Consistency bounds on the Higgs-boson mass

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In this talk we consider the modifications induced by heavy physics on the triviality and vacuum stability bounds on the Higgs-boson mass. We parameterize the heavy interactions using an effective Lagrangian and find that the triviality bound is essentially unaffected for weakly-coupled heavy physics. In contrast there are significant modifications in the stability bound that for a light Higgs boson require a scale of new physics of the order of a few TeV.

a. Introduction

The recent LEP bounds on the Higgs-boson mass [1], $m_H > 113.2\text{GeV}$ together with the standard model (SM) upper limit $m_H < 220\text{GeV}$ [2] (which is highly model-dependent) suggest the existence of a light Higgs boson. Should this be the case, the SM stability and triviality bounds strongly favor the appearance of new physics at scales $\lesssim 100\text{TeV}$. In this talk we review the modifications to these bounds generated by new physics at scales below 50TeV.

b. Triviality and Stability

It is known [3] that some theories (e.g. QED and Φ^4) can be defined at all energy scales in ≥ 4 dimensions only if the bare couplings are zero, *i.e.* they are trivial; interacting versions can be defined only by assuming an ultraviolet cutoff Λ. In perturbation theory this corresponds to the appearance of Landau poles in the running couplings. The SM has this property, so that, for each choice of the Higgs-boson mass m_H there is a cutoff scale Λ beyond which the perturbation expansion breaks down. For fixed Λ this leads to an upper bound on m_H [4] with the corresponding conclusions: the SM is weakly coupled for all scales below a cutoff only if the Higgs-boson is sufficiently light.

A lower bound on m_H can also be derived by a different consistency argument, namely, that the SM vacuum be stable, *i.e.* $V_{\text{eff}}(v) < V_{\text{eff}}(\bar{\varphi})$ for all $|\bar{\varphi}| < \Lambda$, where $v \sim 246\text{GeV}$ is determined (for example) by the Fermi constant. This constraint is satisfied only if m_H is sufficiently large leading to a lower bound on m_H [5].

These calculations are done assuming there are no new-physics effects below Λ. In this talk we extend these results [7]: using an effective Lagrangian we parameterize the effects of the new physics at scales below Λ and use this parameterization to determine the modifications in the stability and triviality bounds described above. We will assume that the scale of new physics Λ is $\gg v$, and that the heavy interactions are decoupling and weakly coupled. Finally we assume that chiral symmetry is natural [8]. With these constraints on the new physics, the terms in the effective Lagrangian [6] that affect the bounds on m_H are generated by the gauge-invariant operators [9] ($\mathcal{O}_{qt}^{(1)}$ affects V_{eff} only through RG mixing and its effects are small; other similar operators were not included for this reason.):

$$\mathcal{O}_\phi = \frac{1}{3}|\phi|^6 \qquad \mathcal{O}_{\partial\phi} = \frac{1}{2}\left(\partial|\phi|^2\right)^2 \qquad \mathcal{O}_\phi^{(1)} = |\phi|^2\,|D\phi|^2$$

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$$\mathcal{O}_{\phi}^{(3)} = \left|\phi^{\dagger} D\phi\right|^{2} \qquad \mathcal{O}_{t\phi} = |\phi|^{2}\left(\bar{q}\tilde{\phi}t + \text{h.c.}\right) \qquad \mathcal{O}_{qt}^{(1)} = \frac{1}{2}\left|\bar{q}t\right|^{2}$$

where ϕ denotes the SM scalar doublet, q the left-handed top-bottom isodoublet and t the right-handed top isosinglet. The Lagrangian we use is then $\mathcal{L}_{SM} + \sum_i \alpha_i \mathcal{O}_i/\Lambda^2$ with the coefficients α_i parameterizing the new-physics effects. We also define $\eta \equiv \lambda v^2/\Lambda^2$.

The triviality constraints are then obtained using the evolution equations for the various couplings:

$$\begin{aligned}
\frac{d\lambda}{dt} &= 12\lambda^2 - 3f^4 + 6\lambda f^2 - \frac{3\lambda}{2}\left(3g^2 + g'^2\right) + \frac{3}{16}\left(g'^4 + 2g^2g'^2 + 3g^4\right) \\
&\quad -2\eta\left[2\alpha_{\phi} + \lambda\left(3\alpha_{\partial\phi} + 4\bar{\alpha} + \alpha_{\phi}^{(3)}\right)\right] \\
\frac{d\eta}{dt} &= 3\eta\left[2\lambda + f^2 - \frac{1}{4}\left(3g^2 + g'^2\right)\right] - 2\eta^2\bar{\alpha} \\
\frac{df}{dt} &= \frac{9f^3}{4} - \frac{f}{2}\left(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2\right) - \frac{f\eta}{2}\left(-6\frac{\alpha_{t\phi}}{f} + \bar{\alpha} + 3\alpha_{qt}^{(1)}\right) \\
\frac{d\alpha_{\phi}}{dt} &= 9\alpha_{\phi}\left(6\lambda + f^2\right) + 12\lambda^2(9\alpha_{\partial\phi} + 6\alpha_{\phi}^{(1)} + 5\alpha_{\phi}^{(3)}) + 36\alpha_{t\phi}f^3 \\
&\quad -\frac{9}{8}\left[2(3g^2 + g'^2)\alpha_{\phi} + 2\alpha_{\phi}^{(1)}g^4 + \left(\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)}\right)(g^2 + g'^2)^2\right] \\
\frac{d\alpha_{\partial\phi}}{dt} &= 2\lambda\left(6\alpha_{\partial\phi} - 3\alpha_{\phi}^{(1)} + \bar{\alpha}\right) + 6f\left(f\alpha_{\partial\phi} - \alpha_{t\phi}\right) \\
\frac{d\alpha_{\phi}^{(1)}}{dt} &= 2\lambda\left(\bar{\alpha} + 3\alpha_{\phi}^{(1)}\right) + 6f\left(f\alpha_{\phi}^{(1)} - \alpha_{t\phi}\right) \\
\frac{d\alpha_{\phi}^{(3)}}{dt} &= 6(\lambda + f^2)\alpha_{\phi}^{(3)} \\
\frac{d\alpha_{t\phi}}{dt} &= -3f(f^2 + \lambda)\alpha_{qt}^{(1)} + (\frac{15}{4}f^2 - 12\lambda)\alpha_{t\phi} - \frac{f^3}{2}\left(\alpha_{\partial\phi} - \alpha_{\phi}^{(1)} + \bar{\alpha}\right) \\
\frac{d\alpha_{qt}^{(1)}}{dt} &= (3/2)\alpha_{qt}^{(1)}f^2
\end{aligned}$$

where $\kappa = M_Z \exp(8\pi^2 t)$ is the renormalization scale, and $\bar{\alpha} = \alpha_{\partial\phi} + 2\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)}$. The evolution of the gauge couplings g, g' and g_s (for the strong interactions) is unaffected by the α_i's. These equations are solved using the following boundary conditions: $\alpha_i(\Lambda) = O(1)$ (with various sign choices); $\langle\phi\rangle = 0.246/\sqrt{2}\,\text{TeV}$ (at $\kappa = v$) and, finally, that the W, Z, t, H masses have their physical values. Requiring that the couplings never leave the perturbative regime for $\kappa < \Lambda$ then yields the triviality bound for this extension of the SM. The plots of the running coupling constants and the triviality bounds are given in Fig.1.

The triviality results are indistinguishable from the SM due to our requirement that the model remains weakly coupled; if this is relaxed our conclusions need not hold [10].

The effective potential at one loop is easily obtained from the above Lagrangian. The result is

$$V\text{eff}(\bar{\varphi}) = -\eta\Lambda^2|\phi|^2 + \lambda|\phi|^4 - \frac{\alpha_{\phi}}{3\Lambda^2}|\phi|^6 + \frac{1}{64\pi^2}\sum_{i=0}^{5} c_i R_i^2[\ln(R_i/\kappa^2) - \nu_i] + O(1/\Lambda^4)$$

where $c_0 = -4$, $c_1 = 1$, $c_{2,4} = 3$, $c_3 = 6$, $c_5 = -12$, $\nu_{0,1,2,5} = 3/2$, $\nu_{3,4} = 5/6$, $R_0 = \eta\Lambda^2$ and

$$\begin{aligned}
R_1 &= \lambda(6|\bar{\varphi}|^2 - v^2)\left[1 - (2\alpha_{\partial\phi} + \alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})|\bar{\varphi}|^2/\Lambda^2\right] - 5\alpha_{\phi}|\bar{\varphi}|^4/\Lambda^2 \\
R_2 &= \lambda(2|\bar{\varphi}|^2 - v^2)\left[1 - (\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)}/3)|\bar{\varphi}|^2/\Lambda^2\right] - \alpha_{\phi}|\bar{\varphi}|^4/\Lambda^2 \\
R_3 &= (g^2/2)|\bar{\varphi}|^2\left(1 + |\bar{\varphi}|^2\alpha_{\phi}^{(1)}/\Lambda^2\right) \\
R_4 &= [(g^2 + g'^2)/2]|\bar{\varphi}|^2\left(1 + |\bar{\varphi}|^2(\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})/\Lambda^2\right) \\
R_5 &= f|\bar{\varphi}|^2\left(f + 2\alpha_{t\phi}|\bar{\varphi}|^2/\Lambda^2\right),
\end{aligned}$$

This has the same *form* as in the SM, but with *modified* R_i. Note that V_{eff} is gauge dependent [11] but the effects of this gauge dependence are small since the RG-improved tree-level effective potential is gauge-invariant. This leads to a variation in the Higgs-boson mass

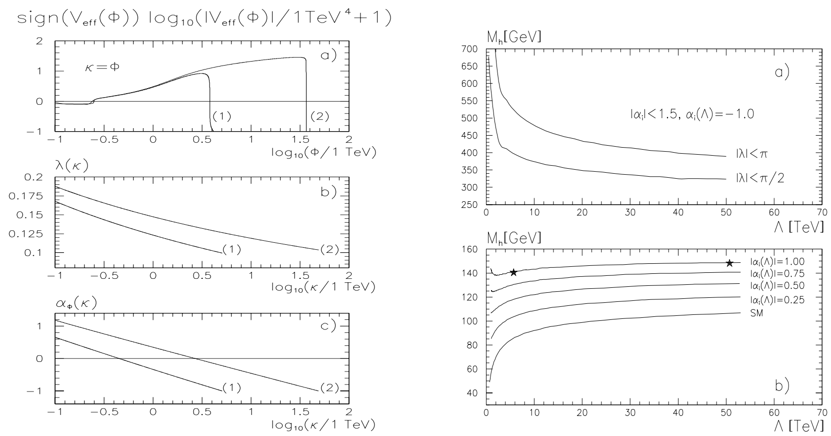

Figure 1: **Left panel:** *(a) $V_{\rm eff}$ at the scale $\kappa = \phi$ as a function of the field strength. The running of λ (b) and α_ϕ (c) when $\alpha_i(\Lambda) = -1$, $m_t = 175$GeV, for $\Lambda = 5.1$TeV, $m_H = 140.4$GeV (curves (1)) and $\Lambda = 48.9$TeV, $m_H = 148.7$GeV (curves (2)).* **Right panel:** *Triviality (a) and stability (b) bounds on m_H for $m_t = 175$GeV. Stars correspond to solutions (1) and (2).*

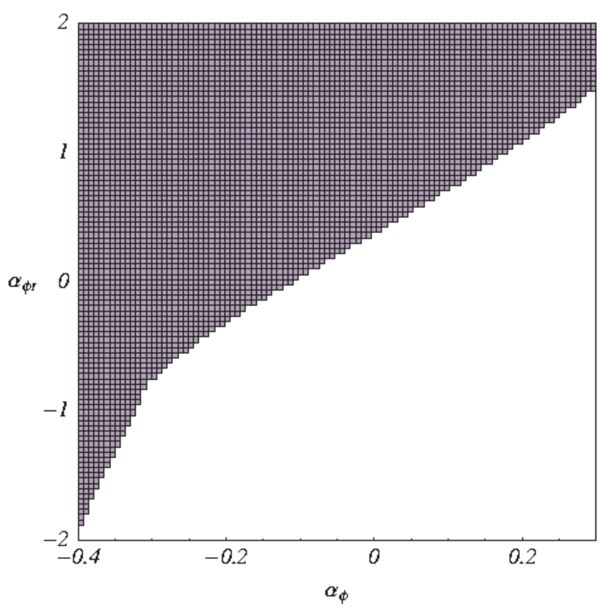

Figure 2: *The unshaded region corresponds to the values of $\alpha_\phi(\Lambda)$, $\alpha_{\phi t}(\Lambda)$ where the effective potential has no SM minimum for fields below 0.75Λ, for any choice of 0.5TeV$< \Lambda < 50$TeV.*

limit: $\Delta m_H \lesssim 0.5$GeV [12]. A plot of the effective potential for some representative values of the parameters is presented in Fig.1. Using the anomalous dimension for the scalar field, $\gamma = 3f^2/2 - 3(3g^2 + g'^2)/8 - \eta\bar{\alpha}/2$, and a careful definition of $V_{\rm eff}(0)$ [13], one can verify that $V_{\rm eff}$ is scale invariant.

In order to insure the stability of the SM vacuum we demand

$$V_{\rm eff}(\bar{\varphi} = 0.75\Lambda)|_{\kappa=0.75\Lambda} \geq V_{\rm eff}(\bar{\varphi} = v_{\rm phys}/\sqrt{2})|_{\kappa=v_{\rm phys}/\sqrt{2}}.$$

The boundary of the stability region corresponds to those values of m_H and Λ that saturate the above inequality. These boundary values are plotted in Figure 1, it is noteworthy that in contrast with the triviality bounds the presence of the effective operators has a significant impact on the stability bounds. For example for a Higgs–boson mass of 115 GeV, $\Lambda \lesssim 4$ TeV for $|\alpha_i| = 0.50$. We also find that the main effects on the stability bound are generated by α_ϕ, $\alpha_{t\phi}$. For example, for α_ϕ large and positive the potential has no minimum for fields below 0.75Λ; more precisely, there is a region in the $\alpha_\phi - \alpha_{t\phi}$, given in Figure 2, where the SM vacuum is either absent or unstable for $\bar{\varphi} < 0.75\Lambda$.

c. Conclusions

The SM triviality upper bound remains unmodified for weakly coupled heavy physics, while the stability bound increases by $\sim$ 50GeV depending on Λ and $\alpha_i(\Lambda)$. For m_H close to its lower LEP limit the constraint on Λ could be decreased dramatically even for modest values of the α_i. These results complement the ones obtained within specific models [14].

Note that, strictly speaking, our expression for $V_{\rm eff}$ is not valid at points where it changes curvature [15]. Still we can make an arguments similar to the one above slightly below the inflection point $|\bar{\varphi}| \sim 0.75\Lambda$; the resulting bounds are essentially unchanged due to the precipitous drop of $V_{\rm eff}$ beyond this point (see Figure 1).

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