

An Alternative Scheme for the Neutrino Factory with a High Power Proton Driver

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We describe a scheme to produce an intense and collimated beam of neutrinos for the neutrino-oscillation experiment. The scheme feature is the presence of a Proton Driver that generates a proton beam at very large power ($10 \text{ mA} \times 15 \text{ GeV}$), considerably higher than that proposed elsewhere for this application. With this scheme, because of the high intensity of the proton beam, to produce neutrinos at the same required rates, it is sufficient to collect π and μ mesons only around a small angle and at reduced momentum spreads. This eliminates the need for the difficult longitudinal manipulations of the protons and mesons, and of the ionization cooling that still needs to be demonstrated. It is also shown, at the end of the paper, that the Neutrino Factory here proposed can also be used as an injector for a $1 \times 1 \text{ TeV}^2 \mu^+ - \mu^-$ collider at large luminosity.

1. Introduction

A Collaboration team, composed of several institutions, has put forward a proposal [1] for a Neutrino Factory that should be located at Brookhaven National Laboratory. The intent is to aim the Neutrino Beam at some location in New Mexico. The production rate of the neutrinos is about $2 \times 10^{14}/\text{s}$, centered around the use of a Proton Driver with an average power of 1 MW [2]. Because of the relatively low intensity of the primary proton beam, and the fact that the beam from the AGS is bunched at low frequency, to arrive at the prefixed production rate, complicate proton bunch manipulation, that includes stacking and compression, are required. Also the operation of the proton source is pulsed with a low duty cycle that causes difficulties to the stability of the target, that has then be taken to be liquid mercury. To accept as many meson particles as possible, large angular and longitudinal acceptance of the secondary particles are required with complicate production and transport channel. Also the secondary beams need to be bunch-rotated before final acceleration. The large transverse spread is problematic and one needs to resort to some cooling, like ionization cooling, that though still needs to be demonstrated.

To avoid most of these problems, the scheme proposed in this report makes use of a novel concept of Proton Driver that generates a continuous proton beam at much higher power (150 MW versus 1 MW of the Collaboration Proposal). Moreover, the protons are accelerated in a sequence of Super-Conducting Linacs, and the beam in exit is bunched at high frequency. The proton bunches have then small dimensions and spreads. RF manipulation of the proton bunches is not required, and target stability problems are not a concern. At production and during transport, the π and μ mesons have the same high frequency bunching of the proton beam. Considering the relatively small dimensions also of the μ -beam, this is directly transferred and accelerated in a sequence of Super-Conducting Linacs to 32 GeV. Ionization cooling to reduce the beam emittance and momentum spread is then not needed. Also not needed is further bunch compression and rotation of the μ -bunches.

The proposed Neutrino Factory can also be used as an injector to a $1 \times 1 - \text{TeV}^2 \mu^+ - \mu^-$ collider. After acceleration to 32 GeV, the muons of both signs are accelerated further in a more efficient Super-Conducting Linac to 1 TeV. The mesons with opposite sign are separated and transferred into a common storage ring where they circulate with a lifetime of 20 ms. Multi-turn stacking of the bunches is done only with momentum painting to preserve the initial small emittance, and to reduce the number of circulating bunches. A luminosity of about $1 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ is expected.

2. Neutrino Production

The neutrino production follows the following chain of reactions and decays:

$$p + X \rightarrow \pi^\pm \rightarrow \mu^\pm \rightarrow \nu.$$

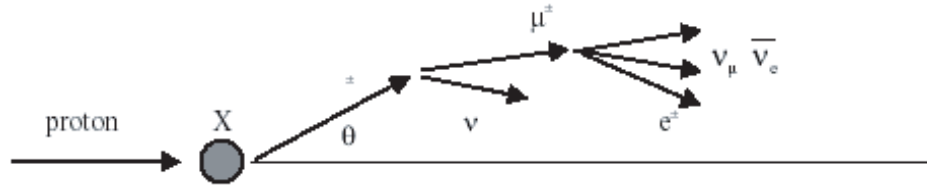


Figure 1: Chain of reactions and decays for the neutrino production

As shown in Figure 1, a proton of momentum p_0 impinges on a heavy nucleus X . From the scattering, a π^\pm meson (mass at rest $m_\pi = 139.5675$ MeV) is produced with momentum p_π and at an angle θ_π from the direction of incidence. The meson decays with a lifetime (at rest) of 2.603×10^{-8} s into a neutrino ν and a μ^\pm meson. In turn the μ^\pm meson (mass at rest $m_\mu = 105.6595$ MeV), that was produced with momentum p_μ , decays in $e^\pm + \nu_\mu + \bar{\nu}_e$ with a lifetime (at rest) of 2.1971×10^{-6} s.

For the neutrino-oscillation experiment, used to determine the neutrino mass with high precision, the neutrinos are required to travel a long distance of several thousand of kilometers. It is then important for this experiment that the neutrino beam is produced well collimated to avoid too much angular dispersion at the arrival point. Because the π^\pm mesons are produced with a large transverse momentum spread, the neutrinos from their decay are thus not very useful. Moreover, the π^\pm mesons have a too short lifetime because one can act on them to reduce their angular spread before they decay. On the other side the lifetime of the μ^\pm mesons, if their energy is large enough, is sufficiently long to operate on them to reduce their transverse spread. Conversely then, it is hoped that the two types of neutrino from the μ^\pm decay form a beam collimated enough to be useful at the long required distance.

The assumption made here is that of course the neutrino mass, if any, is very small and in principle negligible for the estimate of the kinematics of the reactions and decays of Figure 1 where scattering angles have been exaggerated. The angular distribution and momentum spectrum of the π^\pm mesons [3] are shown in Figures 2 and 3. We shall assume, for simplicity, that the μ^\pm mesons will have also about the same distribution and that they follow closely the “footprint” of the π^\pm -meson beam.

3. An alternative Scenario based on a High-Power Proton Driver

The scenario we are here proposing for the Neutrino Factory is sketched in Figure 4. It is made of four parts:

1. a 15-GeV Proton Driver (PD),
2. a $\pi-\mu$ Production Channel ($\pi\mu$ PC), that is a solid target immediately followed by a transport channel made of a super-conducting 20-T solenoid magnet where the π mesons decay and the μ mesons are produced,
3. an accelerating section consisting of a 2-GeV SCL with two re-circulating SCLs (μ SCL) for the acceleration of the μ mesons to 32 GeV, and
4. a 32-GeV muon Storage Ring (μ SR), where the μ mesons circulate until they decay in neutrinos.

No ionization cooling, nor bunch rotation and/or compression of the primary proton beam and of the μ -beam are here required. The main feature of the proposed scheme is the high power of the Proton Beam.

The rate of μ -production dn/dt is in linear approximation proportional to the proton beam intensity I_p , to the momentum bites Δp_π and Δp_μ , and to the square of the transverse angle θ_π accepted of the p -mesons, according to

$$dn/dt = \alpha(p_0, p_\pi, p_\mu, \theta_\pi) I_p \Delta p_\pi \Delta p_\mu \theta_\pi^2 \quad (1)$$

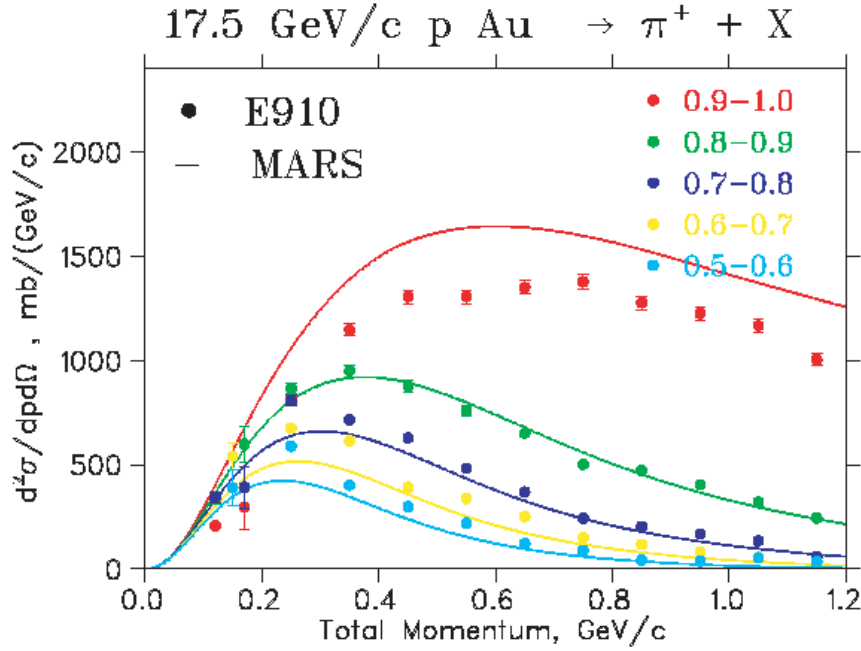


Figure 2: Angular and momentum distribution of the π^+ -meson production.

where $\alpha(p_0, p_\pi, p_\mu, \theta_\pi)$ is a constant that depends on the momentum p_0 of the primary protons, on the central momentum values p_π and p_μ of the collected p and μ mesons, and on the angle θ_π of the production direction of the π^\pm mesons. The constant can be estimated by inspection of Figures 2 and 3. According to Equation (1), in order to obtain the same rate of production one can trade momentum spreads and angular acceptance with proton beam intensity. As an example, proton-beam intensity by a factor 150 higher than that of the Collaboration Proposal [1] (that is 150 MW versus 1 MW) yields the same production rate with a momentum bite 10 times smaller and an angular spread also 4 times smaller. At the same time, as it is possible to see from Figures 2 and 3, by accepting a smaller angle, one optimizes the production, with a proper choice of the central momenta, by eliminating the need to collect particles at the edge of the distribution where the collection is less efficient and makes the beam excessively too large and diluted.

4. The Proton Driver

The Proton Driver (PD) is sketched in Figure 5. It is made of a 2-GeV proton Super-Conducting Linac (pSCL) followed by a re-circulator where the proton beam makes four passes through 3 segments of SCL each for an energy gain of 1 GeV. The final energy is 15 GeV and the average beam intensity is 10 mA (positive ions). The average proton power in exit is thus 150 MW. The proton beam power is essentially determined by the final energy, whereas the beam intensity is relatively modest and well within present technology. Moreover the Proton Driver is operating in a Continuous Wave mode and the proton beam is bunched at 200 MHz. The injector 2-GeV SCL has a front-end operating at 201.25 MHz, followed by an intermediate section at 805 MHz, and a final section at 1.61 GHz. Thus the beam is bunched at 200 MHz and is made of one bunch every eight rf buckets in exit of the injector Linac. The three sectors that make the re-circulator are all operating at 1.61 GHz. The main parameters of the Proton Driver are given in Table 1. In particular, the beam emittance, dimension, and momentum spread in exit are very small. Because the beam intensity in the Linac is relatively low, no space-charge effects are significantly expected. The concern with the possible formation of a beam halo is removed as well as the consequences on activation caused by uncontrolled beam losses. At the end of the traversal of each Linac section of the re-circulator the beam velocity is always very close to the speed of light, and the penalty on

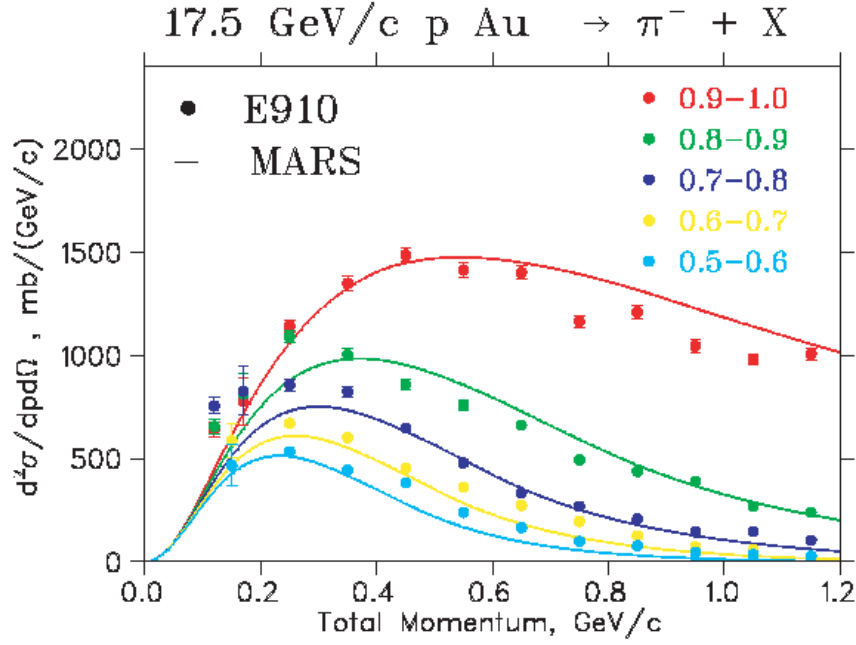


Figure 3: Angular and momentum distribution of the π^- -meson production.



Figure 4: Neutrino Factory

the acceleration rate from the transit-time factor by keeping the cavity cell length fixed, adjusted to exactly a half of the rf wavelength, is minimal.

5. Targetry

The proton beam hits a solid target, for instance made of gold or tungsten; a liquid mercury target is not required for this scheme. The proton beam in exit of the Driver is already bunched at 200 MHz with a spacing between bunches of about 150 cm. The proton bunches have very small length and momentum spread. Also the beam transverse cross-section is small. Thus, the beam does not need to be manipulated further. Since otherwise the beam is continuous, no thermal shock effects on the target are expected. At the turn-on of the operation the beam intensity will be gradually raised at a tolerable rate without creating thermal stress. It is indeed a fact that, as it was determined from studies done for the Accelerator Production of Tritium (APT) project [4], a solid target can absorb considerably more beam power in the continuous mode of operation than in pulsed mode at low duty cycle.

Yet, even in the continuous mode of operation, there is a limit to the energy deposition on the target that we take here to be 150 kW/cm^2 . A solution that allows spreading the proton beam over the target, and, at the same time, re-constructing the small cross-section of the π -meson beam is shown in Figure 6. Two dipole magnets B1 and B2 are located at the upstream side of the target

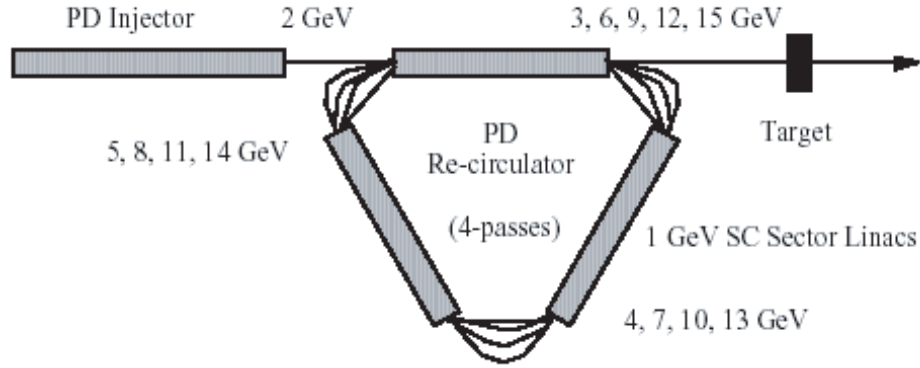


Figure 5: 15-GeV, Proton Driver (150-MW, 10-mA CW)

Table I Proton Driver Parameters

Mode of Operation	Continuous Wave (CW)
Final Kinetic Energy	15 GeV
Average Proton Current	10 mA
Average Proton Power	150 MW
SCL-Injector Energy	2 GeV
SCL-Injector Frequency	201.25 MHz to 116 MeV 805 MHz to 400 MeV 1.61 GHz to 2 GeV
Number of Passes in SCL-Sectors	4 (5 in the first Linac Sector)
Energy Gain for each Pass	1 GeV
Frequency of SCL-Sectors	1.61 GHz
Extracted Beam Emittance	0.3π mm-mrad (rms, norm.)
Extracted Bunch Area	1.0π MeV-degree at 1.61 GHz
rms Momentum Spread	$< 1 \times 10^{-4}$
rms Bunch Length	< 1 mm
Beam Bunching Frequency	201.25 MHz, 1 bunch every 8 rf buckets at 1.61 GHz
Bunch Spacing	150 cm

separated by about 10 m. Both magnets have the length of about one meter and capable of an excitation of ± 7.5 kG. The magnet power supply is oscillating at the frequency of 1 MHz, so that the beam entering the system is swept across the cross-section of the target over a squared area of 33×33 cm². The sweeping actually occurs both horizontally and vertically. At this purpose the magnet system is duplicated with another pair (not shown in the Figure 6) oriented perpendicular to each other. Assuming that the target length is 30 cm, the target traversal time by the beam is about one nanosecond, and the total target area is swept in about one microsecond so that any region of the target does not receive more than 0.1% of the total beam power, that is 150 kW/cm^2 . Beforehand, though, and simultaneously, the beam is focused on the target to an amplitude value $\beta^* = 150$ m, that yields locally a beam spot size of 1.7 mm (rms value). The same operation, but with the sequence inverted, is done on the other side of the target, the receiving end of the π^\pm mesons coming out of the target. A similar sweeping system of magnets, B3 and B4, are located at the downstream end of the target. The beam in exit is counter-swept at the same frequency of 1 MHz and the beam image re-constructed at the exit of the B4 magnet as shown in Figure 6. At the exit of the target the π^\pm mesons enter the solenoid magnet. Because the π^\pm mesons have a considerably lower momentum than the primary protons, it will be easier to bend them, and the second half of the bending system, to the right of the target, is considerably easier to conceive, design and build. The magnet system at the entrance, of course, represents a more challenging

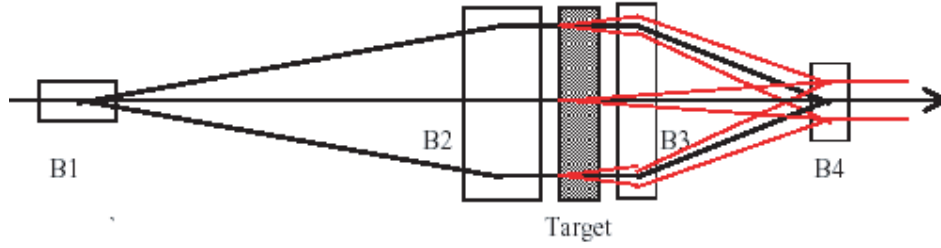


Figure 6: Targetry of the protons and refocusing of the p^\pm mesons with sweeping magnets.

technical difficulty; but it should be possible to build magnets operating with an oscillating power supply at the 1-MHz frequency. Of course, other solution may be possible and can be proposed.

6. The $\pi - \mu$ Production Channel

We shall denote with $d^2\sigma_\pi/dp_\pi d\Omega_\pi$ the differential cross-section for the production of the π^\pm -mesons. An inspection of Figures 2 and 3 shows that $d^2\sigma_\pi/dp_\pi d\Omega_\pi$ 1.5 barn/(GeV/c) for both signs, around the central momentum value p_π 600 MeV/c, and at production angles θ_π having $0.973 < \cos\theta_\pi < 1$; that is $0 < \theta_\pi < 0.233$ rad. We shall accept all the particle momenta in the range $500 < p_\pi < 700$ MeV/c. The resulting total cross-section is then $\sigma_\pi = 0.051$ barn. The production rate of the π^\pm mesons of both sign is given by

$$dn_\pi/dt = 2\sigma_\pi \ell \rho I_p / e \quad (2)$$

where ℓ 30 cm is the target length, $I_p/e = 6.25 \times 10^{16}/s$ is the proton intensity, and ρ 6×10^{22} nucleus/cm³ is the density of the target material (Au). The total production rate of the π^\pm mesons is thus $dn_\pi/dt = 1.12 \times 10^{16}/s$, that is an yield of about $0.18\pi/p$.

The target is located inside a super-conducting solenoid magnet with a field $B = 20$ Tesla that will trap and transport the π^\pm -mesons produced. Inside the magnet the mesons performs helical trajectories with a transverse radius a_π directly related to the production angle θ_π according to the relation

$$a_\pi = 0.3 p_\pi \theta_\pi / B \quad (3)$$

with a_π in mm, B in Tesla, p_π in GeV/c, and θ_π in mrad. With our parameters the radius a_π varies between 1.75 and 2.45 mm. The period of the helical trajectories is 63 cm for the central momentum. Because of their initial divergence, the π -meson beam in exit of the target has a large transverse dimension, equal to the full width of 7 cm with a 30 cm long target, larger than the original proton beam size. The size of the π^\pm -meson beam is thus still reasonable small, and can be accommodated in a sufficiently small magnet aperture. To avoid activation and unnecessary beam loss, the internal diameter can be set to 30 cm. Of course, inspection of Equation (3) shows that there is a large range of values where to trade one parameter with respect to the others. The total emittance of the captured π^\pm -meson beam is estimated to be $\epsilon_\pi = 16\pi m - rad$ at the exit of the target. To this one should add the contribution of the multiple Coulomb scattering of the electrically charged mesons in the target, that raises the total expected emittance of the π -meson beam to $\epsilon_\pi = 23\pi m - rad$.

The decay length of the π -mesons into μ -mesons varies between 28 and 40 m. We adopt the length of 35 m for the transport in the solenoid magnet. Thus about 63% of the π^\pm mesons will have had a chance to decay. At the moment of production the π^\pm -meson beam has the same longitudinal distribution of the beam of the primary protons. At end of the transport, each bunch is tilted because of the velocity difference between the top and the bottom of the momentum distribution. This will cause a total final extension of each bunch of 63 cm, less than half of the bunch spacing. The momentum spread still ranges between 500 and 700 MeV/c. The parameters of the π mesons production are summarized in Table II.

Table II Parameters of Production of the π Mesons

Target Material	Gold (Au)
Target Length	30 cm
Central Momentum	600 MeV/c
Momentum Range	500-700 MeV/c
Angular Acceptance	0.233 rad
Differential Cross-Section	1,500 mb/(GeV/c)
Total Cross-Section	51 mb
Production Rate	$1.12 \times 10^{16} \pi/s$
Yield	0.18 π /proton
Solenoid Field	20 Tesla
Length of Production Channel	35 m
Beam cross-section Radius	3.5 cm
Total Emittance	23 π m-rad
Bunch Spacing	150 cm
Final Long. Extension	± 31 cm

7. μ -Production

Each π meson decays in a μ meson essentially directed forward. The momentum distribution of the μ^\pm mesons is about uniform in the range from 0.6 and 1.0 of the π^\pm mesons momentum value. We shall accept μ^\pm mesons with a central value of 450 MeV/c and a full momentum bite of 22.5 MeV/c (5%). The distribution of the μ^\pm mesons in the transverse phase space follows closely the “footprint” of the π^\pm mesons with a normalized (full) emittance of 27 π m-rad in both planes. To this we should also add the contribution from multiple Coulomb scattering that raises the final μ -meson beam emittance to 27 π m-rad. The longitudinal distribution preserves the original bunching at 200 MHz, and each μ -bunch has the same total extension (63 cm) of the π -bunches at the moment of their decay. The full bunch area is thus expected to be about 0.024 π eV-s, with an energy spread of ± 10 MeV. The μ production rate is finally $0.88 \times 10^{15} \mu/s$, that is a yield of 0.014 μ/p . The yield is about a factor 40 lower than that of the corresponding design of the Collaboration Proposal [1]; but since the proton beam intensity is a factor 150 times larger, actually the overall μ production rate is about a factor 4 higher. At the same time one has avoided, with the present scheme, difficult bunch manipulation and rotation, and ionization cooling. During production in the transport channel, the maximum Larmor radius of the μ -beam cross-section is 5.6 mm, but, like the original π -meson beam, also the μ -mesons spread over a full width of 7cm. Thus it fits well within the available physical aperture of the solenoid magnet. The main parameters of the μ -production are summarized in Table III.

Table III Parameters of Production of the μ Mesons

Central Momentum	450 MeV
Momentum Bite (full)	22.5 MeV
Momentum Spread (full)	5%
Energy Spread	± 10 MeV
Production Rate	$0.88 \times 10^{15} \mu/s$
Yield	0.014 μ /proton
μ -Beam cross-section Radius	3.5 cm
Normal Emittance (full)	37 π m-rad
Bunch Separation	150 cm
Bunching Frequency	201.25 MHz
Bunch Final Extension	± 31 cm
Bunch Area (full)	0.024 π eV-s

8. μ -Acceleration

The interesting feature of the scheme proposed here for the Neutrino Factory is that the μ beam at the end of the transport in the solenoid magnet has dimensions and spreads small enough that it can immediately be injected in a Super-Conducting Linac (μ SCL). The Linac starts with a frequency of 200.125 MHz to capture the beam bunches when they have large longitudinal spreads as at the moment of production. As during acceleration the bunches are shortened, the rf is then switched to 805 MHz for a more efficient acceleration. Also, during the first section of the μ SCL, rf cavities are separated by super-conducting solenoid magnets to provide focusing and the transverse confinement of the beam. In a later accelerating section, after the beam transverse emittance has been sufficiently reduced by adiabatic damping, the solenoid magnets are replaced with ordinary quadrupole magnets. There is no need of bunch rotation or other rf manipulation before final acceleration. There is also no need of ionization cooling since the beam dimensions are already satisfactorily small. Eventually, if necessary, because of the relatively large transverse dimension, the first section of the accelerating device can be an Induction Linac that provides larger transverse aperture and can simultaneously add acceleration and transverse focusing. Then this will be followed by a SCL proper. Actually, it may be possible that the entire acceleration can take place in a SCL from start; a solution that we mostly prefer. The final energy is 32 GeV when the momentum spread is less than 1%, the bunch length of few centimeters, and the actual full betatron emittance 122π mm-mrad.

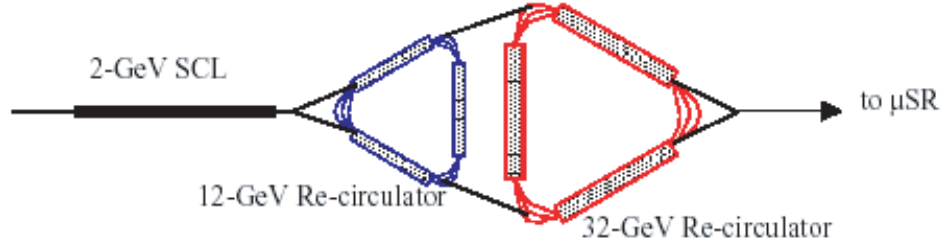


Figure 7: μ -Acceleration to 32 GeV

Table IV Parameters of the Muon SC Linacs

Mode of Operation	Continuous Wave (CW)
Final Kinetic Energy	32 GeV
Average Muon Current	$2 \times 0.14\text{mA}$
Average Muon Power	$2 \times 4.5 \text{ MW}$
SCL-Injector Energy	2 GeV
SCL-Injector Frequency	201.25 MHz to 800 MeV; 805 MHz to 2 GeV
Energy of First Re-Circulator	12 GeV
Energy of Second Re-Circulator	32 GeV
Number of Passes in SCL-Sectors	3 (4 in the first Linac Sector of each Re-Circulator)
Energy Gain for each Pass	1 GeV for 1st Re-Circulator; 2 GeV for 2nd Re-Circulator
Frequency of Re-Circulator Linacs	1.61 GHz
Extracted Beam Emittance	122π mm-mrad (full)
Extracted Bunch Area	0.024π eV-s
Momentum Spread	$\pm 3 \times 10^{-3}$
Bunch Length	± 0.2 ns
Beam Bunching Frequency	201.25 MHz
Bunch Spacing	150 cm

The layout of the μ SCL is sketched in Figure 7 and follows closely the design of the Collaboration Proposal [1]. There are three parts: (1) a 2-GeV SCL, followed by (2) a three-sided re-circulator for acceleration to 12 GeV, and (3) a larger final re-circulator, also three-sided, for acceleration to 32 GeV. Each of the three sides of the re-circulators are made of SCL, and at each pass the energy gain is about 1 GeV in the first and 2 GeV in the second re-circulator. The number of beam passes in each re-circulator is thus 3. The rf in both re-circulators is always 1.61 GHz. The main parameters of the muon Super-Conducting Accelerators to 32 GeV are shown in Table 4.

9. μ -Storage and Decay in Neutrinos

At the end of the acceleration, the muons, that have now a lifetime of 0.67 ms, are first separated according to their charge and then injected into a storage ring in opposite long straights as shown in Figure 8. In the μ SR the muons of opposite sign circulate in opposite directions waiting for their decay into a pair of neutrinos. A full beam divergence of 0.067 mrad is required so that the spot size at the distance of about 3,000 km does not exceed ± 100 m. This requires a final focus of the μ -beam with $\beta^* 110$ km. A single high-beta insertion is provided in each of the two long straight sections of the muon storage ring from where the neutrinos escape toward the desired location. Otherwise the design of the μ SR follows very closely that of the Collaboration Proposal [1]. The storage ring has a periodicity 2. There are two arcs separated by two 500-m long straight sections. In each arc there are eight super-conducting bending magnets with a bending field of 8 Tesla. The ring circumference is about 1.1 kilometer, the transverse width is 35 m, and the shape is elongated to allow that most of the neutrinos (45%) escape along the desired direction from each of the long straight-sections. To preserve during the storage the initial beam bunching and longitudinal shape, it may be necessary to install a 201.25 MHz (or higher) rf cavity system. The main parameters of the 32-GeV muon Storage Ring are summarized in Table V.



Figure 8: Injection and Circulation of the μ^\pm in the 32-GeV Storage Ring.

10. The μ -Collider

The Neutrino Factory we have described can also be used as the muon source to inject into a high-luminosity high-energy $\mu^+ - \mu^-$ Collider. The beam of muons of both sign, at the exit of the 32-GeV μ SCL, by-passes the μ SR, and is accelerated even further to a considerable higher energy that we take here to be 1 TeV for each charge species. The two charges are then separated, individually transported, and injected into one-ring collider proper. The outline of the process is sketched in Figure 9.

The muon High-Energy Accelerator (μ HEA), that accelerates both signs of muons from 32 to 1000 GeV, follows a layout similar to the μ SCL described above that accelerates muon from 0.45 to 32 GeV, with the exception that now a high frequency rf cavity system (1.6–3.2 GHz) is used for more compact and more efficient acceleration (an average of 25 MeV/m, as suggested by the TESLA project). The μ HEA is made of a SC Linac for acceleration to 100 GeV, a Medium-Energy Re-circulator that accelerates to 300 GeV, and a High-Energy Re-circulator for the final acceleration to 1 TeV. As in the μ SCL, also the μ HEA accelerates simultaneously both signs of muons. The main parameters of the μ HEA are given in Table ???. Both re-circulators are 3-sided, and the beam makes three passes in each linac section. In the first Re-circulator the energy gain per SCL-section is 30 GeV, in the second 60 GeV. The average beam current and final power are relatively low. Most of the cost is for the manufacturing of the rf cavities and cryostats.

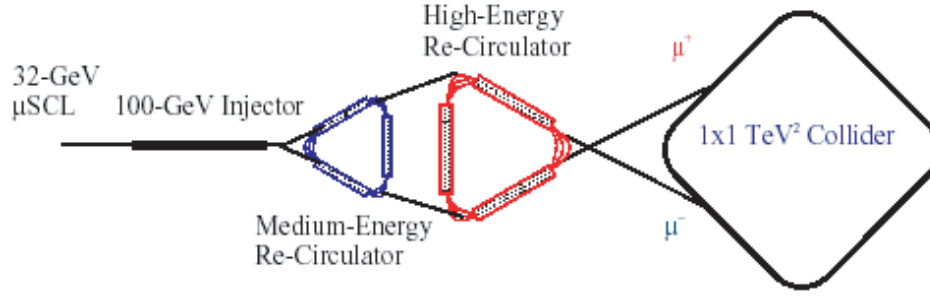


Figure 9: High-Energy SCL Accelerators and $1 \times 1 \text{ TeV}^2 \mu^+ - \mu^-$ Collider.

Table V Parameters of the Muon Storage Ring

Kinetic Energy	32 GeV
Magnetic Rigidity	107.026 T-m
β	0.999994585
γ	303.86
Momentum	32.105 GeV/c
Circumference	1112.077 m
Revolution Frequency	0.2696 MHz
Period	3.7095 μs
Periodicity	2 w/mirror symmetry
Total Number of Dipole Magnets	16
Dipole Field	8
Tesla Magnet Length	5.254 m
Bending Radius	13.378 m
Packing Factor in the Arcs	0.75
Length of an Arc	56.04 m
Width of the Storage Ring	35.675 m
Length of the Insertions	500 m
Number of Hi-Beta Insertions	1 per Long Straight
Hi-Beta value	110 km
Beam Spot Radius at Hi-Beta location	3.6 m
Beam Divergence at Hi-Beta location	± 0.0333 mrad
Muon Lifetime	0.67 ms
Total Number of muons stored	5.9×10^{11}
Circulating Current	25.4 mA

The Collider has a circumference of 3.743 km and is made of 8-Tesla super-conducting bending magnets. The packing factor, that allows also insertions for two low-beta insertions, injection and rf cavities, is 70%. The bending radius of the magnets is 417 m. The ring has a periodicity of four, with internal mirror symmetry. There are four arcs separated by four straight insertions each about 100 meter long. The main parameters of the Collider are given in Table VII.

At 1 TeV the muon lifetime is 20 ms. Injection into the Collider is continuous at the rate of $8.8 \times 10^{14} \mu/\text{s}$; thus the total number of muons circulating at any one time is 1.84×10^{13} and constant. To get the highest value of luminosity possible one requires a small circumference of the storage ring, a fewer number of bunches with a lot of particles, and a small transverse cross-section of the beam. The best strategy is thus to stack successive beam turns in the momentum phase space and preserve the beam emittance to the original value at the source. We shall assume that a low-beta value of $\beta^* = 10$ cm at the crossing is achievable. Also the beam-beam crossing is in dispersion-less locations. The Collider capturing rf cavity system operates at 20 MHz, that is

Table VI Parameters of the Muon High-Energy Accelerator

Mode of Operation	Continuous Wave (CW)
Final Kinetic Energy	1 TeV
Average Muon Current	2×0.14 mA
Average Muon Power	2×141 MW
SCL-Injector Energy	100 GeV
SCL-Injector Frequency	1.6 to 3.2 GHz
Energy of First Re-circulator	300 GeV
Energy of Second Re-circulator	1000 GeV
Number of Passes in SCL-Sectors	3 (4 in the first Linac Sector of each Re-Circulator)
Energy Gain for each Pass	30 GeV for 1st Re-Circulator 60 GeV for 2nd Re-Circulator
Frequency of Re-Circulator Linacs	3.2 GHz
Extracted Beam Emittance	3.9π mm-mrad (full)
Extracted Bunch Area	0.024π MeV-degree at 1.61 GHz
Momentum Spread	$\pm 1.0 \times 10^{-3}$
Bunch Length	± 0.02 ns Beam
Bunching Frequency	201.25 MHz
Bunch Spacing	150 cm

one tenth of the incoming bunching frequency. The beam is injected by performing “longitudinal painting” in the standing-by rf buckets where 10 beam bunches are deposited every turn. The total number of circulating bunches is thus 250 each with a total 7.35×10^{10} muons. In order to reach the final intensity, initially a total of 1,670 turns are required. Injection and “longitudinal painting” continues at the same rate to compensate for the depletion of the circulating muons. Each turn is captured by the rf system and accelerated over to make room for the subsequent turns as in cyclotron or FFAG accelerator. With this mode of operation the expected (constant) luminosity is $0.9 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$.

References

- [1] M.J. Brennan, et al., Collaboration home page <http://www.cap.bnl.gov/mumu/studyii>
- [2] M.J. Brennan, et al., “Upgrading the AGS to 1 MW Proton Beam Power,” presented to PAC 2001, Chicago, Illinois, June 2001.
- [3] Harold Kirk, private communication. BNL, May 2000.
- [4] APT³He Target/Blanket Topical Report, Los Alamos National Laboratory report LA-12670-MS (1993). See also APT Accelerator Topical Report, Los Alamos National Laboratory report LA-12668-MS (1993).

Table VII Parameters of the $\mu^+ - \mu^-$ Collider

Collision Energy	1 TeV per beam
Luminosity	$0.9 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$
Circumference	3,743 m
Periodicity	4 w/ mirror symmetry
Length of Long Straights	100 m
Low-Beta β^*	10 cm (dispersionless)
Muon Lifetime	20.8 ms
Total Muon stored	1.84×10^{13}
rf Cavity System	20.125 MHz
Number of Bunches	250
Emittance, full	$3.9\pi \text{ mm-mrad}$
Revolution Frequency	80.0 kHz
Period	$12.5\mu\text{s}$
Bending Field	8.0
Tesla Bending Radius	417.0 m
Packing Factor	0.70
Number of injected turns/cycle	1,670
Total Bunch Area	$393\pi \text{ eV-s}$
Bunch Half Length	20 ns
Total Beam Momentum Spread	$\pm 2\%$
Circulating Current	235 mA
Method of Injection	Momentum Stacking Cyclotron/FFAG mode