Precision Electroweak Data: Phenomenological Analysis

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The precision electroweak program, including weak neutral current (WNC), Z-pole, and high energy collider experiments, has been the primary prediction and test of electroweak unification. It has established that the standard model (SM) is correct and unique to first approximation, establishing the gauge principle as well as the SM gauge group and representations; shown that the SM is correct at loop level, confirming the basic principles of renormalizable gauge theory and allowing the successful prediction or constraint on m_t, α_s, and M_H; severely constrained new physics at the TeV scale, with the ideas of unification strongly favored over TeV-scale compositeness; and yielded precise values for the gauge couplings, consistent with (supersymmetric) gauge unification.

1. The precision program

The weak neutral current was a critical prediction of the electroweak standard model (SM) [1]. Following its discovery in 1973, there were generations of ever more precise WNC experiments, including pure weak νN and νe scattering processes, weak-electromagnetic interference processes such as polarized $e^{\uparrow\downarrow}D$ or μN, $e^+e^- \to$ (hadron or charged lepton) cross sections and asymmetries below the Z pole, and parity-violating effects in heavy atoms (APV). There were also early direct observations of the W and Z. The program was supported by theoretical efforts in the calculation of QCD and electroweak radiative corrections; the expectations for observables in the standard model, large classes of extensions, and alternative models; and global analyses of the data. Even before the beginning of the Z-pole experiments at LEP and SLC in 1989, this program had established [2]-[6]:

- The SM is correct to first approximation. The four-fermion operators for νq, νe, and eq were uniquely determined, in agreement with the standard model. The W and Z masses agreed with the expectations of the $SU(2)\times U(1)$ gauge group and canonical Higgs mechanism, eliminating contrived alternative models with the same four-fermi interactions as the standard model.
- Electroweak radiative corrections were necessary for the agreement of theory and experiment.
- The weak mixing angle (in the on-shell renormalization scheme) was determined to be $\sin^2\theta_W = 0.229 \pm 0.0064$; consistency of the various observations, including radiative corrections, required $m_t < 200$ GeV.
- Theoretical uncertainties, especially in the c threshold in deep inelastic WCC scattering, dominated.
- The combination of WNC and WCC data uniquely determined the $SU(2)$ representations of all of the known fermions, i.e., of the ν_e and ν_μ, as well as the L and R components of the e, μ, τ, d, s, b, u, and c [7]. In particular, the left-handed b and τ were the lower components of $SU(2)$ doublets, implying unambiguously that the t quark and ν_τ had to exist.
- The electroweak gauge couplings were well-determined, allowing a detailed comparison with the gauge unification predictions of the simplest grand unified theories (GUT). Ordinary

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$SU(5)$ was excluded (consistent with the non-observation of proton decay), but the supersymmetric extension was allowed.

- There were stringent limits on new physics at the TeV scale, including additional Z' bosons, exotic fermions (for which both WNC and WCC constraints were crucial), exotic Higgs representations, leptoquarks, and new four-fermion operators.

The LEP/SLC era greatly improved the precision of the electroweak program. It allowed the differentiation between non-decoupling extensions to the SM (such as most forms of dynamical symmetry breaking and other types of TeV-scale compositeness), which typically predicted several % deviations, and decoupling extensions (such as most of the parameter space for supersymmetry), for which the deviations are typically 0.1%.

The first phase of the LEP/SLC program involved running at the Z pole, $e^+e^- \to Z \to \ell^+\ell^-,\ q\bar{q}$, and $\nu\bar{\nu}$. During the period 1989-1995 the four LEP experiments ALEPH, DELPHI, L3, and OPAL at CERN observed $\sim 2 \times 10^7 Z's$. The SLD experiment at the SLC at SLAC observed some 5×10^5 events. Despite the much lower statistics, the SLC had the considerable advantage of a highly polarized e^- beam, with $P_{e^-} \sim 75\%$. There were quite a few Z pole observables, including:

- The lineshape: M_Z, Γ_Z, and the peak cross section σ.

- The branching ratios for $e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-,\ q\bar{q},\ c\bar{c},\ b\bar{b}$, and $s\bar{s}$. One could also determine the invisible width, $\Gamma(\text{inv})$, from which one can derive the number $N_\nu = 2.985 \pm 0.008$ of active (weak doublet) neutrinos with $m_\nu < M_Z/2$, i.e., there are only 3 conventional families with light neutrinos. $\Gamma(\text{inv})$ also constrains other invisible particles, such as light sneutrinos and the light majorons associated with some models of neutrino mass.

- A number of asymmetries, including forward-backward (FB) asymmetries; the τ polarization, P_τ; the polarization asymmetry A_{LR} associated with P_{e^-}; and mixed polarization-FB asymmetries.

The expressions for the observables are summarized in Appendix A, and the experimental values and SM predictions in Table I. These combinations of observables could be used to isolate many Z-fermion couplings, verify lepton family universality, determine $\sin^2\theta_W$ in numerous ways, and determine or constrain m_t, α_s, and M_H. LEP and SLC simultaneously carried out other programs, most notably studies and tests of QCD, and heavy quark physics.

LEP 2 ran from 1995-2000, with energies gradually increasing from ~ 140 to ~ 208 GeV. The principal electroweak results were precise measurements of the W mass, as well as its width and branching ratios (these were measured independently at the Tevatron); a measurement of $e^+e^- \to W^+W^-$, ZZ, and single W, as a function of center of mass (CM) energy, which tests the cancellations between diagrams that is characteristic of a renormalizable gauge field theory, or, equivalently, probes the triple gauge vertices; limits on anomalous quartic gauge vertices; measurements of various cross sections and asymmetries for $e^+e^- \to f\bar{f}$ for $f = \mu^-, \tau^-, q, b$ and c, in reasonable agreement with SM predictions; a stringent lower limit of 113.5 GeV on the Higgs mass, and even hints of an observation at ~ 115 GeV; and searches for supersymmetric or other exotic particles.

In parallel with the LEP/SLC program, there were much more precise ($< 1\%$) measurements of atomic parity violation (APV) in cesium at Boulder, along with the atomic calculations and related measurements needed for the interpretation; precise new measurements of deep inelastic scattering by the NuTeV collaboration at Fermilab, with a sign-selected beam which allowed them to minimize the effects of the c threshold and reduce uncertainties to around 1%; and few % measurements of $\overset{(-)}{\nu}_\mu e$ by CHARM II at CERN. Although the precision of these WNC processes was lower than the Z pole measurements, they are still of considerable importance: the Z pole experiments are blind to types of new physics that do not directly affect the Z, such as a heavy Z' if there is no $Z - Z'$ mixing, while the WNC experiments are often very sensitive. During the same period there were important electroweak results from CDF and D$\emptyset$ at the Tevatron, most notably a precise value for M_W, competitive with and complementary to the LEP 2 value; a direct measure of m_t, and direct searches for Z', W', exotic fermions, and supersymmetric particles. Many of these non-Z pole results are summarized in Table II.

Quantity	Group(s)	Value	Standard Model	pull
M_Z [GeV]	LEP	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1
Γ_Z [GeV]	LEP	2.4952 ± 0.0023	2.4966 ± 0.0016	-0.6
$\Gamma(\text{had})$ [GeV]	LEP	1.7444 ± 0.0020	1.7429 ± 0.0015	—
$\Gamma(\text{inv})$ [MeV]	LEP	499.0 ± 1.5	501.76 ± 0.14	—
$\Gamma(\ell^+\ell^-)$ [MeV]	LEP	83.984 ± 0.086	84.019 ± 0.027	—
σ_{had} [nb]	LEP	41.541 ± 0.037	41.477 ± 0.014	1.7
R_e	LEP	20.804 ± 0.050	20.744 ± 0.018	1.2
R_μ	LEP	20.785 ± 0.033	20.744 ± 0.018	1.2
R_τ	LEP	20.764 ± 0.045	20.790 ± 0.018	-0.6
$A_{FB}(e)$	LEP	0.0145 ± 0.0025	0.01637 ± 0.00026	-0.8
$A_{FB}(\mu)$	LEP	0.0169 ± 0.0013		0.4
$A_{FB}(\tau)$	LEP	0.0188 ± 0.0017		1.4
R_b	LEP + SLD	0.21664 ± 0.00068	0.21569 ± 0.00016	1.4
R_c	LEP + SLD	0.1729 ± 0.0032	0.17230 ± 0.00007	0.2
$A_{FB}(b)$	LEP	0.0982 ± 0.0017	0.1036 ± 0.0008	-3.2
$A_{FB}(c)$	LEP	0.0689 ± 0.0035	0.0740 ± 0.0006	-1.5
$A_{FB}(s)$	DELPHI,OPAL	0.0976 ± 0.0114	0.1037 ± 0.0008	-0.5
A_b	SLD	0.921 ± 0.020	0.9347 ± 0.0001	-0.7
A_c	SLD	0.667 ± 0.026	0.6681 ± 0.0005	0.0
A_s	SLD	0.895 ± 0.091	0.9357 ± 0.0001	-0.4
A_{LR} (hadrons)	SLD	0.15138 ± 0.00216	0.1478 ± 0.0012	1.7
A_{LR} (leptons)	SLD	0.1544 ± 0.0060		1.1
A_μ	SLD	0.142 ± 0.015		-0.4
A_τ	SLD	0.136 ± 0.015		-0.8
$A_\tau(\mathcal{P}_\tau)$	LEP	0.1439 ± 0.0041		-0.9
$A_e(\mathcal{P}_\tau)$	LEP	0.1498 ± 0.0048		0.4
$\bar{s}_\ell^2(Q_{FB})$	LEP	0.2322 ± 0.0010	0.23143 ± 0.00015	0.8

Table I Principal Z-pole observables, their experimental values, theoretical predictions using the SM parameters from the global best fit [3], and pull (difference from the prediction divided by the uncertainty). $\Gamma(\text{had})$, $\Gamma(\text{inv})$, and $\Gamma(\ell^+\ell^-)$ are not independent, but are included for completeness.

The LEP and (after initial difficulties) SLC programs were remarkably successful, achieving greater precision than had been anticipated in the planning stages, e.g., due to better than expected measurements of the beam energy (using a clever resonant depolarization technique) and luminosity.

The effort required the calculation of the needed electromagnetic, electroweak, QCD, and mixed radiative corrections to the predictions of the SM. Careful consideration of the competing definitions of the renormalized $\sin^2\theta_W$ was needed. The principal theoretical uncertainty is the hadronic contribution $\Delta\alpha^{(5)}_{\text{had}}(M_Z)$ to the running of α from its precisely known value at low energies to the Z-pole, where it is needed to compare the Z mass with the asymmetries and other observables. The radiative corrections, renormalization schemes, and running of α are further discussed in Appendix B. The LEP Electroweak Working Group (LEPEWWG) [8] combined the results of the four LEP experiments, and also those of SLD and some WNC and Tevatron results, taking proper account of common systematic and theoretical uncertainties. Much theoretical effort also went into the development, testing, and comparison of radiative corrections packages, and into the study of how various classes of new physics would modify the observables, and how they could most efficiently be parametrized.

2. Fits to the standard model

Global fits allow uniform theoretical treatment and exploit the fact that the data collectively contain much more information than individual experiments. However, they require a careful

Quantity	Group(s)	Value	Standard Model	pull
m_t [GeV]	Tevatron	174.3 ± 5.1	175.3 ± 4.4	-0.2
M_W [GeV]	LEP	80.446 ± 0.040	80.391 ± 0.019	1.4
M_W [GeV]	Tevatron,UA2	80.451 ± 0.061		1.0
R^-	NuTeV	$0.2277 \pm 0.0021 \pm 0.0007$	0.2300 ± 0.0002	-1.1
κ^ν	CCFR	$0.5820 \pm 0.0027 \pm 0.0031$	0.5833 ± 0.0004	-0.3
R^ν	CDHS	$0.3096 \pm 0.0033 \pm 0.0028$	0.3093 ± 0.0002	0.1
R^ν	CHARM	$0.3021 \pm 0.0031 \pm 0.0026$		-1.7
$R^{\bar\nu}$	CDHS	$0.384 \pm 0.016 \pm 0.007$	0.3862 ± 0.0002	-0.1
$R^{\bar\nu}$	CHARM	$0.403 \pm 0.014 \pm 0.007$		1.0
$R^{\bar\nu}$	CDHS 1979	$0.365 \pm 0.015 \pm 0.007$	0.3817 ± 0.0002	-1.0
$g_V^{\nu e}$	CHARM II	-0.035 ± 0.017	-0.0398 ± 0.0003	—
$g_V^{\nu e}$	all	-0.040 ± 0.015		-0.1
$g_A^{\nu e}$	CHARM II	-0.503 ± 0.017	-0.5065 ± 0.0001	—
$g_A^{\nu e}$	all	-0.507 ± 0.014		0.0
$Q_W(\mathrm{Cs})$	Boulder	$-72.65 \pm 0.28 \pm 0.34$	-73.10 ± 0.03	1.0
$Q_W(\mathrm{Tl})$	Oxford,Seattle	$-114.8 \pm 1.2 \pm 3.4$	-116.67 ± 0.05	0.5
$\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\nu)}$	CLEO	$3.26^{+0.75}_{-0.68} \times 10^{-3}$	$3.14^{+0.17}_{-0.16} \times 10^{-3}$	0.2
$\frac{1}{2}(g_\mu - 2 - \frac{\alpha}{\pi})$	E821	$4510.55 \pm 1.51 \pm 0.51$	4506.55 ± 0.36	2.5

Table II Recent non-Z-pole observables. From [3].

consideration of experimental and theoretical systematics and their correlations. The results here are from work with Jens Erler for the 2001 update of the electroweak review in the *Review of Particle Properties* [3]. They incorporate the full Z-pole, WNC (especially important for constraining some types of new physics), and relevant hadron collider and LEP 2 results. The radiative corrections were calculated with GAPP (*Global Analysis of Particle Properties*) [9]. GAPP is fully $\overline{\mathrm{MS}}$, which minimizes the mixed QCD-EW corrections and their uncertainties and is a complement to ZFITTER [10], which is on-shell. We use a $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z)$ which is properly correlated with α_s [11], and also with the hadronic vacuum polarization contribution to $g_\mu - 2$ [12].

The data are for the most part in excellent agreement with the SM predictions. The best fit values for the SM parameters (as of 07/01) are,

$$\begin{aligned} M_H &= 98^{+51}_{-35}\ \mathrm{GeV}, \\ m_t &= 175.3 \pm 4.4\ \mathrm{GeV}, \\ \alpha_s &= 0.1200 \pm 0.0028, \\ \hat{s}^2_Z &= 0.23113 \pm 0.00015, \\ \Delta\alpha^{(5)}_{\mathrm{had}}(M_Z) &= 0.02778 \pm 0.00020. \end{aligned} \tag{1}$$

- This fit included the direct (Tevatron) measurements of m_t and the theoretical value of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z)$ as constraints, but did not include other determinations of α_s or the LEP 2 direct limits on M_H.

- The $\overline{\mathrm{MS}}$ value of $\sin^2\theta_W$ ($\hat{s}^2_Z$) can be translated into other definitions. The effective angle $\bar{s}^2_\ell = 0.23143 \pm 0.00015$ is closely related to $\hat{s}^2_Z$. The larger uncertainty in the on-shell $s^2_W = 0.22278 \pm 0.00036$ is due to its (somewhat artificial) dependence on M_H and m_t. On the other hand, the Z-mass definition $s^2_{M_Z} = 0.23105 \pm 0.00008$ has no M_H or m_t dependence, but the uncertainties reemerge when comparing with other observables.

- The best fit value $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z) = 0.02778 \pm 0.00020$ is dominated by the theoretical input constraint $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z) = 0.02779 \pm 0.00020$. However, $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z)$ can be determined from the indirect data alone, i.e., from the relation of M_Z and M_W to the other observables, and by its correlation with $g_\mu - 2$. The result, 0.02866 ± 0.00040, is $\sim 1.9\sigma$ above the theoretical value, mainly because of $g_\mu - 2$.

- Similarly, the value $m_t = 175.3 \pm 4.4$ GeV includes the direct Tevatron constraint $m_t = 174.3 \pm 5.1$. However, one can determine $m_t = 178.1^{+10.4}_{-8.3}$ GeV from indirect data (loops) only, in excellent agreement.

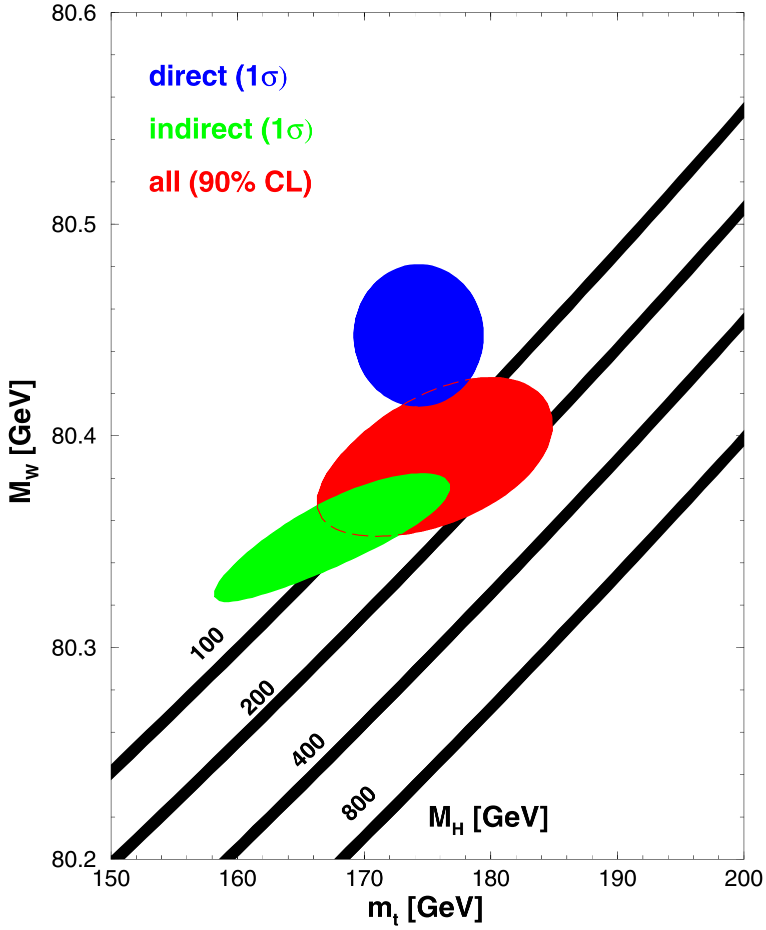

Figure 1: Allowed regions in M_W vs m_t from direct, indirect, and combined data, compared with the standard model expectations as a function of M_H. From [3].

- The value $\alpha_s = 0.1200 \pm 0.0028$ is consistent with other determinations, e.g., from deep inelastic scattering, hadronic τ decays, the charmonium and upsilon spectra, and jet properties.

- The central value of the Higgs mass prediction from the fit, $M_H = 98^{+51}_{-35}$ GeV, is below the direct lower limit from LEP 2 of $\gtrsim 113.5$ GeV, or their candidate events at 115 GeV, but consistent at the 1σ level. Including the direct LEP 2 likelihood function [13, 14] along with the indirect data, one obtains $M_H < 199$ GeV at 95%. Even though M_H only enters the precision data logarithmically (as opposed to the quadratic m_t dependence), the constraints are significant. They are also fairly robust to most, but not all, types of new physics. (The limit on M_H disappears if one allows an arbitrarily large negative S parameter and/or a large positive T (section 3), but most extensions of the SM yield $S > 0$.) One caveat is that M_W and A_{LR} especially favor rather low values of M_H, while $A_{FB}(b)$, which deviates by 3.2σ from the SM (see below), compensates by favoring a high value [15]. The predicted range should be compared with the theoretically expected range in the standard model: 115 GeV $\lesssim M_H \lesssim$ 750 GeV, where the lower (upper) limit is from vacuum stability (triviality). On the other hand, the MSSM predicts $M_H \lesssim 130$ GeV, while the limit increases to around 150 GeV in extensions of the MSSM.

- The results in (1) are in excellent agreement with those of the LEPEWWG [8] up to well-

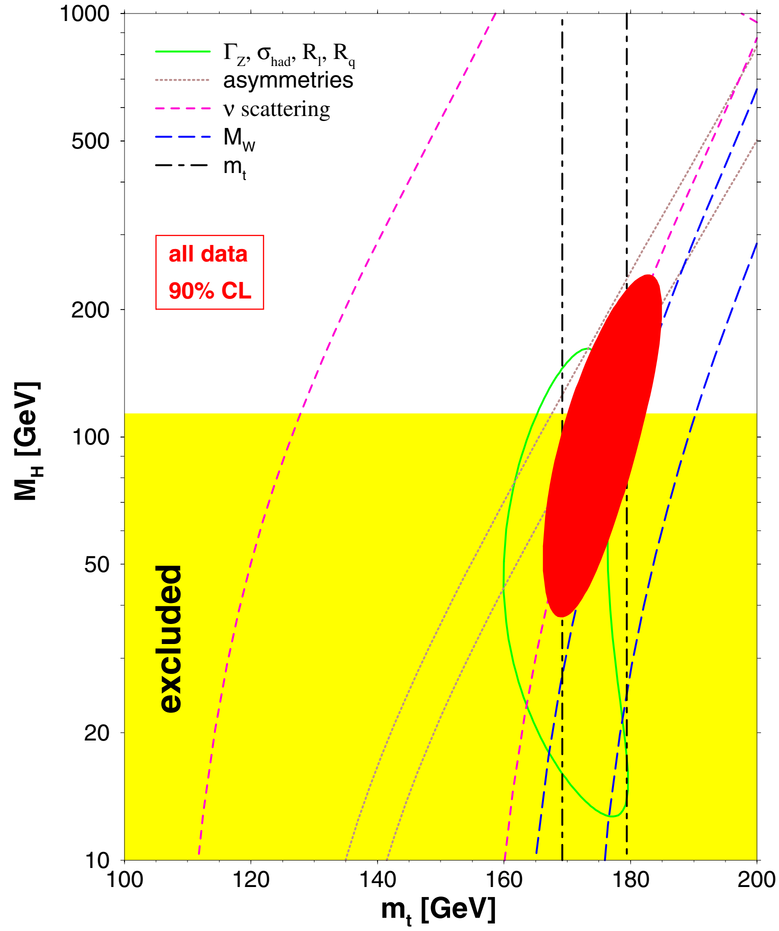

Figure 2: Allowed regions in M_H vs m_t from precision data, compared with the direct exclusion limits from LEP 2. From [3].

understood effects [3], such as more extensive WNC inputs and small differences in higher order terms and $\Delta\alpha^{(5)}_{\rm had}(M_Z)$, despite the different renormalization schemes used. The LEP-EWWG obtains: $\bar{s}^2_\ell = 0.23142 \pm 0.00014$; $\alpha_s = 0.118 \pm 0.003$; $m_t = 175.7^{+4.4}_{-4.3}$ GeV; and $M_H = 98^{+58}_{-38}$ GeV.

- The most significant deviation from the SM is in the forward-backward asymmetry into b quarks, $A_{FB}(b)$, which is 3.2σ below the prediction. If not just a statistical fluctuation or systematic problem, this could be a hint of new physics. However, any such effect should not contribute too much to R_b, which is consistent with the SM. The size of the deviation suggests a tree level effect, such as the mixing of $b_{L,R}$ with exotic quarks [2, 16]. The most recent LEP results on M_W have moved slightly above the SM prediction (table (II) and figure (1)), but even when combined with the Tevatron results (also a bit high) this is only a 1.5σ effect. The muon magnetic moment $g_\mu - 2$ result could point towards new physics, but there are still significant hadronic uncertainties. Within the SM fits, the only affect is the correlation of the theoretical value with $\Delta\alpha^{(5)}_{\rm had}(M_Z)$, which lowers the M_H prediction by ~ 5 GeV [12].

3. Beyond the standard model

The standard model ($SU(3)\times SU(2)\times U(1)$ plus general relativity), extended to include neutrino mass, is the correct description of nature to first approximation down to 10^{-16} cm. However, nobody thinks that the SM is the ultimate description of nature. It has some 28 free parameters;

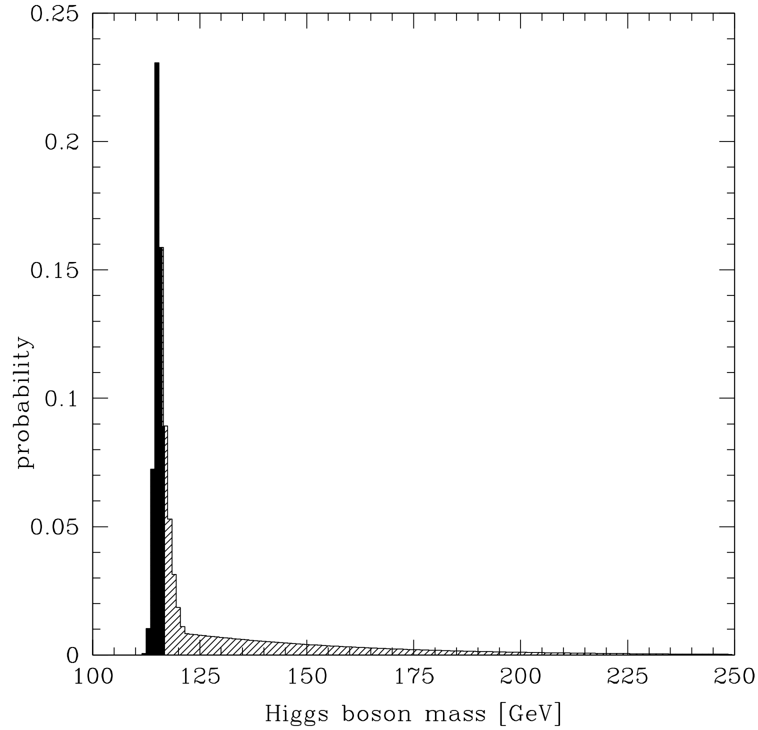

Figure 3: Probability density for M_H, including direct LEP 2 data and indirect constraints. From [13].

has a complicated gauge group and representations; does not explain charge quantization, the fermion families, or their masses and mixings; has several notorious fine tunings associated with the Higgs mass, the strong CP parameter, and the cosmological constant; and does not incorporate quantum gravity.

Many types of possible TeV scale physics are constrained by the precision data. For example,

- S, T, and U parametrize new physics sources which only affect the gauge propagators, as well as Higgs triplets, etc. One expects $T \neq 0$, usually positive and often of order unity, from nondegenerate heavy fermion or scalar doublets, while new chiral fermions (e.g., in extended technicolor (ETC)), lead to $S \neq 0$, again usually positive and often of order unity. The current global fit result is [3]

$$\begin{aligned} S &= -0.03 \pm 0.11(-0.08), \\ T &= -0.02 \pm 0.13(+0.09), \\ U &= 0.24 \pm 0.13(+0.01) \end{aligned} \tag{2}$$

  for $M_H = 115$ (300) GeV. (We use a definition in which S, T, and U are exactly zero in the SM.) The value of S would be $2/3\pi$ for a heavy degenerate ordinary or mirror family, which is therefore excluded at 99.8%. Equivalently, the number of families is $N_{\rm fam} = 2.97 \pm 0.30$. This result assumes $T = U = 0$, and therefore that any new families are degenerate. This restriction can be relaxed by allowing $T \neq 0$, yielding the somewhat weaker constraint $N_{\rm fam} = 3.27 \pm 0.45$ for $T = 0.10 \pm 0.11$. This is complementary to the lineshape result $N_\nu = 2.985 \pm 0.008$, which only applies for ν's lighter than $\sim M_Z/2$. S also eliminates many QCD-like ETC models. T is equivalent to the ρ_0 parameter [2], which is defined to be exactly unity in the SM. For $S = U = 0$, one obtains $\rho_0 \sim 1 + \alpha T = 1.0012^{+0.0023}_{-0.0014}$, with the SM fit value for M_H increasing to $M_H = 211^{+814}_{-139}$ GeV.

- Supersymmetry: in the decoupling limit, in which the sparticles are heavier than $\gtrsim 200-300$ GeV, there is little effect on the precision observables, other than that there is necessarily a

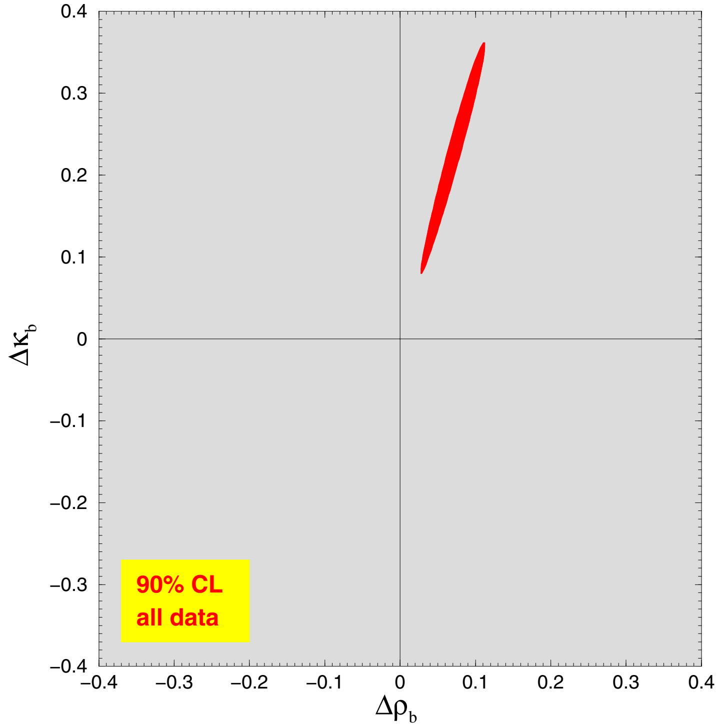

Figure 4: Regions in new physics parameters suggested by $A_{FB}(b)$ and R_b. A very large ($O(20\%)$) vertex correction $\Delta\kappa_b$ would be required to account for the data by loop effects. Courtesy of Jens Erler.

light SM-like Higgs, consistent with the data. There is little improvement on the SM fit, and in fact one can somewhat constrain the supersymmetry breaking parameters [17].

- Heavy Z' bosons are predicted by many grand unified and string theories. Limits on the Z' mass are model dependent, but are typically around $M_{Z'} > 500 - 800$ GeV from indirect constraints from WNC and LEP 2 data, with comparable limits from direct searches at the Tevatron. Z-pole data severely constrains the $Z - Z'$ mixing, typically $|\theta_{Z-Z'}| < \text{few} \times 10^{-3}$.

- Gauge unification is predicted in GUTs and some string theories. The simplest non-supersymmetric unification is excluded by the precision data. For the MSSM, and assuming no new thresholds between 1 TeV and the unification scale, one can use the precisely known α and $\hat{s}^2_Z$ to predict $\alpha_s = 0.130 \pm 0.010$ and a unification scale $M_G \sim 3 \times 10^{16}$ GeV [18]. The α_s uncertainties are mainly theoretical, from the TeV and GUT thresholds, etc. α_s is high compared to the experimental value, but barely consistent given the uncertainties. M_G is reasonable for a GUT (and is consistent with simple seesaw models of neutrino mass), but is somewhat below the expectations $\sim 5 \times 10^{17}$ GeV of the simplest perturbative heterotic string models. However, this is only a 10% effect in the appropriate variable $\ln M_G$. The new exotic particles often present in such models (or higher Kač-Moody levels) can easily shift the $\ln M_G$ and α_s predictions significantly, so the problem is really why the gauge unification works so well. It is always possible that the apparent success is accidental (cf., the discovery of Pluto).

4. Conclusions

The precision Z-pole, LEP 2, WNC, and Tevatron experiments have successfully tested the SM at the 0.1% level, including electroweak loops, thus confirming the gauge principle, SM group, representations, and the basic structure of renormalizable field theory. The standard model parameters $\sin^2\theta_W$, m_t, and α_s were precisely determined. In fact, m_t was successfully predicted from its indirect loop effects prior to the direct discovery at the Tevatron, while the indirect value

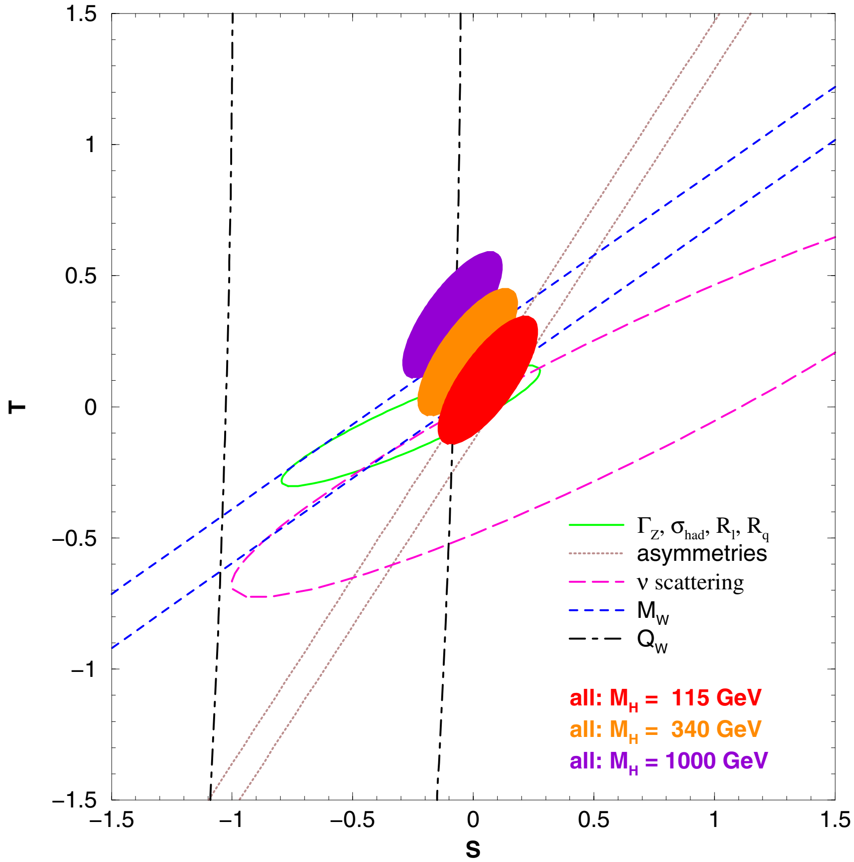

Figure 5: Allowed regions in S vs T. From [3].

of α_s, mainly from the Z-lineshape, agreed with more direct QCD determinations. Similarly, $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ and M_H were constrained. The indirect (loop) effects implied $M_H \lesssim 191$ GeV, while direct searches at LEP 2 yielded $M_H > 113.5$ GeV, with a hint of a signal at 115 GeV. This range is consistent with, but does not prove, the expectations of the supersymmetric extension of the SM (MSSM), which predicts a light SM-like Higgs for much of its parameter space. The agreement of the data with the SM imposes a severe constraint on possible new physics at the TeV scale, and points towards decoupling theories (such as most versions of supersymmetry and unification), which typically lead to 0.1% effects, rather than TeV-scale compositeness (e.g., dynamical symmetry breaking or composite fermions), which usually imply deviations of several % (and often large flavor changing neutral currents). Finally, the precisely measured gauge couplings were consistent with the simplest form of grand unification if the SM is extended to the MSSM.

Although the Z-pole program has ended for the time being, there are prospects for future programs using the Giga-Z option at TESLA or possible other linear colliders, which might yield a factor 10^2 more events. This would enormously improve the sensitivity [19], but would also require a large theoretical effort to improve the radiative correction calculations.

: Appendix AThe Z Lineshape and Asymmetries

The Z lineshape measurements determine the cross section $e^+e^- \to f\bar{f}$ for $f = e, \mu, \tau, s, b, c$, or hadrons as a function of $s = E^2_{CM}$. To lowest order,

$$\sigma_f(s) \sim \sigma_f \frac{s\Gamma_Z^2}{\left(s - M_Z^2\right)^2 + \frac{s^2\Gamma_Z^2}{M_Z^2}}, \tag{A1}$$

where significant initial state radiative corrections are not displayed.

The peak cross section σ_f is related to the Z mass and partial widths by

$$\sigma_f = \frac{12\pi}{M_Z^2} \ \frac{\Gamma(e^+e^-)\Gamma(f\bar{f})}{\Gamma_Z^2}. \tag{A2}$$

The widths are expressed in terms of the effective $Zf\bar{f}$ vector and axial couplings $\bar{g}_{V,Af}$ by

$$\Gamma(f\bar{f}) \sim \frac{C_f G_F M_Z^3}{6\sqrt{2}\pi}\left[|\bar{g}_{Vf}|^2 + |\bar{g}_{Af}|^2\right], \tag{A3}$$

where $C_\ell = 1$ and $C_q = 3$. Electroweak radiative corrections are absorbed into the $\bar{g}_{V,Af}$. There are fermion mass, QED, and QCD corrections to (A3).

The effective couplings in (A3) are defined in the SM by

$$\bar{g}_{Af} = \sqrt{\rho_f} t_{3f}, \quad \bar{g}_{Vf} = \sqrt{\rho_f}\left[t_{3f} - 2\bar{s}_f^2 q_f\right], \tag{A4}$$

where q_f is the electric charge and t_{3f} is the weak isospin of fermion f, and $\bar{s}_f^2$ is the effective weak angle. It is related by (f-dependent) vertex corrections to the on-shell or $\overline{\mathrm{MS}}$ definitions of $\sin^2\theta_W$ by

$$\bar{s}_f^2 = \kappa_f s_W^2 \ \ (\mathrm{on-shell}) = \hat{\kappa}_f \hat{s}_Z^2 \ \ (\overline{\mathrm{MS}}). \tag{A5}$$

$\rho_f - 1$, $\kappa_f - 1$, and $\hat{\kappa}_f - 1$ are electroweak corrections. For $f = e$ and the known ranges for m_t and M_H, $\bar{s}_e^2 \sim \hat{s}_Z^2 + 0.00029$.

It is convenient to define the ratios

$$R_{q_i} \equiv \frac{\Gamma(q_i\bar{q}_i)}{\Gamma(\mathrm{had})}, \quad R_{\ell_i} \equiv \frac{\Gamma(\mathrm{had})}{\Gamma(\ell_i\bar{\ell}_i)}, \tag{A6}$$

which isolate the weak vertices (including the effects of α_s for R_{ℓ_i}). In (A6) $q_i = b, c, s$; $\ell_i = e, \mu, \tau$; and $\Gamma(\mathrm{had})$ is the width into hadrons. The data are consistent with lepton universality, i.e., with $R_e = R_\mu = R_\tau \equiv R_\ell$. The partial width into neutrinos or other invisible states is defined by $\Gamma(\mathrm{inv}) = \Gamma_Z - \Gamma(\mathrm{had}) - \sum_i \Gamma(\ell_i\bar{\ell}_i)$, where Γ_Z is obtained from the width of the cross section and the others from the peak heights. This allows the determination of the number of neutrinos by $\Gamma(\mathrm{inv}) \equiv N_\nu\Gamma(\nu\bar{\nu})$, where $\Gamma(\nu\bar{\nu})$ is the partial width into a single neutrino flavor. It has become conventional to work with the parameters $M_Z, \Gamma_Z, \sigma_{\mathrm{had}}, R_\ell, R_b, R_c$, for which the correlations are relatively small (but still must be included).

The experimenters have generally presented the Born asymmetries, A^0, for which the off-pole, γ exchange, P_{e^-}, and (small) box effects have been removed from the data. Important asymmetries include:

$$\begin{aligned}
\mathrm{forward-backward:} \quad & A_{FB}^{0f} \simeq \frac{3}{4}A_e A_f, \\
\tau \ \ \mathrm{polarization:} \quad & P_\tau^0 = -\frac{A_\tau + A_e\frac{2z}{1+z^2}}{1 + A_\tau A_e\frac{2z}{1+z^2}}, \\
e^-\mathrm{polarization(SLD):} \quad & A_{LR}^0 = A_e, \\
\mathrm{mixed \ \ (SLD):} \quad & A_{LR}^{0FB} = \frac{3}{4}A_f.
\end{aligned} \tag{A7}$$

The LEP experiments also measure a hadronic forward-backward charge asymmetry Q_{FB}. In (A7), A_f is defined as the ratio

$$A_f \equiv \frac{2\bar{g}_{Vf}\bar{g}_{Af}}{\bar{g}_{Vf}^2 + \bar{g}_{Af}^2} \tag{A8}$$

for fermion f. The forward-backward asymmetries into leptons allow another (successful) test of lepton family universality, by $A_{FB}^{0e} = A_{FB}^{0\mu} = A_{FB}^{0\tau} \equiv A_{FB}^{0\ell}$. In the τ polarization, $z = \cos\theta$, where θ is the scattering angle. The SLD polarization asymmetry A_{LR}^0 for hadrons (or leptons) projects out the initial electron couplings. It is especially sensitive to $\sin^2\theta_W$ because it is linear in the small $\bar{g}_{Ve}$, while the leptonic $A_{FB}^{0\ell}$ are quadratic. The mixed polarization-FB asymmetry A_{LR}^{0FB} projects out the final fermion coupling.

: Appendix BRadiative Corrections

The data are sufficiently precise that one must include high-order radiative corrections, including the dominant two-loop electroweak ($\alpha^2 m_t^4,\ \alpha^2 m_t^2$), dominant 3 loop QCD (and 4 loop estimate), dominant 3 loop mixed QCD-EW, and 2 loop $\alpha\alpha_s$ vertex corrections.

In including EW corrections, one must choose a definition of the renormalized $\sin^2\theta_W$. There are several popular choices, which are equivalent at tree-level, but differ by finite (m_t and M_H dependent) terms at higher order. These include

- On shell: $s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}$,
- Z mass: $s_{M_Z}^2\left(1 - s_{M_Z}^2\right) \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2}$,
- $\overline{\rm MS}$: $\hat{s}_Z^2 \equiv \frac{\hat{g}'^2(M_Z)}{\hat{g}'^2(M_Z)+\hat{g}^2(M_Z)}$,
- Effective (Z-pole): $\bar{s}_f^2 \equiv \frac{1}{4}\left(1 - \frac{\bar{g}_{Vf}}{\bar{g}_{Af}}\right)$.

The first two are *defined* in terms of the Z and W masses; the $\overline{\rm MS}$ from the renormalized couplings $\hat{g}$, $\hat{g}'$; and the effective from the observed vertices. Of course, each can be determined experimentally from any observable, given the appropriate SM expressions. s_W^2 is especially simple conceptually, but the value extracted from Z pole observables has a large m_t and M_H dependence. It (along with $s_{M_Z}^2$) is also awkward in the presence of any type of new physics which shifts the values of the physical boson masses. The Z-pole $\bar{s}_f^2$ depends on the fermion f in the final state. The $\overline{\rm MS}$ definition is especially useful for comparing with theoretical predictions and for describing non Z-pole experiments. The values of $\hat{s}_Z^2$ and $\bar{s}_f^2$ are less sensitive to most types of new physics than the on-shell definitions. The advantages and drawbacks of each scheme are discussed in more detail in [1, 2].

The expressions for M_W and M_Z in the on-shell and $\overline{\rm MS}$ schemes are

$$M_W^2 = \frac{(\pi\alpha/\sqrt{2}G_F)}{s_W^2(1-\Delta r)} = \frac{(\pi\alpha/\sqrt{2}G_F)}{\hat{s}_Z^2(1-\Delta\hat{r}_W)} \tag{B1}$$

and

$$M_Z^2 = \frac{M_W^2}{c_W^2} = \frac{M_W^2}{\hat{\rho}\hat{c}_Z^2}, \tag{B2}$$

where the other renormalized parameters are the fine structure constant α (from QED) and the Fermi constant G_F, defined in terms of the μ lifetime. Δr, $\Delta\hat{r}_W$, and $\hat{\rho}-1$ collect the radiative corrections involving μ decay, M_W, M_Z, and the running of α up to the Z pole. In $\overline{\rm MS}$, $\Delta\hat{r}_W$ has only weak m_t and M_H dependence, and is dominated by the running of α, i.e, $\Delta\hat{r}_W \sim \Delta\alpha + \cdots \sim 0.066 + \cdots$. In contrast, the on-shell Δr has an additional large (quadratic) m_t dependence, which results in a large sensitivity of the observed value of s_W^2 to m_t. The $\overline{\rm MS}$ scheme isolates the large effects in the explicit parameter $\hat{\rho} \sim 1 + \frac{3G_F\hat{m}_t^2}{8\sqrt{2}\pi^2} + \cdots$. The various definitions are related by (m_t and M_H dependent) form factors κ, e.g., $\bar{s}_f^2 = \kappa_f s_W^2 = \hat{\kappa}_f\hat{s}_Z^2$.

The $\overline{\rm MS}$ weak angle $\hat{s}_Z^2$ can be obtained cleanly from the weak asymmetries. Comparison with M_Z and M_W is important for constraining M_H and new physics. The largest theory uncertainty in the $M_Z - \hat{s}_Z^2$ relation is the hadronic contribution to the running of α from its precisely known value $\alpha^{-1} \sim 137.036$ at low energies, to the electroweak scale, where one expects $\alpha^{-1}(M_Z) \sim \hat{\alpha}^{-1}(M_Z) + 0.99 \sim 129$. ($\hat{\alpha}$ refers to the $\overline{\rm MS}$ scheme.) There is a related uncertainty in the hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon. More explicitly, one can define $\Delta\alpha$ by

$$\alpha(M_Z^2) = \frac{\alpha}{1-\Delta\alpha}. \tag{B3}$$

Then,

$$\Delta\alpha = \Delta\alpha_\ell + \Delta\alpha_t + \Delta\alpha_{\rm had}^{(5)} \sim 0.031497 - 0.000070 + \Delta\alpha_{\rm had}^{(5)}. \tag{B4}$$

The leptonic and t loops are reliably calculated in perturbation theory, but not $\Delta\alpha^{(5)}_{\rm had}$ from the lighter quarks. $\Delta\alpha^{(5)}_{\rm had}$ can be expressed by a dispersion integral involving $R_{\rm had}$ (the cross section for $e^+e^-\to$ hadrons relative to $e^+e^-\to\mu^+\mu^-$). Until recently, most calculations were data driven, using experimental values for $R_{\rm had}$ up to CM energies ~ 40 GeV, with perturbative QCD (PQCD) at higher energies. However, there are significant experimental uncertainties (and some discrepancies) in the low energy data. A number of recent studies have argued that one could reliably use a combination of theoretical estimates using PQCD and such non-perturbative techniques as sum rules and operator product expansions down to ~ 1.8 GeV, leading to lower uncertainties. The on-shell evaluations use the new resonance data from BES [20] as further input. The recent estimates, which are in very good agreement, are summarized in [3]. One can also determine $\Delta\alpha^{(5)}_{\rm had}$ directly from the precision fits (Section 2).

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