Precision Energy Measurements for Linear Colliders

Eric Torrence*
University of Oregon

The requirements and prospects for precision beam energy measurements at a future e^+e^- linear collider are reviewed.

1. Introduction

One of the great advantages to doing physics at an e^+e^- collider is the well defined initial state. Knowledge of the center of mass collision energy ($\sqrt{s}$) does not come for free, however, and some technique for measuring the incoming beam energy (E_{beam}) must be devised. The precision to which E_{beam} must be measured depends upon the particular analysis under consideration, and for a future linear collider operating anywhere from the Z^0 resonance up to 1 TeV there is a multitude of possibilities. Broadly speaking, however, there are two distinct precision scales required at a future linear collider.

Firstly, for all analyses envisioned at the top quark pair-production threshold and above, a relative precision of 2×10^{-4} (or 200 ppm) on E_{beam} appears to be adequate. This precision will lead to a 35 MeV uncertainty on the top quark mass from a threshold scan, which is below the statistical precision of 40 MeV expected in 10fb^{-1}, and is also probably well below the limiting theoretical uncertainties estimated for this method [1]. This precision is also suitable for other mass measurements, the Standard Model Higgs boson for instance, which can be best determined by direct reconstruction to a precision of roughly 50 MeV [2].

Secondly, for analyses proposed at lower energy to measure the electroweak parameters $\sin^2\theta_W^{\text{eff}}$ and M_W, the requirements on the beam energy precision are significantly tighter. Measuring M_W by means of a threshold scan is only interesting if the total error can be reduced to around 5 MeV, which requires an equivalent precision of 50 ppm on E_{beam} or better. For measurements of $\sin^2\theta_W^{\text{eff}}$ from A_{LR}, the knowledge of E_{beam} is required to correct the measured value back to the theoretically useful Z^0 pole value. The precision required depends upon the ultimate precision attainable on the weak mixing angle as shown in Table I [3]. If the Blondel scheme can be realized with $\mathcal{P}_{e^-} = 80\%$ and $\mathcal{P}_{e^+} = 50\%$, for example, the beam energy must be known to better than 2 MeV (40 ppm) to avoid being a limiting uncertainty.

It should be stressed that the requirements on ΔE_{beam} listed above are realistic targets which are *required* to carry out the physics program at a high energy e^+e^- collider. More precise measurements of E_{beam} are, of course, always a good thing, but the actual physics impact of any improvement is not clearly apparent without more detailed studies of particular analyses.

Table I Beam energy requirements for weak mixing angle measurements

	$\Delta\sin^2\theta_W^{\text{eff}}F$	ΔE_{beam} [MeV]	ΔE_{beam} [ppm]
SLD final	0.00027	25	500
$\mathcal{P}_{e^-} = 80\%$ only	0.00005	~ 5	100
$(\mathcal{P}_{e^-}/\mathcal{P}_{e^+}) = (80/50)\%$	0.00002	~ 2	40

*torrence@physics.uoregon.edu; `http://physics.uoregon.edu/˜torrence/`

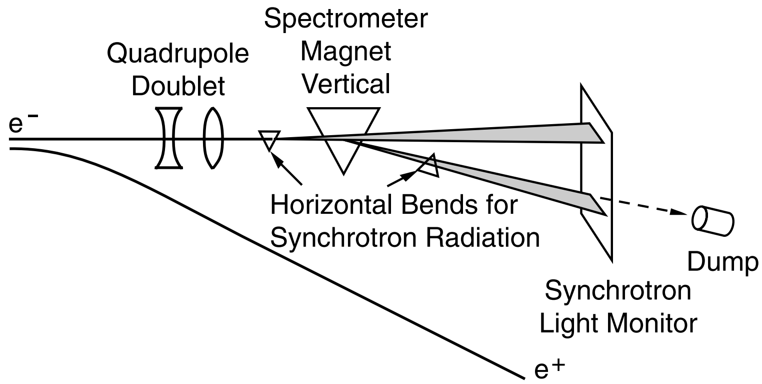

Figure 1: The SLC WISRD beam energy spectrometer.

2. Spectrometric Methods

The deflection of an charged particle traversing a magnetic field is a well established method for measuring a particle's momentum. At the SLC, the WISRD spectrometer was successfully used to make beam energy measurements at 120 Hz with a precision of 250 ppm at $E_{\text{beam}} = 45$ GeV[4]. As shown in Figure 1, the WISRD consists of a strong vertical analyzing dipole flanked by two weaker horizontal dipole magnets. The synchrotron radiation stripes produced by these two weaker dipoles are detected downstream on wire arrays, such that the deflection angle of the beam in the analyzing magnet can be directly monitored. The beam energy is then calculated as $E_{\text{beam}} = l/x \int B \cdot dl$, where l is the distance from the analyzing magnet to the wire screen, x is the separation between the synchrotron stripes, and $\int Bdl$ is the integrated bending field of the analysis magnet.

The systematic uncertainties of the SLC WISRD are dominated by the alignment tolerances of the detector screens, which contribute 190 ppm to the total error. The total bending field of $\int Bdl = 3\ Tm$ is known to 100 ppm. In addition to these experimental uncertainties, the precision to which the energy measured at the spectrometer (installed in the dump lines) can be related to the luminosity weighted collision energy at the SLD interaction point is uncertain at a level of 135 ppm, mainly due to the limited knowledge of misalignments and dispersion in the colliding beams.

The WISRD, with some modifications, is an ideal scheme to meet the 200 ppm goal of a high energy linear collider. It provides many benefits, including the possibility of bunch-by-bunch measurements, in a simple passive device which can be operated with essentially 100% duty factor. To improve the precision of the device, improvements can be made in the magnetic field survey, the synchrotron radiation detector design, and the overall geometry of the device.

The issue of the detector-IP difference is an important one, but beyond the scope of this document. This is a universal problem for any method of measuring the beam energy which contributes additional uncertainty to the method. In the end, it may well limit the achievable precision on the luminosity weighted beam energy, but without a precise measure of the beam energy somewhere to start with, it becomes a moot point. It is expected that in the natural operating mode of a high energy linear collider, this uncertainty can be brought far below the limit quoted for the SLC. For particularly sensitive analyses, like the precision electroweak measurements at low energy, luminosity can be traded against beam energy spread such that this uncertainty can be reduced to the level of 1 MeV.

3. Spin Precession

It is often asked whether spin precession can be a useful tool for beam energy measurements at a linear collider. The success of resonant depolarization at LEP [5], plus the need to have a longitudinal polarimeter, make this an attractive option at first glance. The physics of spin precession argue otherwise, however.

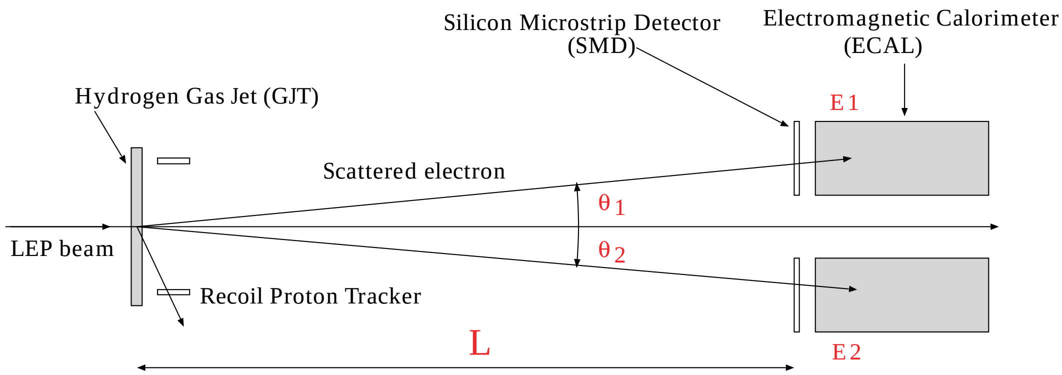

Figure 2: The Møller detector proposed for LEP2.

At $E_{\text{beam}} = 500\,\text{GeV}$, the spin precession frequency of an electron is $\nu_s = 1135$. This should be a good thing, as it presents a large amplification of the spin precession frequency over the revolution frequency, and it grows with increasing beam energy. One complete precession of the spin can be accomplished with a bending angle $\theta_b = 5.6$ mRad. The problem is that through a fixed magnetic bending field, the product $\nu_s\theta_b$ is invariant, so the observed spin precession is independent of energy. One could try to measure ν_s directly, but to do so requires a 100 ppm measurement of θ_b in addition to an equally precise measurement of the beam polarization. Since it is easier to measure $\int B \cdot dl$ precisely (which along with θ_b provide a direct spectrometric measurement) there is no advantage to using the beam polarization instead.

4. Kinematic Methods

There are a variety of kinematic methods which have been proposed to measure the energy of an electron beam. The most interesting, which was also proposed but never realized for LEP2, is to use the kinematics of Møller or Bhabha scattering [6]. The method involves scattering electrons of a hydrogen gas jet target, and reconstructing the beam energy from the properties of the scattered electrons. As shown in Figure 2, the scattered Møller electrons are reconstructed in both an electromagnetic calorimeter and a silicon microstrip detector to give excellent position and energy resolution.

The beam energy is reconstructed as

$$E_{\text{beam}} = \frac{8m_e}{(\tan\theta_1 + \tan\theta_2)^2}\frac{1}{1-\kappa^2} - m_e, \tag{1}$$

where κ can be reconstructed as either $\kappa = \frac{E_1 - E_2}{E_1 + E_2}$ or $\kappa = \frac{\tan\theta_1 - \tan\theta_2}{\tan\theta_1 + \tan\theta_2}$. In the first case, the angles and energies of the scattered electrons must be measured to very high precision. In the second case, only the angles of the scattered electrons are needed, although a precise knowledge of the interaction point is also required. This can be measured with the use of the recoil proton tracker shown in Figure 2.

In the LEP2 study, the detector acceptance is from 2–6 mRad at a distance of 20 meters. With typical silicon microstrip detectors and a crystal calorimeter capable of $\sim 1\%$ energy resolution, a statistical error of 2 MeV is achievable in 30 minutes of running.

The limiting systematic uncertainty for the Møller method results from the Fermi momentum of the target electrons. It is estimated that this will limit the precision of this method to $\simeq 2$ MeV, providing that the systematic uncertainties related to the detectors can be controlled to below this level.

How the performance of a Møller device would translate to a linear collider, where the repetition rate is much lower than LEP2 and multiple interaction pileup would have to be controlled, requires further study. The feasibility of placing precision detectors within a few centimeters of a high energy electron beam also could pose serious design problems. This appears to be the only method, however, with the potential to make an absolute energy measurement at the few

MeV level. The use of Compton scattering rather than Møller/Bhabha scattering should also be considered.

Another method which is potentially very useful is to use the kinematics of $e^+e^- \to f\bar{f}\gamma$ radiative return events to reconstruct the collision energy. In the most likely tree-level process, the photon is emitted collinear to the incoming particles, and the invariant mass of the $f\bar{f}$ system can be reconstructed as

$$\frac{s'}{s} = \frac{\sin\theta_1 + \sin\theta_2 - |sin(\theta_1 + \theta_2|}{\sin\theta_1 + \sin\theta_2 + |sin(\theta_1 + \theta_2|}. \quad (2)$$

Due to the strong resonance of the Z^0 pole, there will be a peak in the s' spectrum which can be calibrated to the known Z^0 mass measured at LEP1, leading to a direct determination of s. This analysis is currently being performed by the LEP collaborations, and more details can be found, for example, here [7].

The $q\bar{q}\gamma$ final state has the best statistical sensitivity, but uncertainties in the hadronization process probably limit the precision in this channel to $\sim$ 50 MeV near the W-pair threshold. This problem is avoided in the $\mu^+\mu^-\gamma$ final state, although the requirements for the tracking detector are rather stringent. To approach a 5 MeV uncertainty requires an absolute θ measurement of the muon tracks with a precision below 100 ppm. One distinct advantage of this method is that it allows a direct measurement of the luminosity weighted beam energy at the interaction point, which is exactly what is needed.

5. Summary

The requirements for measuring the beam energy at a future e^+e^- linear collider are twofold. At high energy, a precision of $\sim$ 200 ppm appears to be adequate for the physics analyses envisioned. A spectrometer based on the design of the SLC WISRD appears to be a very suitable technology for this level of precision.

At lower energies, where precision electroweak measurements are desired at the Z^0 pole and W^+W^- threshold, a more stringent target of less than 50 ppm must be achieved. At the Z^0 pole, a suitable scanning strategy could be conceived to allow cross-calibration against the known Z^0 mass, although to monitor the variation of the beam energy with time, it is still probably necessary to have some method which approaches this precision. At the W^+W^- threshold, radiative return events may be useful, but a more direct method would also be needed.

One method which may be suitable to reach this level of precision is based on Møller scattering off a gas jet target. It is also possible with careful calibration against the Z^0 pole that a WISRD-style device may be able to provide a relative measure with the desired precision.

There are other beam energy measurement methods in the literature, most notably using synchrotron radiation [8] or resonant photon absorption [9] which should be considered. It is not at all clear, however, whether these methods are suitable for beam energies above a few GeV.

The issue of relating an external beam energy measurement to the luminosity-weighted collision energy is important, but not discussed in detail here. Low dispersion machine operation should reduce this effect to negligible levels, although for high energy operation it could become a limiting systematic.

References

[1] T. Abe et al. Linear collider physics resource book for snowmass 2001. pages 239–254, 2001. Resource book for Snowmass 2001, 30 Jun - 21 Jul, Snowmass, Colorado.

[2] T. Abe et al. Linear collider physics resource book for snowmass 2001. pages 109–111, 2001. Resource book for Snowmass 2001, 30 Jun - 21 Jul, Snowmass, Colorado.

[3] P. C. Rowson and M. Woods. Experimental issues for precision electroweak physics at a high-luminosity z factory. 2000.

[4] J. Kent et al. Precision measurements of the slc beam energy. 1989. Presented at IEEE Particle Accelerator Conf., Chicago, Ill., Mar 20-23 1989.

[5] R. Assmann et al. Calibration of centre-of-mass energies at lep1 for precise measurements of z properties. *Eur. Phys. J.*, C6:187–223, 1999.

[6] C. Cecchi, J. H. Field, and T. Kawamoto. Beam energy measurement at lep2 using moeller scattering. *Nucl. Instrum. Meth.*, A385:445–455, 1997.

[7] G. Abbiendi et al. Determination of the lep beam energy using radiative fermion-pair events. 2001. OPAL–PN493.

[8] S. Henderson, V. Boisvert, K. Finkelstein, D. Rice, and J. White. A technique for measuring the relative cesr beam energy. 1999. Contributed to IEEE Particle Accelerator Conference (PAC 99), New York, 29 Mar - 2 Apr 1999.

[9] R. A. Melikian and D. P. Barber. A resonance method for the measurement of electron beam energy up to tev-energies. 1998. Prepared for 13th International Symposium on High-Energy Spin Physics (SPIN 98), Protvino, Russia, 8-12 Sep 1998.