Run Scenarios for the Linear Collider

M. Battaglia
CERN
J. Barron, M. Dima, L. Hamilton, A. Johnson, U. Nauenberg, M. Route, D. Staszak, M. Stolte, T. Turner, C. Veeneman
University of Colorado
J. Wells
University of California, Davis
J. Butler, H. E. Montgomery
Fermi National Accelerator Laboratory
R. N. Cahn, I. Hinchliffe
Lawrence Berkeley National Laboratory
G. Bernardi
LPNHE. Universities of Paris VI and VII
J. K. Mizukoshi
University of Hawaii
G. W. Wilson
University of Kansas
G. A. Blair
Royal Holloway, University of London
J. Jaros
Stanford Linear Accelerator Laboratory
P. D. Grannis*
State University of New York at Stony Brook

Scenarios are developed for runs at a Linear Collider, in the case that there is a rich program of new physics.

1. Introduction

The physics program of the linear e^+e^- collider LC is potentially very extensive, particularly in the case that a Higgs boson with mass below 300 GeV is found and relatively low energy scale supersymmetry (SUSY) exists. For such a case, we have examined a possible run plan for the LC to explore the new states and their masses, and estimated the precision on measured parameters that can be attained in a reasonable time span.

For this study, we have examined a scenario with a light SM-like Higgs boson of mass 120 GeV and two minimal supergravity (mSUGRA) models with many low mass sparticles. This scenario is conservative; with many particles to study there are many desired operational conditions for the collider (different energies and beam polarizations). We have not assumed that positron polarization is available, again a conservative assumption from the point of view of the running time required.

Table I Profile by year of the luminosity accumulation. The luminosity is given in fb^{-1} assuming 500 GeV operation.

Year	1	2	3	4	5	6	7
$\int \mathcal{L}dt$	10	40	100	150	200	250	250

*Electronic address: pgrannis@sunysb.edu

Table II Minimal Sugra parameters for the two SUSY benchmark points used in this study.

	TESLA RR1	SPS1
m_0	100 GeV	100 GeV
$m_{1/2}$	200 GeV	250 GeV
$\tan\beta$	3	10
A_0	0 GeV	0 GeV
sgn(μ)	+	+

We have taken the total time for the runs to be that required to accumulate 1 ab^{-1} (1000 fb^{-1}) at 500 GeV. Based on estimates [1] of the luminosity that could be delivered by the LC summarized in Table I, we estimate that this represents a program for the first 6 – 7 years of LC operation. Such an estimate is only qualitative and depends more upon the ultimate luminosity of the accelerator than upon the details of early low luminosity during commissioning.

We have chosen two SUSY benchmarks shown in Table II: the TESLA TDR RR1 [2] and the Snowmass E3 working group [3] point, also known as benchmark SPS1. They provide a rich spectrum of sparticles at relatively low masses. The TESLA RR1 scenario has been used for a variety of previous studies, but is now ruled out by LEP data. The SPS1 point gives low sparticle masses, but emphasizes decays via τ's and thus provides additional experimental challenges. For both scenarios, LHC should have discovered SUSY and explored some of its aspects prior to the LC operation. However, the precision measurements available at the LC, and its access to states unobservable at the LHC, will be needed for the full exploration of the new physics.

In devising a proposed run plan, one should keep in mind that it is entirely possible, even likely, that there is new physics to be explored that depends on operation of the LC near its highest energy. Thus a run plan that devotes excessive operation at lower energies may be counterproductive.

2. Run Plan

For any physics scenario, studies of the Higgs boson and the top quark will be high priorities for the linear collider. The Higgs studies are possible at any energy above the associated ZH production threshold but, due to the fall of the cross-section with energy, are best optimized not too far above threshold. The top studies require operation in the vicinity of the $t\bar{t}$ threshold at 350 GeV.

If supersymmetry exists, the desired LC energies and electron beam polarizations depend sensitively on the specific realization of the supersymmetric sector. In many cases, the best resolution for the sparticle masses is obtained by dedicated runs near the threshold for producing a specific particle. In this case, however, it is necessary to establish these particle thresholds with some accuracy prior to making a scan. In some cases these may be determined from LHC experiments; in others it is necessary to establish masses from LC operation at its maximum energy through the use of kinematic end points in SUSY decay chains. For this study, we have imagined that the mass estimates must be obtained from the LC, so the first order of business for SUSY studies is operation at the full machine energy.

The luminosity required for subsequent threshold runs is dictated by the threshold behavior of the cross-sections (typically proportional to β for fermion-antifermion (*e.g.* gaugino pair) production and to β^3 for boson (*e.g.* sfermion) pair production in e^+e^- collisions), and by the dependence of cross-sections on electron beam polarization. In some cases, we have concluded that particular sparticle production cross-sections are too small, or that decay chains yield too few easily reconstructed particles, to warrant spending time on a dedicated threshold scan.

The suggested run plans for the two assumed SUSY benchmark points are shown in Tables III and IV. In both plans, we assume that special e^-e^- runs are taken to obtain high precision measurements of the $\tilde{e}_R$ mass. For the SPS1 point, the $\tilde{\chi}_2^\pm$ mass is such that the $\tilde{\chi}_1^+\tilde{\chi}_2^-$ threshold is above the nominal maximum 500 GeV LC energy; to reach this crucial state in the SPS1 point, the machine would need to operate at 580 GeV, a possibility if one trades luminosity for energy by higher rf loading of the accelerator structures [1]. For the TESLA RR1 benchmark, we have envisioned two energies with high integrated luminosity from which mass measurements may be

Table III Run allocations for the TESLA RR1 Minimal Sugra parameters.

Beams	Energy	Pol.	$\int \mathcal{L}dt$	$[\int \mathcal{L}dt]_{\text{equiv}}$	Comments
e^+e^-	500	L/R	245	245	Sit at top energy for heavy sparticle end point measurements
e^+e^-	320	L/R	160	250	End point measurements for light sparticles Scan $\tilde{\nu}$ pair thresholds
e^+e^-	255	L/R	20	40	Scan $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ threshold
e^+e^-	265	R	20	40	Scan $\tilde{\mu}_R$ and $\tilde{\tau}_1$ pair thresholds
e^+e^-	310	L	20	30	Scan $\tilde{e}_L\ \tilde{e}_R$ threshold
e^+e^-	350	L/R	20	30	Scan $t\bar{t}$ threshold Scan $\tilde{\tau}_2$ pair threshold
e^+e^-	450	L	100	110	Scan $\tilde{\chi}_2^0\ \tilde{\chi}_3^0$ threshold
e^+e^-	470	L/R	100	105	Scan $\tilde{\chi}_1^\pm\ \tilde{\chi}_2^\mp$ threshold
e^-e^-	265	RR	10	95	Scan with e^-e^- collisions for $\tilde{e}_R$ mass

Table IV Run allocations for the SPS1 Minimal Sugra parameters.

Beams	Energy	Pol.	$\int \mathcal{L}dt$	$[\int \mathcal{L}dt]_{\text{equiv}}$	Comments
e^+e^-	500	L/R	335	335	Sit at top energy for sparticle end point measurements
e^+e^-	270	L/R	100	185	Scan $\tilde{\chi}_1^0\ \tilde{\chi}_2^0$ threshold (R pol.) Scan $\tilde{\tau}_1\ \tilde{\tau}_1$ threshold (L pol.)
e^+e^-	285	R	50	85	Scan $\tilde{\mu}_R^+\ \tilde{\mu}_R^-$ threshold
e^+e^-	350	L/R	40	60	Scan $t\bar{t}$ threshold Scan $\tilde{e}_R\ \tilde{e}_L$ threshold (L & R pol.) Scan $\tilde{\chi}_1^+\ \tilde{\chi}_1^-$ threshold (L pol.)
e^+e^-	410	L/R	100	120	Scan $\tilde{\tau}_2\ \tilde{\tau}_2$ threshold
e^+e^-	580	L/R	90	120	Sit above $\tilde{\chi}_1^\pm\ \tilde{\chi}_2^\mp$ threshold for $\tilde{\chi}_2^\pm$ end point mass
e^-e^-	285	RR	10	95	Scan with e^-e^- collisions for $\tilde{e}_R$ mass

made from kinematic end point studies. The 320 GeV run also serves in this scenario for a scan of $\tilde{\nu}$ thresholds. Recent work however suggests that the precision of end point mass measurements is optimized by running at the highest available energy. For the SPS1 point, the $\tilde{\nu}$ threshold runs are of limited utility in any case (the branching ratios into low background final states are too small), so for this point we have included only one run at 500 GeV for end point measurements.

Note that for both run plans, at least two thirds of the accumulated luminosity is acquired at an energy within 80% of the maximum LC energy, so searches for new phenomena beyond the Higgs and supersymmetry studies posited here should be possible.

Tables III and IV show the desired electron polarization states for each energy setting. We assume an e^- beam polarization of 80% (and no e^+ polarization). In some cases, we suggest an equal luminosity for left and right polarized electrons, either to examine backgrounds or to access different particle states. For some of the dedicated threshold runs, we specify the dominant beam polarization that maximizes the desired reaction rate; in these cases, we imagine that perhaps 90% of the data is taken with this preferred polarization and the remaining ~10% of the data with the other polarization. The e^-e^- operation assumes that both beams are polarized as indicated.

We should warn that a rigorous optimization of the run scenarios has not been made, and indeed the guidelines used for evaluating the two benchmark points are somewhat different. This study can be no more than an example that a reasonable length run can provide good precision observables, since the multitude of possible SUSY (and Higgs) models still permitted is huge and the strategy and physics reach for each is likely to be quite different.

Tables III and IV indicate both the luminosity allocated at a particular energy, and the 'equivalent luminosity' defined as that accumulation that would have been made for the equivalent time spent at 500 GeV. The 500 GeV equivalent luminosity is the appropriate unit to account for the time spent. It is the sum of the equivalent luminosities that is set to 1 ab^{-1} in this study, and is the unit indicated in Table I.

Table V Sparticle masses and dominant branching fractions for the SPS1 benchmark.

Particle	m(GeV)	Final state / (BR(%))							
$\tilde{e}_R$	143	$\tilde{\chi}_1{}^0 e$ (100)							
$\tilde{e}_L$	202	$\tilde{\chi}_1{}^0 e$ (45)	$\tilde{\chi}_1{}^\pm \nu_e$ (34)	$\tilde{\chi}_2{}^0 e$ (20)					
$\tilde{\mu}_R$	143	$\tilde{\chi}_1{}^0 \mu$ (100)							
$\tilde{\mu}_L$	202	$\tilde{\chi}_1{}^0 \mu$ (45)	$\tilde{\chi}_1{}^\pm \nu_\mu$ (34)	$\tilde{\chi}_2{}^0 \mu$ (20)					
$\tilde{\tau}_1$	135	$\tilde{\chi}_1{}^0 \tau$ (100)							
$\tilde{\tau}_2$	206	$\tilde{\chi}_1{}^0 \tau$ (49)	$\tilde{\chi}_1{}^- \nu_\tau$ (32)	$\tilde{\chi}_2{}^0 \tau$ (19)					
$\tilde{\nu}_e$	186	$\tilde{\chi}_1{}^0 \nu_e$ (85)	$\tilde{\chi}_1{}^\pm e^\mp$ (11)	$\tilde{\chi}_2{}^0 \nu_e$ (4)					
$\tilde{\nu}_\mu$	186	$\tilde{\chi}_1{}^0 \nu_\mu$ (85)	$\tilde{\chi}_1{}^\pm \mu^\mp$ (11)	$\tilde{\chi}_2{}^0 \nu_\mu$ (4)					
$\tilde{\nu}_\tau$	185	$\tilde{\chi}_1{}^0 \nu_\tau$ (86)	$\tilde{\chi}_1{}^\pm \tau^\mp$ (10)	$\tilde{\chi}_2{}^0 \nu_\tau$ (4)					
$\tilde{\chi}_1{}^0$	96	stable							
$\tilde{\chi}_2{}^0$	175	$\tilde{\tau}_1 \tau$ (83)	$\tilde{e}_R e$ (8)	$\tilde{\mu}_R \mu$ (8)					
$\tilde{\chi}_3{}^0$	343	$\tilde{\chi}_1{}^\pm W^\mp$ (59)	$\tilde{\chi}_2{}^0 Z$ (21)	$\tilde{\chi}_1{}^0 Z$ (12)	$\tilde{\chi}_2{}^0 h$ (1)	$\tilde{\chi}_1{}^0 h$ (2)			
$\tilde{\chi}_4{}^0$	364	$\tilde{\chi}_1{}^\pm W^\mp$ (52)	$\tilde{\nu}\nu$ (17)	$\tilde{\tau}_2 \tau$ (3)	$\tilde{\chi}_1{}^0 Z$ (2)	$\tilde{\chi}_2{}^0 Z$ (2)	$\tilde{e}_L e$ (2)	$\tilde{\mu}_L \mu$ (2)	$\tilde{\ell}_R \ell$ (2)
$\tilde{\chi}_1{}^\pm$	175	$\tilde{\tau}_1 \tau$ (97)	$\tilde{\chi}_1{}^0 q\bar{q}$ (2)	$\tilde{\chi}_1{}^0 e\nu$ (0.4)	$\tilde{\chi}_1{}^0 \mu\nu$ (0.4)	$\tilde{\chi}_1{}^0 \tau\nu$ (0.4)			
$\tilde{\chi}_2{}^\pm$	364	$\tilde{\chi}_2{}^0 W$ (29)	$\tilde{\chi}_1{}^\pm Z$ (24)	$\tilde{\ell}\nu_\ell$ (18)	$\tilde{\chi}_1{}^\pm h$ (15)	$\tilde{\nu}_\ell \ell$ (8)	$\tilde{\chi}_1{}^0 W$ (6)		

Table VI Selected cross sections in femtobarns for the SPS1 benchmark. Electron beam L and R polarizations have magnitude 80%. Unless otherwise noted, the energy is 500 GeV.

Reaction	σ_L	σ_R	Reaction	σ_L	σ_R
$\tilde{\chi}_1{}^0\tilde{\chi}_2{}^0$	105	25	$\tilde{e}_L{}^+\tilde{e}_L{}^-$	105	17
$\tilde{\chi}_1{}^0\tilde{\chi}_3{}^0$	4	16	$\tilde{e}_R{}^+\tilde{e}_R{}^-$	81	546
$\tilde{\chi}_1{}^0\tilde{\chi}_4{}^0$	2	4	$\tilde{e}_R{}^+\tilde{e}_L{}^-$	17	151
$\tilde{\chi}_2{}^0\tilde{\chi}_2{}^0$	139	16	$\tilde{e}_L{}^+\tilde{e}_R{}^-$	152	17
$\tilde{\chi}_1{}^\pm\tilde{\chi}_1{}^\mp$	310	36	$\tilde{\mu}_R{}^+\tilde{\mu}_R{}^-$	30	87
$\tilde{\chi}_1{}^\pm\tilde{\chi}_2{}^{\mp a}$	7	2	$\tilde{\mu}_L{}^+\tilde{\mu}_L{}^-$	38	12
$\tilde{\chi}_1{}^\pm\tilde{\chi}_2{}^{\mp b}$	37	10	$\tilde{\tau}_1^+\tilde{\tau}_1^-$	35	88
$\tilde{\chi}_1{}^\pm\tilde{\chi}_2{}^{\mp c}$	43	11	$\tilde{\tau}_1^\pm\tilde{\tau}_2^\mp$	2	1
$\tilde{\nu}_e\tilde{\nu}_e^*$	929	115	$\tilde{\tau}_2^+\tilde{\tau}_2^-$	31	11
$\tilde{\nu}_\mu\tilde{\nu}_\mu^*$	18	14			
$\tilde{\nu}_\tau\tilde{\nu}_\tau^*$	18	14			

[a] For 540 GeV operation
[b] For 580 GeV operation
[c] For 620 GeV operation

3. Studies of supersymmetry

In this section we discuss the determination of the sparticle mass precisions to be expected for the SPS1 benchmark run plan; similar considerations were applied during the workshop to the TESLA RR1 benchmark point and the results are simply summarized here.

The sparticle masses and dominant branching fractions for the SPS1 point are given in Table V. The cross-sections at $\sqrt{s}$ = 500 GeV for relevant two body processes in SPS1 are shown in Table VI. Not shown in the Table are the corresponding squark, gluino and higgs masses and decay channels. The lighter squarks and gluino have masses $\sim$ 530 and 595 GeV respectively; the two stop states have masses of 393 and 572 GeV. The $(h^0, H^0, A^0, H^\pm)$ masses are respectively (113, 380, 379 and 388) GeV.

As indicated in Section 2, the measurement of SUSY particle masses relies on a first run at a high energy where many sparticle pairs are produced. In many of these cases, the produced sparticles decay to two particles, one of which is a well-measured SM particle, and the other is a sparticle (stable or unstable). Since the original sparticles are mono-energetic (in the absence of radiative losses from the incoming beams), their decay products have flat energy distributions between lower and upper end points fixed by the parent and decay sparticle masses. Observation of these end points then determines the masses of parent and decay sparticles. The effects of initial state radiation and detector resolutions will smear the energy distributions to some degree, but the end points can still be determined.

3.1. Energy end point mass measurements

There have been several studies [4][5] of the precisions obtainable in end point studies, incorporating smearing effects, for both the RR1 and SPS1 benchmark points, typically at 500 GeV. Our estimates of end point mass precisions are based upon simple scaling of statistical errors from these studies to the number of events expected with our run plan. This is perhaps an over-simplified model in the case where non-negligible backgrounds for a particular process from SM or other SUSY processes are present. For the precisions to be expected in the SPS1 benchmark, we have used, wherever possible, the results from Ref. [5] which were done for the same SUSY scenario and thus have the appropriate SUSY backgrounds.

The $\tilde{e}_R$ and $\tilde{e}_L$ end point studies are particularly rich, with distinct upper and lower edges coming from the distinct $\tilde{e}_R$ and $\tilde{e}_L$ decays to $\tilde{\chi}_1^0 e^\pm$. The relative sizes and locations of these end point edges in both e^+ and e^- depend on the four cross sections for $\tilde{e}_R^+\tilde{e}_R^-$, $\tilde{e}_L^+\tilde{e}_L^-$, $\tilde{e}_L^+\tilde{e}_R^-$ and $\tilde{e}_R^+\tilde{e}_L^-$ for the different electron beam polarizations. The energy spectra are further complicated by the presence of backgrounds from the SM and SUSY processes, and by the fact that the $\tilde{e}_L$ typically has other decays besides the $\tilde{\chi}_1^0 e$. A new method to facilitate these analyses by taking differences between distributions with opposite beam polarizations, and between emitted positron and electron distributions was developed in this workshop [6].

The $\tilde{\tau}$ and $\tilde{\chi}_2^0$ end point measurements studies are complicated by the fact that they decay dominantly into τ final states, and thus have missing neutrinos that wash out the energy end points. Nevertheless, the energy of the hadron from τ 1-prong decays does carry some information on the parent τ energy. We have guessed, without direct confirmation, that this may be sufficient to locate the energy for the $\tilde{\tau}$ and $\tilde{\chi}_2^0$ threshold scans to within 1 – 2 GeV.

The $\tilde{\chi}_3^0$ case is special for the SPS1 benchmark; the $\tilde{\chi}_3^0$ has an observable and distinctive decay into $\tilde{\chi}_1^0 Z$. Using the well reconstructed $Z \to \ell^+\ell^-$, the usual end point method works, albeit with low statistics. The $\tilde{\chi}_4^0$ is produced with insufficient rate at 500 GeV in this benchmark to be observable.

The charged states $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$ pose special problems also for end point measurements in the SPS1 benchmark. The dominant $\tilde{\chi}_1^\pm$ decay is $\tilde{\tau}_1 \nu_\tau$, which does not produce sharp end points. However, $\tilde{\nu}_e \tilde{\nu}_e^*$ production with $\tilde{\nu}_e \to \tilde{\chi}_1^\pm e^\mp$ is observable and permits a determination of the $\tilde{\chi}_1^\pm$ mass. The $\tilde{\chi}_2^\pm$ decay into $\tilde{\chi}_1^\pm Z$ gives a useful, but statistically limited, method for determining its mass from the run at 580 GeV, above the $\tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ threshold. (Without some model assumptions, it is of course not possible to know what energy is appropriate for the production of $\tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$, but the knowledge of the $\tilde{\chi}_1^\pm$ mass and the measured $\tilde{\nu}_e$ pair cross-section, sensitive to both $\tilde{\chi}^\pm$ states, would give a good indication of the $\tilde{\chi}_2^\pm$ mass.)

The end point analyses assume that it is possible to find a final state that can be clearly identified as arising from a particular two body reaction. This assumption needs to be examined carefully, as in practice many two-body processes can feed the same final state. The details vary strongly with benchmark point.

We have looked at the competing reactions feeding particular final states of 2 or 4 leptons (e, μ, τ) plus missing energy ($\not{E}$) for the SPS1 benchmark [7]. We have required here that the final states contain no strongly interacting particles. For example, after taking all the cross sections and branching ratios into account, the contributions to the $e^\pm \tau^\mp \not{E}$ final state with right polarized electron beam are spread over the initial channels: $\tilde{e}_L \tilde{e}_L$ (5); $\tilde{e}_R \tilde{e}_L$ (56); $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ (0.3); $\tilde{\nu}_e \tilde{\nu}_e^*$ (21); and $\tilde{\chi}_1^0 \tilde{\chi}_3^0$ (0.8) (where the numbers in parentheses are $\sigma \times$BR in fb). For such cases with multiple competing reactions, attributing structures in the energy distributions to particular SUSY particles will be difficult. In general, the end point analyses are likely to require iterative approaches to separate effects of the different sparticles.

Nevertheless, for benchmark SPS1, we find that most sleptons and gauginos can be reasonably well isolated in specific channels. The $\tilde{e}_R$ and $\tilde{e}_L$ are mixed in the $e^+e^-\not{E}$ final states, but Ref. [6] shows that they can be disentangled. Table VII shows some of the final states that are dominated by specific sparticle production processes. We see that apart from the $\tilde{\nu}_\tau, \tilde{\chi}_2^0$ and

Table VII The dominant contributors to some specific final states, with specified initial electron beam polarization. N is the number events expected (before acceptance and efficiency cuts) in the 335 fb^{-1} allocated to the 500 GeV run. The percentage of these events from the dominant reaction is $\mathcal{F}$. The L and R e^- beam polarizations were taken with magnitude 80%.

Final state	Pol (e^-)	N	dominant reaction(s)	$\mathcal{F}$	SM particles used	masses measured
$e^+e^-\not{E}$	R/L	210K/65K	$\tilde{e}_L\tilde{e}_L$, $\tilde{e}_R\tilde{e}_R$, $\tilde{e}_L\tilde{e}_R$	92	$e^\pm$	$\tilde{e}_L, \tilde{e}_R, \tilde{\chi}_1{}^0$
$\mu^+\mu^-\not{E}$	R	31K	$\tilde{\mu}_R\tilde{\mu}_R$	95	$\mu^\pm$	$\tilde{\mu}_R, \tilde{\chi}_1{}^0$
$\tau^+\tau^-\not{E}$	L	152K	$\tilde{\chi}_1{}^\pm\tilde{\chi}_1{}^\mp$	64	$\tau^\pm$	$\tilde{\chi}_1{}^\pm, \tilde{\tau}_1$
$e^\pm\tau^\mp\not{E}$	L	88K	$\tilde{\nu}_e\tilde{\nu}_e^*$	65	$e^\pm$	$\tilde{\chi}_1{}^\pm, \tilde{\nu}_e$
$\mu^+\mu^-\tau^+\tau^-\not{E}$	L	2K	$\tilde{\mu}_L\tilde{\mu}_L$	97	$\mu^\pm$	$\tilde{\mu}_L, \tilde{\chi}_1{}^0, \tilde{\chi}_2{}^0$
$e^+e^-\tau^+\tau^-\not{E}$	R	10K	$\tilde{e}_L\tilde{e}_R$	91	$e^\pm$	$\tilde{e}_L, \tilde{\chi}_1{}^0, \tilde{e}_R$
$\tau^+\tau^-\tau^\pm\mu^\mp\not{E}$	R	8K	$\tilde{\nu}_\mu\tilde{\nu}_\mu^*$ ($\tilde{\mu}_L\tilde{\mu}_L$)	43 (57)	$\mu^\pm$	$\tilde{\nu}_\mu, \tilde{\chi}_1{}^\pm$

perhaps $\tilde{\nu}_\mu$, there is at least one process that allows relatively clean access to each of the sparticle masses through end point measurements. (Recall that the $\tilde{\chi}_3{}^0$ mass can be accessed in the SPS1 benchmark through its decay into $\tilde{\chi}_1{}^0 Z$).

We caution however that, although for the SPS1 case examined here one can find channels that are specifically sensitive to particular sparticle masses, it is by no means clear that that one will easily deduce the sparticles responsible for the observed end points when one does not *a priori* know the SUSY model.

Table VIII shows our estimates of the precisions obtainable by end point measurements for the SPS1 benchmark, based on the run plan shown in Table IV. The caveats of the preceding paragraphs suggest these should be taken only as educated guesses; a complete Monte Carlo calculation including all SUSY processes and SM backgrounds should be made.

3.2. Threshold scan mass measurements

Once one has an estimate of sparticle masses from the end point measurements, refined determinations can often be obtained by performing a scan across the threshold of a reaction involving that sparticle. For these studies, it is not necessary to restrict attention to easily reconstructed final states; it is sufficient that the final states are observable in the detector and that other thresholds in the same polarization and final state do not occur in the same region. Studies of such threshold scans have been made in Ref. [4] for benchmark RR1. In that study, 100 fb^{-1} were devoted to each scan, with runs distributed over 10 equidistant energy points. This strategy is almost surely not ideal; an optimized scan algorithm should depend upon the amount of background in the channels observed, the total cross-section times branching ratio, the uncertainty in $\sigma\times$BR, and on the steepness of the threshold curve as a function of energy. Ref [8] has studied an optimization for $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$ thresholds where the cross sections are small and find that two points on the rise of the cross-section and one well above threshold are more suitable. A study performed for this workshop [9] has investigated how to obtain both sparticle masses and total widths, and finds that a two point scan may be optimum. This analysis also concludes that the widths for many of the states may be accessible at the 35 – 50% level.

As part of this study, we have made an analytic estimate of the accuracy available in a threshold scan [10] for the case that equal luminosities are collected at N scan points, spaced at equal energy, δE. The threshold is assumed to be within δE below the first of the scan points. No background is included in these studies. The presence of beamstrahlung should not affect the threshold turn-on markedly, since the collisions at the dominant peak at the full beam energy give an unsmeared threshold behavior. Minimizing the likelihood function formed from the Poisson probabilities to give the observed numbers of events at each energy point, we can determine the most likely value of the threshold energy and hence sparticle mass. The analytic results can be approximated as:

$$\delta m \approx \Delta E(1 + 0.36/\sqrt{N})/\sqrt{18N\mathcal{L}\sigma_u}$$

for a β^3 p-wave threshold, and

$$\delta m \approx \Delta E N^{-1/4}(1 + 0.38/\sqrt{N})/\sqrt{2.6 N \mathcal{L} \sigma_u}$$

for a β^1 s-wave threshold. Here, ΔE is the full energy interval over which the scan is made, $\mathcal{L}$ is the total luminosity devoted to each point of the scan, N is the number of energy settings, and σ_u is the cross-section at the upper energy of the scan. Note that the p-wave threshold benefits little from increasing the number of energy settings above 3 to 4, while an s-wave threshold precision continues to improve weakly as $N^{-1/4}$ with the number of points in the scan. These analytic approximations are in good agreement with the Monte Carlo precisions for the p-wave $e^+e^- \to \tilde{\mu}_R{}^+\tilde{\mu}_R{}^-$ and s-wave $e^+e^- \to \tilde{\chi}_1{}^+\tilde{\chi}_1{}^-$ threshold scans of Ref. [4].

The run plans for the RR1 and SPS1 benchmark points call for scans as indicated in Tables III and IV. In both, a special scan at the $\tilde{e}_R$ threshold is called for using right polarized e^- beams; this strategy [11] is dictated by the fact that the $\tilde{e}_R\tilde{e}_R$ threshold energy cross section rises as β in e^-e^-, whereas in e^+e^- it rises as β^3. The sharper rise, even after inclusion of the effects of beamsstrahlung gives a better determination of the $\tilde{e}_R$ mass.

In the RR1 scenario, the $\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$ states are observable through their decays into $\tilde{\chi}_1{}^\pm\ell^\mp$ with subsequent $\tilde{\chi}_1{}^\pm \to \tilde{\chi}_1{}^0 q\bar{q}$ and $\tilde{\chi}_1{}^\pm \to \tilde{\chi}_1{}^0\ell\nu$, although the event rates are small. We thus include a scan at the $\tilde{\nu}$ pair threshold around 320 GeV to get some mass information, estimated on the basis of the analysis in Ref [8]. (The more precise $\tilde{\nu}$ mass determination in Ref [4] seems to be too optimistic for this channel.) In the SPS1 benchmark, the $\tilde{\chi}_1{}^\pm$ decays dominantly into $\tilde{\tau}_1\nu_\tau$ and the signature is hard to dig out from background. In the SPS1 case, we thus do not call for a $\tilde{\nu}$ pair threshold scan.

We have estimated sparticle mass precisions from threshold scans by simple statistical scaling of the results of Ref [4], based on the ratio of $\sigma \times \mathrm{BR} \times \mathcal{L}$ for the appropriate reaction in our run scenario to that used in Ref [4]. We use the reaction cross-sections at 500 GeV for this scaling. This simple estimating procedure is doubtless too naive, since it ignores details of the backgrounds at different benchmark points, and has not incorporated the effects of uncertainties in the knowledge of $\sigma \times \mathrm{BR}$.

The resulting estimates of the mass precisions from the scans in benchmark SPS1 and run plan of Table IV are given in Table VIII, together with the combination in quadrature for the end point and scan mass errors, where both are available.

Similar mass error estimates for the benchmark RR1, worked out in less detail at the Workshop for the run plan in Table III, are also given in Table VIII. In general, we expect that the precisions for the RR1 case will be better than for SPS1, owing to the smaller sparticle masses (and higher cross-sections), and to the smaller decay branching ratios into τ's. Specific differences between any two benchmarks always exist. The decay $\tilde{\chi}_3{}^0 \to \tilde{\chi}_1{}^0 Z$, open in the SPS1 case but not the RR1 case, is illustrative of this.

3.3. SUSY model parameter determination

Once we have measured sparticle masses, we will want to estimate the underlying supersymmetry parameters, and to probe the character of the SUSY-breaking. We have noted above that in general it will be a challenge to determine the nature of the SUSY-breaking model, but the totality of information from LHC and LC should give us good indicators. The recent work [12] analyzing the renormalization group evolution of masses suggests that at least it is possible to cleanly distinguish the class of SUSY model (e.g. mSUGRA *vs.* gauge mediated SUSY).

It is a separate matter to ask, given the hypothesis of the SUSY model, how well its parameters may be determined. For the two SUSY points considered here, we have made an estimate of the precision on the underlying SUSY parameters assuming that mSUGRA is at work. For the mSUGRA scenarios considered here, we expect that the errors on m_0 and $m_{1/2}$ are mainly determined by the errors on the $(\tilde{e}_R, \tilde{\mu}_R)$ and $(\tilde{\chi}_1{}^\pm, \tilde{\chi}_2{}^\pm)$ masses respectively. The errors on A_0 and $\tan\beta$ should be primarily controlled by the errors on $(\tilde{\tau}_1, \tilde{\tau}_2)$ and $(\tilde{\chi}_1{}^\pm, \tilde{\chi}_1{}^0)$ masses respectively.

We use the full set of mass error estimates of Table VIII for the SPS1 benchmark point and propagate them to give the mSUGRA parameter errors. These agree well with those given in [4] by the above simplified relations, after scaling for the number of observed events. The resulting mSUGRA parameter estimates are given in Table IX. These estimates are conservative since they

Table VIII Mass precision estimates in GeV for benchmark point SPS1 for end point (EP), threshold scan (TH) and combined measurements, and the combined estimates for the RR1 point.

	SPS1			RR1
particle	δM_{EP}	δM_{TH}	δM_{SPS1}	δM_{RR1}
$\tilde{e}_R$	0.19	0.02	0.02	0.02
$\tilde{e}_L$	0.27	0.30	0.20	0.20
$\tilde{\mu}_R$	0.08	0.13	0.07	0.13
$\tilde{\mu}_L$	0.70	0.76	0.51	0.30
$\tilde{\tau}_1$	$\sim 1-2$	0.64	0.64	0.85
$\tilde{\tau}_2$	–	0.86	0.86	1.34
$\tilde{\nu}_e$	0.23	–	0.23	0.4
$\tilde{\nu}_\mu$	7.0	–	7.0	0.5
$\tilde{\nu}_\tau$	–	–	–	10.0
$\tilde{\chi}_1^0$	0.07	–	0.07	0.07
$\tilde{\chi}_2^0$	$\sim 1-2$	0.12	0.12	0.30
$\tilde{\chi}_3^0$	8.5	–	8.5	0.30
$\tilde{\chi}_4^0$	–	–	–	observed
$\tilde{\chi}_1^\pm$	0.19	0.18	0.13	0.09
$\tilde{\chi}_2^\pm$	4.1	–	4.1	0.25

Table IX Errors on mSUGRA mass parameters for the SPS1 and RR1 hypotheses.

parameter	SPS1	RR1
m_0	100 ± 0.08 GeV	100 ± 0.04 GeV
$m_{1/2}$	250 ± 0.20 GeV	200 ± 0.22 GeV
A_0	0 ± 13 GeV	0 ± 18 GeV
$\tan\beta$	10 ± 0.47	3 ± 0.05

do not include potential information from stop masses, nor from the heavier Higgs sector and these may be expected to help materially. Similarly, information on the polarized cross-sections should help to further constrain A_0.

4. Higgs boson

The Higgs boson properties should be determined with as high accuracy as possible to seek departures from the SM and constrain the parameters of potential new physics models. Previous studies [2][13] have estimated the errors on the Higgs mass, cross-sections, total and partial widths, and branching ratios for $m_H = 120$ GeV. These studies have used the Higgs bosons produced in the reaction $e^+e^- \to ZH$ only. They use multivariate analyses based on information from jet topology and separated vertex information to extract the fermionic branching ratios statistically. Combination of the cross-sections for ZH and $\nu\nu H$ (WW fusion) and total width measurements allows the determination of the bosonic couplings.

Using the ZH cross-section as a function of $\sqrt{s}$ from Pythia, and the scenarios proposed in Section 2, we find that our run plans produce as many ZH as would be obtained in dedicated running at 350 GeV with 550 (650) fb^{-1}or with 1280 (1350) fb^{-1}at 500 GeV for the run plans of Table IV (Table III). We do not expect that operation of the collider at several different energies, as envisioned in our run plans, will materially degrade the Higgs studies, as it is mostly just the number of ZH events that matters. In Table X we show the estimated Higgs parameter errors obtained by statistical scaling from the number of ZH events in the run scenario of Table IV. The errors for the run plan of Table III are about 10% better.

We note that the top quark cross section near threshold depends upon the $t\bar{t}H$ Yukawa coupling. Ref [13] indicates that a 14% variation of top Yukawa coupling results in a 2% change in $\sigma_{t\bar{t}}$. However, if LC operation above the $t\bar{t}H$ threshold is possible, direct measurement of the cross-

Table X Relative errors (in %) on Higgs mass, cross-section, total width, branching ratios and Yukawa couplings (λ) for the run plan of Table IV.

Parameter	error	Parameter	error
Mass	0.03	$\Gamma_{\rm tot}$	7
$\sigma(ZH)$	3	λ_{ZZH}	1
$\sigma(WW)$	3	λ_{WWH}	1
BR($b\bar{b}$)	2	λ_{bbH}	2
BR($c\bar{c}$)	8	λ_{ccH}	4
BR($\tau^+\tau^-$)	5	$\lambda_{\tau\tau H}$	2
BR(gg)	5	λ_{ttH}	30

section will give an improved precision. References [2][13] indicate that 1000 fb^{-1} at $\sqrt{s}$ = 800 GeV will result in $\delta\lambda_{t\bar{t}H}$ = 5.5% for m_H = 120 GeV, degrading to about 25% at 500 GeV.

5. Top Quark

The top quark parameters are determined from the scan near the $t\bar{t}$ threshold at 350 GeV. The statistical errors [2] [13] are small compared to the uncertainties in the theoretical errors arising from the QCD theory.

The top quark mass parameter may be defined in several ways; the pole mass used in the Tevatron experimental studies is uncertain at the level of 0.5 GeV due to non-perturbative renormalon effects. If one uses alternate mass definitions, such as one half of the toponium quasi-bound state mass, the non-perturbative effects are reduced.

The top quark width can be determined from the $t\bar{t}$ threshold scan since the cross-section at the 1S bound state energy is proportional to $1/\Gamma_t$. Added information on the width can be obtained from the forward-backward asymmetry which is non-zero due to interference of diagrams involving the $t\bar{t}\gamma$, $t\bar{t}Z$ and $t\bar{t}H$ couplings.

Ref. [2] [13] suggest that the top quark mass should be measured with an error of 150 MeV. The width, expected to be about 1.4 GeV in the SM, should be determined to within 5%.

6. Summary

We have examined how a Linear Collider program of 1000 fb^{-1}could be constructed in the case that a very rich program of new physics is accessible at $\sqrt{s} \leq 500$ GeV. We have examined possible run plans that would allow the measurement of the parameters of a 120 GeV Higgs boson, the top quark, and could give information on the sparticle masses in SUSY scenarios in which many states are accessible.

We find that the construction of the run plan (the specific energies for collider operation, the mix of initial state electron polarization states, and the use of special e^-e^- runs) will depend quite sensitively on the specifics of the supersymmetry model, as the decay channels open to particular sparticles vary drastically and discontinuously as the underlying SUSY model parameters are varied. We have explored this dependence somewhat by considering two rather closely related SUSY model points. We have called for operation at a high energy to study kinematic end points, followed by runs in the vicinity of several two body production thresholds once their location is determined by the end point studies. For our benchmarks, the end point runs are capable of disentangling most sparticle states through the use of specific final states and beam polarizations. The estimated sparticle mass precisions, combined from end point and scan data, are given in Table VIII and the corresponding estimates for the mSUGRA parameters are in Table IX.

The precision for the Higgs boson mass, width, cross-sections, branching ratios and couplings are given in Table X. The errors on the top quark mass and width are expected to be dominated by the systematic limits imposed by QCD non-perturbative effects.

The run plan devotes at least two thirds of the accumulated luminosity near the maximum LC energy, so that the program would be sensitive to unexpected new phenomena at high mass scales.

We conclude that with a 1 ab^{-1} program, expected to take the first 6 – 7 years of LC operation, one can do an excellent job of providing high precision measurements with which to probe the nature of the new physics, and which will give complementary and improved information over that obtained at the LHC.

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