Helical Channels for Longitudinal Compression of Muon Beams

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A key ingredient in producing muons beams appropriate for a collider is a reduction of the longitudinal emittance, which is not possible through ionization cooling without introducing some coupling between longitudinal and transverse motion. Typical schemes involve creating dispersion, and introducing a wedge-shaped absorber in the channel. Here, we summarize the results of simulations using helical wiggler channels.

1. Helical Wigglers for Emittance Exchange

Current designs for muon cooling channels are successful at reducing the transverse emittance of the beam, but generate incidental increases of the longitudinal emittance, which eventually cause particle losses as particles fall out of the RF bucket. Longitudinal cooling would be helpful in curing these losses, and in addition is necessary for achieving beams that can be used in a muon collider experiment [1]. Typically, the longitudinal cooling is achieved without altering the combined 6D cooling rate, so that either transverse emittance is increased in a distinct emittance exchange section, or the transverse cooling rate is reduced in a 6D cooling channel. For helical wigglers, both cases have been tried, and are discussed below.

Helical wigglers are composed of a solenoid (with field B_0) plus a rotating dipole field (of magnitude B_W). The dipole field rotates with a period L_W, and we define $\kappa_W = 2\pi/L_W$. Then the magnetic field is given by

$$\begin{aligned} B_r &= 2B_W I_1'(\kappa_W r)\sin(\phi - \kappa_W z) \\ B_\phi &= 2B_W[I_1(\kappa_W r)/\kappa_W r]\cos(\phi - \kappa_W z) \\ B_z &= B_0 - \kappa_W r B_\phi, \end{aligned} \tag{1}$$

where I_1 is the modified Bessel function, and $I_1(x) \approx x/2$ for small x. The stable orbit corresponds to a fixed radius at phase $\phi - \kappa_W z = 0$ or π. The stable orbit radius varies with energy, roughly as $v_\perp \sim B_W/\gamma$ and so $r \sim B_W/(\kappa_W\gamma)$, yielding radial dispersion. The only resonance occurs when the cyclotron motion is synchronous with the rotating dipole field, corresponding to $P_R = eB_0L_W/2\pi$; the dispersion increases as this resonance is approached, allowing for large dispersions with reasonable magnetic fields. A slow ramp of the rotating dipole field yields a straightforward matching of the beam in and out of the wiggler field. For momenta below resonance, there is an interesting feature of helical wiggler channels, that the forward velocity reaches a maximum value and then decreases as the momentum approaches resonance; this is referred to as a "negative mass" regime. Near the point of maximum velocity, there is a broad range of energies with nearly identical forward velocity. For this configuration, no RF should be needed to maintain the bunched beam, or alternatively the RF could be tuned on crest for maximum acceleration.

The results for one of the helical channels used below are plotted. The parameters are $B_0 = 3.2$ T, $L_W = 2.5$ m, and $B_W = 0.2$ T; to gain a sense of variation with wiggler field and how the beam behaves in the adiabatic ramp, the radius is also given for $B_W = 0.1$ T. In Fig. 1, the radius of the fixed helical orbit is given as a function of muon momentum. The focussing of the beam can be approximated as deriving from the uniform solenoid field alone; close to resonance, however, there is less focussing and more nonlinear motion. The dispersion varies from 20 to 60 cm in the region of interest, while the orbit radius remains below 20 cm. Above resonance, dispersions remain below 20 cm until very close to resonance, so channels are operated at momenta below P_R.

Specialized diagnostics have been developed for post-processing of simulation results. These include a "corrected energy," removing energy-amplitude correlations, and corresponding emittances. In addition, the energy dispersion vector is calculated, as well as a radial dispersion which is of particular use in helical wiggler channels.

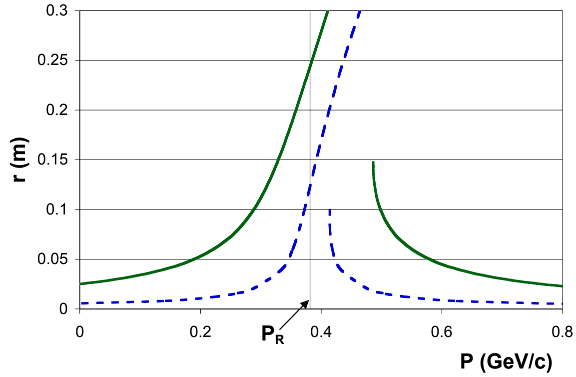

Figure 1: Radius of stable orbit as a function of momentum for dipole field = 0.2 T (green, solid curve) and 0.1 T (blue, dashed curve). The resonant momentum P_R is indicated as well.

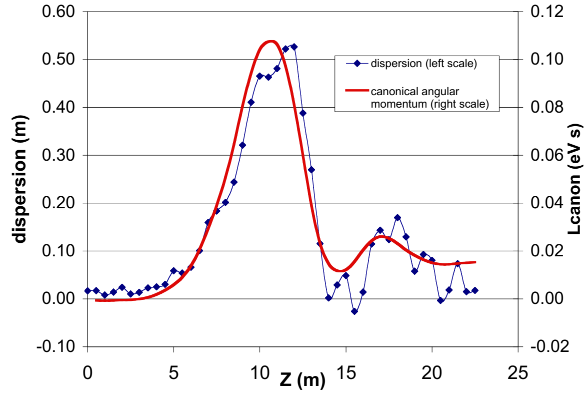

Figure 2: Dispersion and net canonical angular momentum of beam in helical wiggler channel.

2. Results—Emittance Exchange Only

In this example, the above helical wiggler channel is used, with a 22.5 m long channel, 2/3 of which is used for the ramp in and out of the wiggler field. The initial beam has a momentum of 300 MeV/c but otherwise roughly corresponds to the beam at the end of the cooling channel for the Brookhaven Feasibility Study for a Neutrino Factory [2], with transverse emittance of 2.1 mm and longitudinal emittance of 28 mm, and is matched into a 201 MHz RF bucket. The beam energy is chosen to lead to nearly isochronous motion in the helical wiggler, although the forward velocity still has nonlinear dependences. No RF is used in the channel, which has the additional benefit of reducing the necessary aperture for the helical wiggler channel; a 40 cm diameter beampipe suffices to contain the beam, and the magnets can be located just beyond the beampipe. Thin, radially symmetric wedges of beryllium are placed uniformly in the channel in between the ramps, with a minimum radius of 6 cm.

This channel was simulated using ICOOL [3]; at the end of the simulation, the average momentum decreases from 300 to 270 MeV/c, and the longitudinal emittance decreases from 28 to 18 mm. The energy spread is reduced even further, from 24 MeV to 12 MeV. The beam is not compressed in time and thus will have to be rematched into the downstream RF system. Beam losses, including decay, amounted to 5%.

The transverse emittance increases from 2.1 to 4.6 mm; thus the 6D emittance grows by a factor of 3.2, whereas it should ideally remain constant. Multiple scatter and imperfect matching are responsible for most of this emittance growth; however, the reduction in longitudinal emittance appears to be remain at a factor of two regardless of matching. This is because the dispersion in the beam does not remain at the matched value, but instead rapidly decreases once the energy spread has been reduced significantly. In Figure 2, the loss of dispersion is seen to be correlated with the elimination of the angular momentum imparted to the beam by the wiggler field; this suggests that either reversals of the solenoid field or reacceleration would be necessary to obtain further emittance exchange. However, putting RF cavities into the channel would require much larger apertures, and would naturally lead to attempts to cool the beam simultaneously with emittance exchange.

3. Results—Combined 6D Cooling

A combined 6D cooling channel, based on adding rotating dipole fields to a transverse cooling section using a uniform solenoid field of 5 T, with an aperture having 1.4 m diameter in order to contain RF cavities at 201 MHz. The channel was designed by V. Balbekov [4] and simulated using GEANT4 [5] by V.D. Elvira *et al.* [6]. Triangular wedges made of lithium hydride were placed between the RF cavities, and situated so that the RF would have to be phased at 60 degrees off crest to maintain the average beam momentum, yielding significant transverse cooling. The rotating dipole field had a strength of 0.3 T and period 1.8 m. The total length of the channel was 72 m,

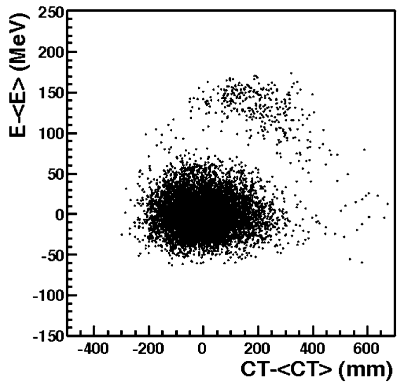

Figure 3: Longitudinal particle distribution at end of cooling channel using helical wiggler.

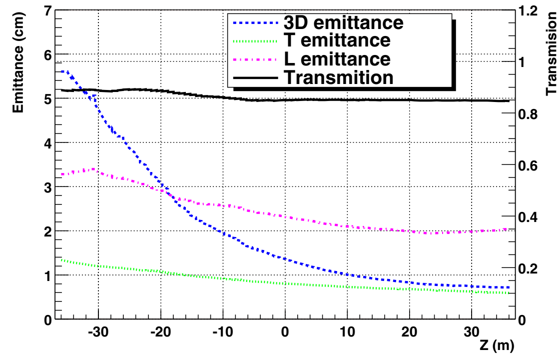

Figure 4: Beam emittances in combined transverse and longitudinal cooling channel using helical wiggler.

including 14.4 m ramps in and out of the rotating dipole field. The only idealized aspect of the simulation is the RF, which was replaced with a single thin, high-voltage section. With room for 75 cm cavities, and taking transit time effects into account, the gradient assumed was roughly 18 MV/m.

The reference momentum of the beam was 240 MeV/c, and the initial beam had transverse emittance 15 mm and longitudinal emittance 46 mm. The reference momentum is below the point of maximum forward velocity, and a secondary RF bucket at higher momentum but the same velocity is observed in Fig. 3 to become occupied by the end of the simulation. The results are shown in Fig. 4; note that due to post-processing cuts, the initial beam emittances are not accurate in the graph. The final emittances were 5.9 mm transverse and 20 mm longitudinal. The initial rms longitudinal emittance was overstated because of nonlinear correlations; subtracting these out yields an emittance of 23.7 mm, leading to a 6D cooling factor of 7.5. This is close to ideal given the total amount of reacceleration. Beam losses were 15%.

4. Conclusions

Emittance exchange channels using helical wigglers have been tried both as adjuncts to transverse cooling, and as combined systems to reduce all beam emittances simultaneously. These systems generally rely on creating dispersion in the beam, and placing wedge-shaped absorbers in high-dispersion regions. Both simultaneous cooling in all dimensions for a large beam, and pure emittance exchange for a smaller beam have been demonstrated. So far, results have been limited to a factor of 2 or 3 in longitudinal cooling.

Detailed simulations of emittance exchange channels are now routinely being performed, and the tools are in place to effectively interpret the results. These channels are an encouraging step in the development of useful longitudinal cooling that can lead to beams appropriate for muon colliders. Furthermore, the cooling performance achieved thus far could be implemented in a neutrino factory design for cheaper acceleration of the final beam, or to reduce particle losses.

Acknowledgments

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References

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