Halo formation and Equilibrium in High-Intensity Hadron Rings: The Role of Nonlinear Parametric Resonances Excited by the Intrinsic Beam–Core Oscillations

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Halo formation under a non-equilibrium state for a 2D Gaussian beam in a FODO lattice was examined. Nonlinear resonant-interactions between individual particles and intrinsic beam–core oscillations result in a beam halo. The location of the halo is analytically tractable using canonical equations derived from an isolated resonance Hamiltonian. Halo formation and achievement to equilibrium can be explained by the transition of time-varying nonlinear resonances.

1. Introduction

One of the major issues in high-power hadron accelerators is activation of the environment surrounding an accelerator due to beam loss. Beam loss must be reduced to a sufficiently low level to allow hands-on-maintenance. In order to produce an acceptable design, it is important to understand the mechanisms of emittance growth and halo formation that result in beam loss. From this point of view, halo formation has been studied by simulations and theoretical analyses. Especially, particle-in-cell (PIC) simulation codes [1] and analysis using particle-core-models (PCM) [2] have greatly facilitated the understanding of space-charge effects. In these studies, a resonant interaction between the individual particles and intrinsic beam-core oscillations has been found to be a driving mechanism of halo formation. However, an analysis using PCM has been made on an equilibrium state, where the r.m.s emittance is constant. The beam-property in a non-equilibrium condition, which takes a key role in the resonant interaction of an injected beam, is different from that in equilibrium. Therefore, the PCM should be misleading when a non-equilibrium state is discussed. In addition, it is inaccurate to apply a simulation analysis, such as an FFT analysis and a Poincar map analysis, for a non-equilibrium like that shown in this paper, because these analyses need to track over 100 turns, but the beam distribution varies through the non-equilibrium state in a much shorter time-period.

The purpose of this paper is to examine halo formation under a non-equilibrium condition in a circular accelerator. In this context, we have been developing a useful analytic model, which is based on IRH (Isolated Resonance Hamiltonian). The theory proves that the transition from non-equilibrium to an equilibrium state associated with halo formation can be explained in terms of time-varying nonlinear resonances.

Assumptions concerning the calculations and the example discussed here are noted as follows. The calculations were carried out for 2-D mismatched beams with a Gaussian distribution in a typical FODO lattice. Most of the beam/machine parameters are taken from KEK 12GeV PS, where the injection energy is 500 MeV and the circumference is 340 m. No external nonlinear fields, except for space-charge originated fields, were included in the present calculations. The momentum spread was assumed to be 0%. The combination of bare tunes (ν_x, ν_y) chosen in the present study were close to the operational parameters, (A $(7.123, 5.229)$ and B $(7.203, 5.229)$). In the case of A, a structure resonance due to a space-charge effect in the horizontal direction has been pointed out in past simulation results, but no resonance was shown in the case of B [3].

2. Formalism of an isolated resonance Hamiltonian for a Gaussian beam

The space-charge potential Φ generated by a beam with the Gaussian distribution is written as

$$\Phi(x, y; s) = -\frac{eN}{4\pi\epsilon_0}\int_0^\infty \frac{\left(1 - e^{-\frac{1}{t+2\sigma_x^2}x^2 - \frac{1}{t+2\sigma_y^2}y^2}\right)dt}{\sqrt{t+2\sigma_x^2}\sqrt{t+2\sigma_y^2}}, \tag{1}$$

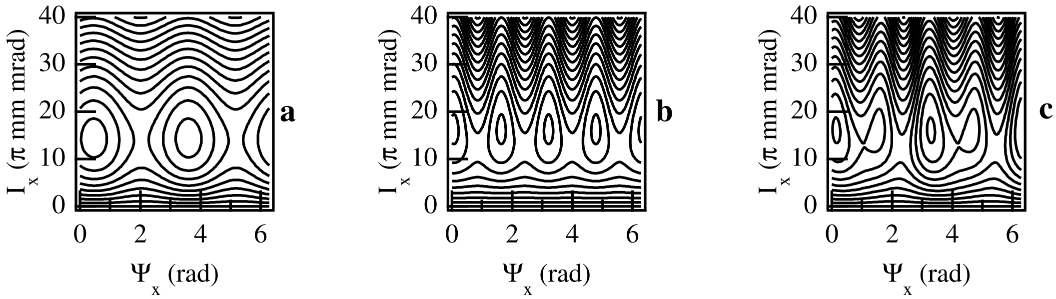

Figure 1: Phase-space structure of H_{iso} in case A. The nonlinear resonance caused (a) mismatching ($imax = 1$, $a = 1$ and $b = 14$), (b) by the lattice structure ($imax = 1$, $a = 2$ and $b = 28$) and (c) the superposition of (a) and (b) ($imax = 10$, $a = 1$ and $b = 14$).

where N is the total number of particles per unit length and ϵ_0 is the permittivity; σ_x and σ_y are the horizontal and vertical r.m.s beam sizes, respectively. By introducing action-angle variables $(\psi_x, \psi_y, I_x, I_y)$ and an independent variable $\theta = s/R_0$ [4], where $x = \sqrt{2\beta_x I_x} \cos(\psi_x + \psi_{0,x})$, $y = \sqrt{2\beta_y I_y} \cos\left(\psi_y + \psi_{0,y}\right)$, R_0 is the averaged orbit radius, β_x and β_y are Twiss parameters, and $\psi_{0,x}$ and $\psi_{0,y}$ are the flutters of the betatron phase with respect to the averaged phase advance of the unperturbed betatron oscillation, the Hamiltonian describing the betatron oscillation perturbed by the space-charge effects is given by

$$H(\psi_x, \psi_y, I_x, I_y; \theta) = \nu_x I_x + \nu_y I_y + \frac{eR_0}{\gamma^2 p v} \Phi(\psi_x, \psi_y, I_x, I_y; \theta), \tag{2}$$

where γ, p and v are the relativistic mass factor, the momentum and the velocity of the on-momentum particle, respectively.

The space-charge potential can be rewritten as the combination of the oscillating terms with the angle variable and the oscillating terms with θ originating from the flutter, r.m.s beam size and Twiss parameter. The parametric nonlinear resonances between an individual particle and the intrinsic beam-core oscillation are known to be excited when the phase of Φ slowly varies. Because the past simulation results have shown nonlinear resonances in the horizontal direction [3], we focus on the lowest slowly oscillating phase, $2a\psi_x - b\theta$, where a and b are integers. The other slowly oscillating phases are given by $i(2a\psi_x - b\theta)$, where i is an integer. The IRH is obtained by averaging the Hamiltonian of Eq. (2) with respect to θ [5]. In this process, rapidly oscillating terms disappear. Furthermore, since $\langle H \rangle$ is not a constant of the motion, the canonical transformation from (ψ_x, I_x) to $(\Psi_x = \psi_x - b\theta/(2a), I_x)$ is made. Finally, we arrive at the IRH describing the parametric nonlinear resonance between the betatron oscillation and the oscillating space-charge forces,

$$H_{iso}(\Psi_x, I_x, I_y) = \left(\nu_x - \frac{b}{2a}\right) I_x + \frac{eR_0}{\gamma^2 p v} \left\langle \Phi(\Psi_x, I_x, I_y) \right\rangle, \tag{3}$$

where $\left\langle \Phi(\Psi_x, I_x, I_y) \right\rangle$ is the time-averaged space charge potential. H_{iso} and I_y in Eq. (3) become constants of motion. Details concerning the evaluation of Eq. (3) will be given in [6].

3. Transition between nonlinear resonances and halo formation

Introducing the realistic time-varying r.m.s beam size under non-equilibrium, which is past simulation results [3], into Eq. (3), the time-varying nonlinear resonances in cases of A and B were examined by the IRH for the early ten turns.

The IRH for case A gives the phase-space structure of the nonlinear resonances, as shown in Figure 1. The beam core is known to oscillate due to both of the lattice structure and mismatching. In Figure 1(a), b is 14, which is the beam core oscillation frequency due to the mismatching per 1 turn. The two resonance islands induced by mismatching are recognized in Figure 1(a). In Figure 1(b), b is 28, which corresponds to the periodicity of the lattice structure. The four

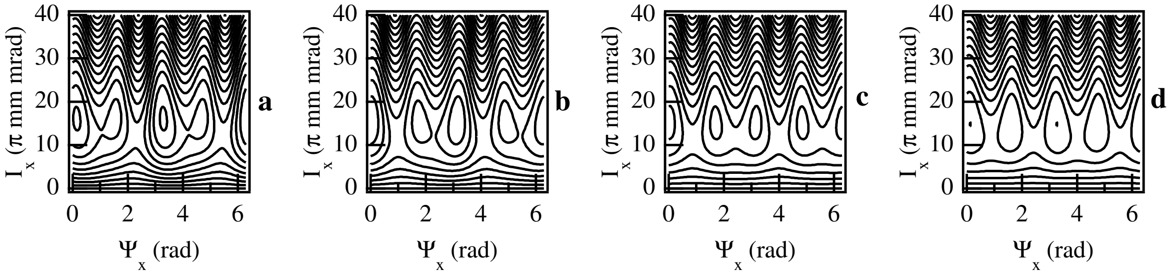

Figure 2: Time varying of H_{iso} in case A. (a) 1st turn, (b) 3rd turn, (c) 5th turn and (d) 7th turn. $imax = 10$, $a = 1$ and $b = 14$.

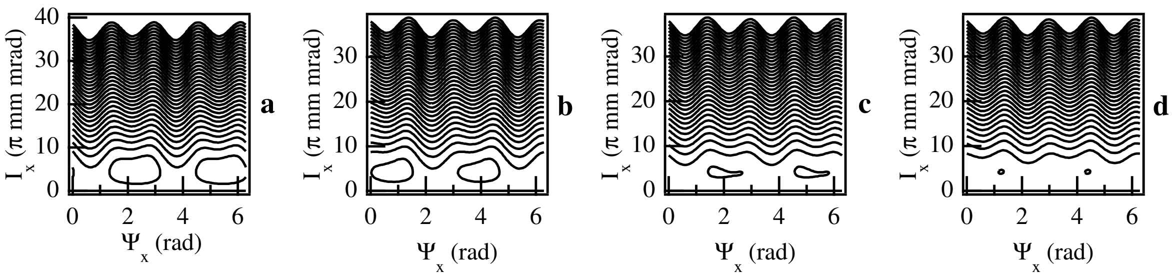

Figure 3: Time varying of H_{iso} in case B. (a) 1st turn, (b) 3rd turn, (c) 5th turn and (d) 7th turn. $imax = 10$, $a = 1$ and $b = 14$.

resonance islands induced due to the lattice structure are confirmed in Figure 1(b). Including the multiple-beam core oscillation, the nonlinear resonances caused by the lattice structure and mismatching overlap, as shown in Figure 1(c).

Next, the IRH for cases A and B was calculated every turn. The phase-space structures for case A are shown in Figure 2. The resonance caused by mismatching is dominant at early few turns because the mismatching remains there. Furthermore, the nonlinear resonance is switched to the structure resonance, after the decay of mismatching due to the growth of filamentation. Thus, the halo tends to grow in the tune pair of case A. The phase-space structures for case B are shown in Figure 3. The resonance caused by mismatching is dominant, similar to that of case A. However, because the condition of the structure resonance is not satisfied since the depressed tune is far from 7, the nonlinear resonance is rapidly lost after decay of the mismatching. The particles moving to the resonance island caused by mismatching are thought to be smeared out due to the nonlinear space-charge fields. Therefore, the beam distribution achieves an equilibrium state.

4. Conclusion

An isolated nonlinear resonance theory has been established to examine halo formation under a non-equilibrium condition in a circular accelerator, which can consistently explain the phase-space dynamics from the early stage of injection to arriving at the equilibrium state. The isolated nonlinear resonance Hamiltonian has been proved to be a useful tool to estimate the position and size of the halo, which is quite important in a practical sense. It has been concluded that the halo is driven by a time-varying nonlinear resonance excited by the intrinsic beam core oscillation at the non-equilibrium state; in addition, the beam distribution achieves an equilibrium state through the decay process of the nonlinear resonances.

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