsino partners of the Higgs bosons cancel. Two doublets (and any number of singlets) yield perfect coupling constant unification if the SUSY scale is $m_{\rm SUSY} \sim 1$ TeV. (Actually, significant MSSM matter superpartner content at 10 TeV is advantageous for obtaining $\alpha_s(m_Z) < 0.12$.) More doublets, triplets, *etc.* would imply a need for intermediate-scale matter between the TeV and M_U scales in order to achieve coupling constant unification. But, if there are extra dimensions, unification at M_U may be irrelevant! As is well known, there are strong theoretical bounds on m_{h^0} deriving from the structure of the MSSM. (In discussing these bounds, we will take $m_{\tilde{t}} \leq 1$ TeV, but one should keep in mind the earlier remark regarding some motivation for sparticle masses that are much higher.)

In the two-doublet MSSM, $m_{h^0} \lesssim 130 - 135$ GeV is predicted, although extra dimension effects might allow additional flexibility. Adding singlets, as in the NMSSM [38] (where one complex Higgs singlet field is added), relaxes this upper bound on m_{h^0} to roughly 140 GeV [39], *assuming perturbativity for the new coupling(s) up to M_U*. Adding more doublets lowers the upper bound. Adding the most general structure ($Y = 2$ triplets being the 'worst' for moving up the mass bound), and allowing the most general mixings *etc.*, one finds (assuming perturbativity up to M_U) an upper bound of ~ 200 GeV [40].

Experimental limits from LEP2 on MSSM Higgs bosons are significant. For maximal mixing ($X_t \equiv A_t - \mu \cot\beta = \sqrt{6}m_{\tilde{t}}$), one finds that $m_{h^0}, m_{A^0} \lesssim 91$ GeV [26], implying that $\tan\beta \lesssim 2.5$ is excluded. But, this analysis assumes $m_{\tilde{t}} < 1$ TeV, a CPC Higgs sector, and the absence of invisible decays. For higher $m_{\tilde{t}}$, the value of m_{h^0} predicted for a given $[m_{A^0}, \tan\beta]$ choice increases and less of the parameter space is excluded. Allowing for CP-violation weakens the lower bounds on the MSSM Higgs boson mixed states and the lower bound on $\tan\beta$ [41, 42]. Allowing for the h^0 and A^0 to have substantial invisible decays might substantially weaken the constraints on the h^0A^0 cross section. The $e^+e^- \to ZX$ channel would have to be relied upon much more heavily. A LEP2 study of this scenario would be worthwhile.

Prospects for discovery of at least one MSSM Higgs boson at future colliders are excellent. Our discussion will focus on the 'decoupling' limit of $m_{A^0} > 2m_Z$ in which the h^0 is quite SM-like with full VV coupling strength. At the Tevatron, 5σ discovery of the h^0 will be possible in $q'\bar{q} \to Vh^0$ ($V = W, Z$, $h^0 \to b\overline{b}$) with $L > 20 - 35$ fb^{-1} of accumulated luminosity, the larger L's being required for higher m_{A^0}, depending upon the SUSY-breaking scenario. For $\tan\beta = 20 - 50$ and $m_{A^0} = 100 - 200$ GeV (the higher $\tan\beta$ values being required for the higher m_{A^0} values), the H^0, A^0 can be discovered at the Tevatron in $gg, q\bar{q} \to b\bar{b}H^0, b\bar{b}A^0 \to b\bar{b}b\bar{b}$ [43, 44]. (The $H^0, A^0 \to \tau^+\tau^-$
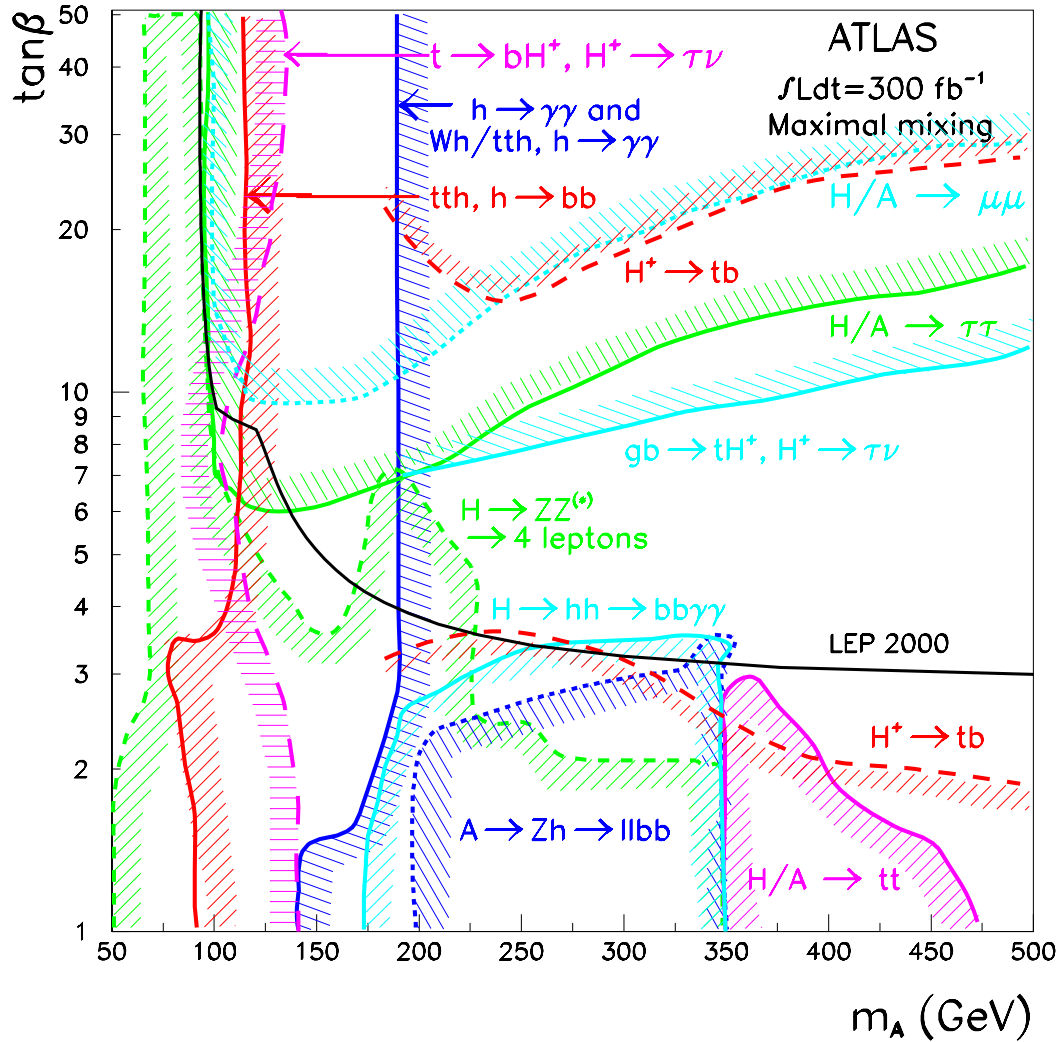
Figure 4: 5σ discovery contours for MSSM Higgs boson detection at the LHC in various channels are shown in the $[m_{A^0}, \tan\beta]$ parameter plane, assuming maximal mixing and an integrated luminosity of $L = 300$ fb^{-1} for the ATLAS detector. This figure is preliminary.

decay modes have not been studied yet.) Given that $m_{A^0} > 2m_Z$ is a very common implication of RGE induced EWSB scenarios, it is most likely that only the h^0 will be seen at the Tevatron. The LHC has somewhat greater sensitivity to the full complement of MSSM Higgs bosons. The best signals for the h^0 are expected in the $gg \to h^0 \to \gamma\gamma$, $t\bar{t}h^0 \to t\bar{t}b\bar{b}$ and $WW \to h^0 \to \tau^+\tau^-$ channels. (Large rates for h^0 production in the decay chains of heavy SUSY particles, such as the gluinos and squarks, are also possible in some scenarios and would produce dramatic signals.) At high enough $\tan\beta$ values, the $gg, q\bar{q} \to b\bar{b}H^0, b\bar{b}A^0$, with $H^0, A^0 \to \tau^+\tau^-$ or $\mu^+\mu^-$ and $gb \to H^\pm t$ with $H^\pm \to \tau^\pm\nu$ will provide good signals for the heavier Higgs bosons. These signals have been studied by both the ATLAS and CMS collaborations. The exact reach of these primary channels is illustrated by the ATLAS results [45] of Fig. 4. Note that for maximal mixing, LEP2 limits exclude $\tan\beta \lesssim 2.5$, the region where other MSSM Higgs discovery modes could be important at the LHC. For $\tan\beta \gtrsim 3$ but below 8 to 15 (for $m_{A^0} = 250$ GeV to 500 GeV, respectively), there is a wedge of parameter space in which only the h^0 will be detected. An important question then becomes whether a high energy linear collider or a possible future muon collider could detect the

H^0, A^0 in this wedge region of parameter space, or at least give some indication of the value of $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ if their masses are large.

Discovery of the h^0 will be straightforward at a LC, using the same production/decay modes as for a light h_{SM}. The high rates imply that precision measurements of the couplings of the h^0 will be possible, possibly allowing the detection of deviations from expectations for the h_{SM} even when m_{A^0} is fairly large [46, 47]. In the simpler SUSY-breaking scenarios (*e.g.* maximal mixing or minimal mixing), detection of such deviations will be possible for $m_{A^0} \lesssim 500 - 600$ GeV for $L = 1$ ab^{-1} of integrated luminosity at $\sqrt{s} \sim 500$ GeV, and would provide a crucial indication of where in mass to search for the H^0, A^0 and $H^\pm$. This will be particularly important if $m_{A^0} + m_{H^0}, 2m_{H^\pm} > 2\sqrt{s}$ (so that $H^0 A^0$ and $H^+ H^-$ pair production is impossible at the LC) and if, in addition, $[m_{A^0}, \tan\beta]$ lie in the LHC no-discovery wedge. Analogous to our discussion for a decoupled h of a general 2HDM, very high $\tan\beta$ is required for an observable signal in the $e^+ e^- \to b\bar{b}A^0, b\bar{b}H^0, btH^\pm$ channels. If SUSY particles and a light SM-like h^0 are detected, even if the $H^0, A^0, H^\pm$ are not detected we will be quite certain that a set of heavier Higgs bosons must exist. The challenge is to zero-in on colliders/techniques for discovering them.

In this regard, production of these heavy Higgs bosons in the s-channel at both $\gamma\gamma$ [48] and $\mu^+ \mu^-$ [49] colliders could provide detectable signals. If we have some indication of the value of m_{A^0} (*e.g.* from detection of h^0 vs. h_{SM} deviations), then we will know exactly what energy to employ. The expectations for a $\gamma\gamma$ collider are explored in some detail in [13, 50]. If m_{A^0} is known within ~ 50 GeV, less than one year of operation of the $\gamma\gamma$ collider with $E_{\gamma\gamma}$ luminosity peaked at $E_{\gamma\gamma} \sim m_{A^0}$ will be needed to detect the H^0, A^0 signal. But, if the indirect determination of m_{A^0} is believed to be unreliable, or the SUSY scenario is such that no deviations will be present regardless of the value of m_{A^0}, one must employ a different strategy. One possibility is $\gamma\gamma$ collisions for LC operation at maximum energy, presumed in [13] to be $\sqrt{s} = 630$ GeV so as to allow substantial luminosity for $E_{\gamma\gamma}$ up to 500 GeV. By running for two years with laser and electron polarizations and orientation such as to yield a broad $E_{\gamma\gamma}$ spectrum and for one year with the $E_{\gamma\gamma}$ spectrum peaked at 500 GeV, detection of the $\gamma\gamma \to H^0, A^0 \to b\bar{b}$ signal will be possible throughout much of the LHC no-discovery wedge region. This is illustrated in Fig. 5. The results shown assume that 50% of the H^0, A^0 signal events will fall into a single $m_{b\bar{b}}$ mass bin of size 10 GeV, as consistent with expected mass resolutions and predicted Higgs widths.

Because of lack of space, I only summarize expectations for a muon collider Higgs factory with energy in the $250 - 500$ GeV range. A 4σ $\mu^+ \mu^- \to H^0, A^0 \to b\bar{b}$ signal could be found, either using operation at $\sqrt{s} \sim 500$ GeV and the bremsstrahlung (radiative return) tail or by employing an appropriate scan strategy, for almost all values of $[m_{A^0}, \tan\beta]$ in the LHC wedge region [15, 49, 51].

Of course, there are variants of these 'standard' results that temper this relatively optimistic outlook.

- Invisible decays will probably allow non-detection scenarios at hadron colliders. This is important even for the h^0. Indeed, $h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is still possible given the LEP2 data. For large $B(h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0)$, it is necessary to have $m_{\tilde{\chi}_1^0}/m_{\tilde{\chi}_1^\pm} \sim M_1/M_2 < 1/2$, *i.e.* smaller than predicted by universal boundary conditions, in order that $h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ not be kinematically suppressed given the $m_{\tilde{\chi}_1^\pm} > 103$ GeV lower limit from LEP2. In the study of [52, 53] (see also [54, 55]), it is found that the universal boundary condition prediction of $M_1/M_2 = 1/2$ allows for at most $B(h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0) \sim 20\%$, whereas $M_1/M_2 = 1/10 - 1/5$ allows $B(h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0) > 50\%$. One also needs substantial $h^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ coupling, which in turn requires that the $\tilde{\chi}_1^0$ have substantial higgsino content. This latter is possible when μ (and M_2) are not large. (One should recall that small M_1, M_2 and μ are preferred by current results for a_μ.)

- Similarly, the usual LHC contours for $H^0, A^0, H^\pm$ discovery in various modes will be modified (at low to moderate $\tan\beta$ when $m_{A^0} > 2m_Z$) if $\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^+ \tilde{\chi}_1^-, \tilde{\tau}^+ \tilde{\tau}^-, \tilde{\nu}\bar{\tilde{\nu}}, \ldots$ decays are kinematically allowed [2, 56, 57]. However, at high $\tan\beta$ the usual dominance of decays to $b\bar{b}$ and $\tau^+ \tau^-$ will be preserved. This, implies that even if SUSY particles are light the widening of the h^0-only LHC wedge at high $\tan\beta$ will be moderate (and the LEP2 limits mean that we do not need to worry very much about low $\tan\beta$).

- Stop loop correction to gg and $\gamma\gamma$ couplings of the MSSM Higgs bosons can be substantial [2, 58]. In particular, stop and top loop contributions to gg fusion negatively interfere, implying
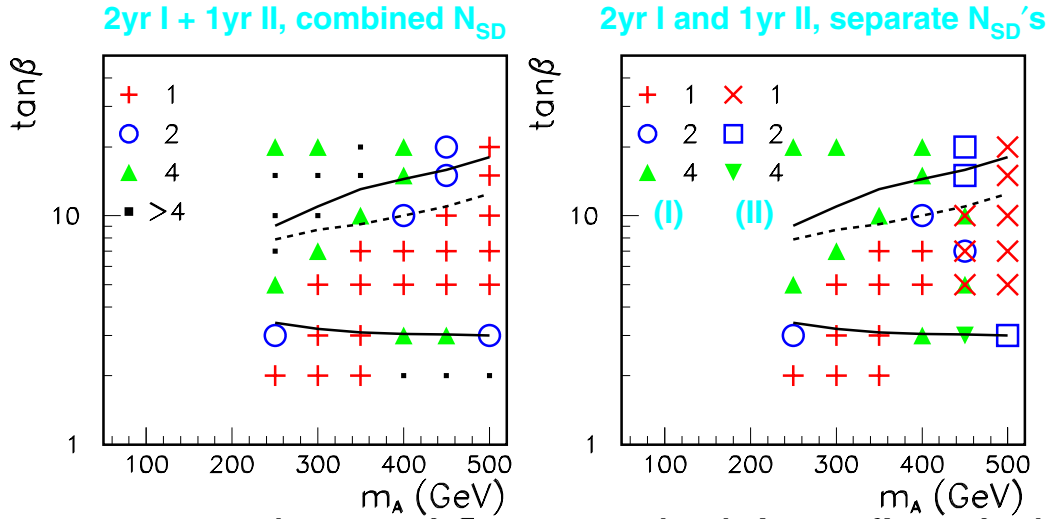
Luminosity Factor Required for 4σ Discovery

Figure 5: Assuming a machine energy of $\sqrt{s} = 630$ GeV, we show the $[m_{A^0}, \tan\beta]$ points for which two 10^7 sec years of running using a broad $E_{\gamma\gamma}$ spectrum (I) and one 10^7 sec year of running using a spectrum peaked at $E_{\gamma\gamma} \sim 500$ GeV (II) will yield $S/\sqrt{B} \geq 4$. In the left-hand window we have combined results from the type-I and type-II running using $S/\sqrt{B} = \sqrt{S_I^2/B_I + S_{II}^2/B_{II}}$. In the right-hand window, we show the separate results for $S_I/\sqrt{B_I}$ and $S_{II}/\sqrt{B_{II}}$. The solid curves indicate the wedge region from the LHC plot of Fig. 4 — the lower black curve is that from the LEP (maximal-mixing) limits, but is somewhat higher than that currently claimed by the LEPEWWG, while the upper solid curve is that above which $H^0, A^0 \to \tau^+\tau^-$ can be directly detected at the LHC. For parameter choices above the dashed curve, $H^\pm \to \tau^\pm\nu_\tau$ can be directly detected at the LHC. Also shown are the additional points for which a 4σ signal level is achieved if the total luminosity is doubled or quadrupled (the '2' and '4' symbol cases) relative to the one-year luminosities we are employing. (The small black squares in the LH window indicate the additional points sampled for which even a luminosity increase of a factor of 4 for both types of running does not yield a 4σ signal.) Such luminosity increases could be achieved for some combination of longer running time and/or improved technical designs. For example, the factor of '2' results probably roughly apply to TESLA.

some reduction of gg fusion production of the h^0 when stops are light, but also some increase in $B(h^0 \to \gamma\gamma)$.

· Radiative corrections to Higgs couplings can result in early or even exact decoupling, *i.e.* $\cos^2(\beta - \alpha) = 0$ independent of m_{A^0}.

· Radiative corrections can also greatly modify expectations for $h^0 \to b\bar{b}$ decays [47]. The important loops here do not decouple when SUSY masses are large. In one extreme, for special, but not unreasonable, parameter choices, one finds $h^0 \sim H_u$, where H_u is the MSSM doublet field that couples to up quarks (only), and $B(h^0 \to b\bar{b}) \sim 0$. In another extreme, substantial enhancement of the $h^0 \to b\bar{b}$ coupling occurs.

In either case, there are many implications for h^0 discovery. For example, suppressed $\Gamma(h^0 \to b\bar{b})$ implies enhanced $B(h^0 \to \gamma\gamma), B(h^0 \to WW^*)$, and it is even possible that detection of the h^0 in its $\gamma\gamma$ decay mode could be possible at the Tevatron for some range of m_{h^0} if $h^0 \sim H_u$ [59]. Since λ_b can be either enhanced or suppressed, it is useful to note [44] that the LHC $gg \to h^0 \to \gamma\gamma$ and Tevatron $Wh^0[\to WW^*]$ modes improve when the LHC, Tevatron $W, Zh^0[\to b\bar{b}]$ modes deteriorate. There is also complementarity between the Tevatron and LHC in that as the $b\bar{b}h^0$ coupling and m_{h^0} vary one finds that h^0 discovery will occur at one or the other machine, even if not at both.

Of course, at the LC the $e^+e^- \to Zh^0$ mode is robust regardless of how the h^0 decays. Further, at the LC, H^0A^0 and H^+H^- detection are quite robust against complicated decays if pair production is not too near the kinematic limit [60, 61, 62]. In fact, the precise decay mixtures provide an

immensely powerful probe of the soft SUSY breaking parameters. It is only necessary to separate different final state channels ($[3\ell, 2b]$, $[1\ell, 0b]$, ... — maybe 15 or 20 different channels) from one another and have precise knowledge of the efficiencies for different channels.

The above discussion was restricted to the MSSM. There is good reason to suppose that the Higgs sector could have one or more singlets beyond the required two-doublets. Singlet Higgs fields do not disturb coupling constant unification and lead to some very attractive improvements to the MSSM. The simplest model is the NMSSM in which a single Higgs singlet is introduced [38]. (See [2] for a review and further details.) The new attractive feature of this model is that the superpotential can contain the term $W \ni \lambda \hat{H}_1 \hat{H}_2 \hat{N}$, such that for $\langle N \rangle \neq 0$ there is a natural source and appropriate magnitude, $\lambda \langle N \rangle = \mu$, for the somewhat mysterious $\mu \hat{H}_1 \hat{H}_2$ superpotential term of the MSSM. In the NMSSM, there are three CP-even Higgs bosons ($h_{1,2,3}$) and two CP-odd Higgs bosons ($a_{1,2}$), assuming no CP violation. As we have already discussed, we can add any number of singlets and still find a Higgs boson signal for $e^+e^- \to Z^* \to Zh_i$ production at a LC, even if the signals overlap. At the LHC, establishing a corresponding guarantee is quite challenging. Indeed, it was shown in [63] that parameters of the NMSSM could be chosen so that no Higgs boson would be detected in the modes for which definitive experimental results were available at the time of the Snowmass 1996 workshop. The modes employed in 1996 were: 1) $Z^\star \to Zh$ at LEP2; 2) $Z^\star \to ha$ at LEP2; 3) $gg \to h \to \gamma\gamma$ at the LHC; 4) $gg \to h \to ZZ^\star$ or $ZZ \to 4\ell$ at the LHC; 5) $t \to H^+b$ at the LHC; 6) $gg \to b\bar{b}h, b\bar{b}a \to b\bar{b}\tau^+\tau^-$ at LHC; 7) $gg \to h, a \to \tau^+\tau^-$ at the LHC. The regions of parameter space in which no Higgs bosons would be detected were characterized by substantial mixing among all the Higgs bosons and moderate $\tan\beta$ values. This study has been updated as part of the Snowmass01 and LesHouches01 workshops [64]. One important discovery mode not confirmed by the experimental groups at the time of Snowmass96 is $tt \to t\bar{t}h_i \to t\bar{t}b\bar{b}$ [65]. The experimental groups now believe that this will be visible [66, 67] if the h_i coupling to $t\bar{t}$ is comparable to the $h_{\rm SM}t\bar{t}$ coupling. In [64], the full NMSSM parameter space (excluding regions for which SUSY pairs or Higgs bosons appear in Higgs decays) was rescanned including the $t\bar{t}h$ mode with the result that most (but not all) parameter choices for which Higgs discovery would not have been possible in the 1996 analysis *would* lead to one of the Higgs bosons being visible in this mode. In addition, we find (using the theoretical estimates of [68]) that essentially all of the remaining 'bad' portions of parameter space would lead to visible signals in the modes where one of the h_i is produced via WW-fusion and then decays to $\tau^+\tau^-$. This illustrates the great importance that the ATLAS and CMS groups should attach to further improving their Higgs discovery techniques, particularly by adding new modes complementary to those already considered.

3. Determining the CP of an observed Higgs boson

Determination of the CP properties of the Higgs bosons could prove very crucial to sorting out a complex Higgs sector. At a LC there are many techniques based on WW and/or ZZ couplings for verifying a substantial CP=+ component. But the VV couplings are only sensitive to the CP=− component of a Higgs boson at one-loop level. As a result, using such couplings it is very hard to see a CP=− coupling even if it is present. Since CP=+ and CP=− couplings to $t\bar{t}$ of any h are both tree-level ($\bar{t}(a + ib\gamma_5)t$, where a,b is the CP-even, -odd Higgs component), angular distributions of the $t, \bar{t}$ and h relative to one another in the $t\bar{t}h$ final state allow determination of the relative sizes of a and b for lighter h's [69]. The best approach is to use the optimal observable technique [70]. At a LC, as long as there is reasonable event rate (which requires $\sqrt{s} > 800$ GeV for $m_h \sim 100 - 200$ GeV) this is straightforward [70]. At the LHC, there will be a high event rate, but reconstruction and identification of the t and $\bar{t}$ is trickier and backgrounds will be larger. Still, there is considerable promise [69, 71].

The CP=+ and CP=− components of a Higgs boson also couple with similar *magnitude* but different structure to $\gamma\gamma$ (via 1-loop diagrams). Thus, determination of the CP properties of any Higgs boson that can be seen in $\gamma\gamma$ collisions at the LC will be possible [72, 73, 74] by comparing production rates for different orientations of the polarizations of the colliding γ's. Very briefly, we have

$$\mathcal{A}_{CP=+} \propto \vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2 \,, \quad \mathcal{A}_{CP=-} \propto (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2) \cdot \hat{p}_{\rm beam} \,. \tag{2}$$

For pure CP states, one will want to maximize the linear polarization of the back-scattered photons and adjust the orientation ($\perp$ for CP odd dominance, $\parallel$ for CP even dominance) to determine the CP

nature of the Higgs boson being produced. For a light SM-like Higgs boson, a detailed study [13] can determine that $C\mathcal{P} = +1$ with an error of $\delta C\mathcal{P}/C\mathcal{P} \sim 0.11$. One can also separate the A^0 from the H^0 when these are closely degenerate (as typical for $\tan\beta \gtrsim 4$ and $m_{A^0} > 2m_Z$). For mixed CP states, one achieves better statistics by using circularly polarized photons and employing helicity asymmetries to determine the CP mixture.

At a muon collider Higgs factory there is a particularly appealing approach using asymmetries involving transversely polarized muon beams [49, 75]. For resonance, R, production with $\overline{\mu}(a + ib\gamma_5)\mu$ coupling to the muon, one has

$$\overline{\sigma}_S(\zeta) \;=\; \overline{\sigma}_S^0 \left(1 + P_L^+ P_L^- + P_T^+ P_T^- \left[\frac{a^2 - b^2}{a^2 + b^2} \cos\zeta - \frac{2ab}{a^2 + b^2} \sin\zeta \right] \right) , \qquad (3)$$

where P_T (P_L) is the degree of transverse (longitudinal) polarization of the colliding μ^+ and μ^-, and ζ is the angle of the μ^+ transverse polarization relative to that of the μ^- as measured using the the direction of the μ^-'s momentum as the $\hat{z}$ axis. Only the $\sin\zeta$ term is truly CP-violating, but the dependence on $\cos\zeta$ is also sensitive to a/b. One must take into account the precession of the μ^+ and μ^- as they circulate around the storage ring. Fortunately, this is easy to do and very decent accuracy is possible for the determination of b/a for a Higgs boson after a few years of operation [76], provided the Higgs factory can achieve luminosities about a factor of two larger than the current benchmarks.

4. Conclusions

There are a large variety of very viable Higgs sector models. Experiment will be required to determine the correct theory. The current and future machines and the related tools and techniques that have been developed have reached a high enough level of sophistication that we should have a good chance of detecting and studying the Higgs bosons of even rather unusual Higgs sectors.

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