Study of $K_L^0 \to \pi^0\mu^+\mu^-$ Decays

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We have examined the decay $K_L^0 \to \pi^0\mu^+\mu^-$ in which the branching ratio, the muon energy asymmetry and the muon decay asymmetry could be measured. In particular, we find that within the Standard Model the longitudinal polarization (P_L) of the muon is proportional to the direct CP violating amplitude. On the other hand the energy asymmetry and the out-of-plane polarization (P_N) depend on both indirect and direct CP violating amplitudes. Although the branching ratio is small and difficult to measure because of background, the asymmetries could be large $\mathcal{O}(1)$ in the Standard Model. A combined analysis of the energy asymmetry, P_L and P_N could be used to separate indirect CPV, direct CPV, and CP conserving contributions to the decay.

There are three possible contributions to the $K_L^0 \to \pi^0 l^+ l^-$ decay amplitude: 1) direct CP-violating contribution from electroweak penguin and W-box diagrams, 2) indirect CP-violating amplitude from the $K_1 \to \pi^0 l^+ l^-$ component in K_L, and 3) CP-conserving amplitude from the $\pi^0\gamma\gamma$ intermediate state [1, 2]. The sizes of the three contributions depend on whether the final-state lepton is an electron or a muon. The CP-conserving two-photon contribution to the electron mode is expected to be $(1-4)\times 10^{-12}$, based on $K_L \to \pi^0\gamma\gamma$ data. Although suppressed in phase space, this contribution to the muon mode is comparable to the electron mode because of the scalar form factor which is proportional to lepton mass[3]. The interesting direct CP-violating component must be extracted from any signal found for $K_L \to \pi^0 l^+ l^-$ in the presence of two formidable obstacles: the theoretical uncertainty on contamination from indirect CP-violating and CP-conserving contributions, and the experimental background from $l^+l^-\gamma\gamma$ [4].

To subtract the indirect CP-violating and CP conserving contributions, several authors have examined the use of measurements from $K_L \to \pi^0\gamma\gamma$, $K_S \to \pi^0 e^+ e^-$, as well as the lepton energy asymmetry[2, 5, 6, 7, 8]. With better measurements expected in the near future, it is useful to reexamine $K_L^0 \to \pi^0 l^+ l^-$decays [9, 10].

In this paper, we examine if the muon decay asymmetries give additional constraints on CP violation in $K_L^0 \to \pi^0\mu^+\mu^-$. Indeed, we find that within the Standard Model the P-odd longitudinal polarization of the muon is non-zero only if the direct CP violating amplitudes are non-zero. We also find that other asymmetries that involve the polarization of both muons can be constructed to isolate the direct and indirect contributions to CP violation.

We will proceed in analogy to the charged version of the decay, $K^+ \to \pi^+\mu^+\mu^-$ which has been analyzed extensively [11, 12, 13, 14, 15, 16]. The structure of $K_L \to \pi^0 l^+ l^-$ decays is more complex than that of $K^+ \to \pi^+ l^+ l^-$ decays because of CP suppression. In particular, the one-photon intermediate state contribution to the vector form factor (F_V), which is expected to be almost real and dominant for $K^+ \to \pi^+ l^+ l^-$ decays, is CP-suppressed in $K_L^0 \to \pi^0 l^+ l^-$ decays.

The $K_L^0 \to \pi^0 l^+ l^-$decay process can have contributions from scalar, vector, pseudo-scalar and axial-vector interactions, with corresponding form factors, F_S, F_V, F_P, and F_A[11]:

$$\begin{aligned}\mathcal{M} &= F_S\bar{u}(p_l,s)v(\bar{p}_l,\bar{s}) + F_V p_k^\mu \bar{u}(p_l,s)\gamma_\mu v(\bar{p}_l,\bar{s})\\ &+ F_P\bar{u}(p_l,s)\gamma_5 v(\bar{p}_l,\bar{s}) + F_A p_k^\mu \bar{u}(p_l,s)\gamma_\mu\gamma_5 v(\bar{p}_l,\bar{s})\end{aligned} \tag{1}$$

Here p_k, p_π, p_l, and $\bar{p}_l$ are the kaon, pion, lepton, and antilepton 4-momenta. Transverse muon polarization effects arise from interference between terms with non-zero phase differences, while longitudinal polarization results from interference between terms with the same phase.

The scalar form factor, F_S, is expected to get a contribution from only the two-photon intermediate state, $K_L \to \pi^0\gamma\gamma \to \pi^0\mu^+\mu^-$. Pseudo-scalar, F_P, and axial-vector, F_A, get contributions from the short-distance "Z-penguin" and "W-box" diagrams only, where the dominant term arises from t-quark exchange; both form factors therefore depend on $V_{ts}V_{td}^*$, with a small contribution from the charm quark. $F_V = F_V^+ + F_V^-$ where F_V^+ is the CP-even contribution from the two photon process and is proportional to $p_k \cdot (p_l - \bar{p}_l)$. F_V^- is the CP-odd contribution, the sum of F_V^{MM}, the indirect CP violation in K_L decays, and F_V^{dir}, the short distance, direct CP violating contribution. Unlike K^+, all the amplitudes in the K_L decay are likely to be of the same order of magnitude, and polarization effects should therefore be large ($\mathcal{O}(1)$), unless there are strong cancellations [17].

In reference [18] we show the symmetries of the decay in the $l^- l^+$ CM frame in detail. In that analysis we show that it is possible to get information about CP violating amplitudes by measuring asymmetries for only one of the leptons. It is well-known[19] that the out-of-plane polarization of the muon, transverse to the plane formed by the π and the lepton momenta, gets contributions from CP-violating amplitudes, but several effects are mixed up and it is not possible to give a separation of the direct and the indirect pieces. It is, however, not well-known that more information can be obtained from the parity-violating single lepton longitudinal asymmetries. This longitudinal polarization has the property that although it is not CP violating, it is zero if there is no direct CP violation.

We will show our numerical results in the kaon rest frame. We concentrate on four measurable quantities: the total decay rate, the energy asymmetry between the muon and the anti-muon, the out-of-plane, and the longitudinal components of the muon polarization. The in-plane transverse polarization should also be considered when designing an experiment.

For the purposes of discussing possible experiments, it is useful to have the lepton polarization given in a covariant way. The form given in Eq. 2 is the same as for $K^+ \to \pi^+ l^- l^+$ [11]. In Eq. 2, $q^2 = (p_l + \bar{p}_l)^2$, and s denotes the covariant spin vector of the lepton. The decay rate in the kaon rest frame is given in Eq. 3 [11].

$$\begin{aligned}
P(s) &= \{-2Re(F_S F_P^*) m (s \cdot \bar{p}_l) + 2Re(F_V F_A^*) m [2(s \cdot p_k)(\bar{p}_l \cdot p_k) - m_K^2 (s \cdot \bar{p}_l)] \\
&- 2Re(F_P F_V^*)[-\frac{1}{2} q^2 (p_k \cdot s) + (p_k \cdot p_l)(\bar{p}_l \cdot s)] \\
&+ 2Re(F_S F_A^*)[\frac{1}{2}(q^2 - 4m^2)(p_k \cdot s) - (\bar{p}_l \cdot s)(p_l \cdot p_k)] \\
&+ [2Im(F_P F_A^*) + 2Im(F_S F_V^*)] \varepsilon^{\mu\nu\rho\sigma} p_{k\mu} p_{l\nu} \bar{p}_{l\rho} s_\sigma \}/(m^2 \rho)\,, \qquad (2)
\end{aligned}$$

$$\begin{aligned}
m^2 \rho_0(E_l, \bar{E}_l) &= |F_S|^2 \frac{1}{2}(q^2 - 4m^2) + |F_P|^2 \frac{1}{2} q^2 + |F_V|^2 m_k^2 (2E_l \bar{E}_l - \frac{1}{2} q^2) \\
&+ |F_A|^2 m_k^2 [2E_l \bar{E}_l - \frac{1}{2}(q^2 - 4m^2)] + 2Re(F_S F_V^*) m m_k (\bar{E}_l - E_l) \\
&+ 2Re(F_P F_A^*) \frac{m}{2} (m_k^2 - M_\pi^2 + q^2)]. \qquad (3)
\end{aligned}$$

The total decay rate is given by

$$\Gamma = \frac{m^2}{2m_k} \int \rho_0(E_l, \bar{E}_l) \frac{dE_l d\bar{E}_l}{(2\pi)^3}. \qquad (4)$$

The spin vector s_L in the direction of the μ^- momentum in the kaon rest frame is

$$s_L = (p, E_l \sin\theta_{l\bar{l}}, 0, E_l \cos\theta_{l\bar{l}})/m \qquad (5)$$

where we take the decay to be in the $x-z$ plane with $\vec{p}_l$ pointing in the z-direction. Then the longitudinal polarization of μ^- in the kaon rest frame is given as

$$\begin{aligned}
P_L &= [-2Re(F_S F_P^*)(\bar{E}_l - \frac{E_l}{p_l^2} \vec{p}_l \cdot \vec{\bar{p}}_l)) \\
&+ 2Re(F_V F_A^*) m_k^2 (\bar{E}_l + \frac{E_l}{p_l^2} \vec{p}_l \cdot \vec{\bar{p}}_l)) \\
&+ 2Re(F_P F_V^*) m_k (m + \frac{m}{p_l^2} \vec{p}_l \cdot \vec{\bar{p}}_l)) \\
&+ 2Re(F_S F_A^*) m_k (-m + \frac{m}{p_l^2} \vec{p}_l \cdot \vec{\bar{p}}_l))] \frac{p_l}{(m^2 \rho_0)} \qquad (6)
\end{aligned}$$

where $\vec{p}_l(\vec{\bar{p}}_l)$ and $E_l(\bar{E}_L)$ are the momentum and energy of $\mu^-(\mu^+)$. Notice that in this frame F_V^+ is odd under $E_l \leftrightarrow E_{\bar{l}}$, while F_V^- and the other form factors are even under the same interchange. Above equation also shows explicitly that P_L is zero if the direct amplitudes F_A and F_P are zero.

The transverse (out of decay plane) polarization perpendicular to the muon momentum vector in the kaon center of mass frame is given as

$$P_N = 2[Im(F_S F_V^*) + Im(F_P F_A^*)]\, p_l \frac{\bar{p}_l \sin\theta_{l\bar{l}}}{(m^2\rho_0)}. \tag{7}$$

It should be noted that F_P and F_A are in phase and therefore do not contribute to P_N.

For completeness, we also give the expression for the transverse, in-the-plane polarization P_T. For this the spin vector is $s_T = (0, \cos\theta_{l\bar{l}}, 0, -\sin\theta_{l\bar{l}})$, and the expression is as follows:

$$\begin{aligned} P_T &= -[2Re(F_S F_P^*)\, m \\ &+ 2Re(F_P F_V^*)\, m_K\, E_l \\ &+ 2Re(F_S F_A^*)\, m_K\, E_l \\ &+ 2Re(F_V F_A^*)\, m\, m_K^2\,]\, \bar{p}_l \sin\theta_{l\bar{l}}/(m^2\rho_0). \end{aligned} \tag{8}$$

It depends on the same quantities as P_L and, depending on the experimental configuration, the measured quantity may be a linear combination of the two.

We have considered the form-factors in detail in reference [18]. The values that we have chosen to use come from earlier work by other authors. Briefly, we have used the analysis of D'Ambrosio *et al.*[6] for F_V^{MM}, which is parameterized in terms of a single parameter a_S. We have used the analysis of Donoghue and Gabbiani[5] to evaluate the short distance form factors: F_V^{dir}, F_A, and F_P. We have also used the chiral perturbation theory calculation of Donoghue and Gabbiani[5] for F_V^+, which is expressed in terms of a single parameter, α_V. Since [5] is concerned only with electron final states, the scalar contribution, F_S, is negligible and they do not calculate it. Therefore we use the earlier result of Ecker *et al*[19]. F_S is the largest contribution for our purposes.

Using the form factors discussed in [18] the total decay rate is about $\sim 6.6 \times 10^{-12}$, and it is dominated by the scalar interaction which makes most of the contribution for mu-mu mass above 280 MeV. This is because of the two pion loop contribution embodied in the form factor F_S. Unlike F_S, the rest of the form factors are expected to give contributions that fall with q^2. The contribution from F_P is small because of the suppression due to the lepton mass. The region $\sqrt{q^2} < 280$ MeV will be affected by interference effects as shown in figure 1. It is interesting to note that, using the above described form factors and parameters [18], the destructive interference causes the decay rate to be almost zero in the region where the μ^+ is at rest in the kaon rest frame. The longitudinal, in-plane transverse, and the out-of-plane polarizations of the μ^+ are shown in figures 2, 3, and 4. The energy asymmetry and the polarizations are large, but they also have large dependence on the parameter a_S and F_S. The $\mathcal{O}(p^4)$ calculation for F_S that we have used is likely to be inadequate. Other contributions to F_S, especially the vector meson contribution could easily change the form factor by a factor of 2. This serious problem must be overcome before the important parameters can be extracted from data because F_S is the dominant amplitude over much of the phase space. There could be a number of ways of constraining F_S as well as a_S. The branching ratio above some cut on $\mu - \mu$ mass could be used to fix F_S. The new expected measurement of $K_S \to \pi^0 e^+ e^-$[10] will lead to constraints on the magnitude of a_S. The out-of-plane polarization could be used to fix the sign as well as magnitude of a_S, and the short distance physics could be extracted by fitting the measured distribution of the longitudinal polarization which mainly comes from $Re(F_S F_A^*)$ and $Re(F_A F_V^*)$.

The modes $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ have not as yet been observed; the current best limits on the branching ratios for $K_L \to \pi^0 l^+ l^-$ were obtained by the KTeV experiment at FNAL; $B(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}$ and $B(K_L \to \pi^0 e^+ e^-) < 5.1 \times 10^{-10}$ [24, 25]. These limits were based on 2 observed events in each case, and expected backgrounds of 0.87 ± 0.15 for the muon mode and 1.06 ± 0.41 for the electron mode. The main backgrounds for the muon mode were estimated to be from $\mu^+\mu^-\gamma\gamma$ (0.37 ± 0.03) and $\pi^+\pi^-\pi^0$ (0.25 ± 0.09), in which both charged pions decay in flight. Of these, the former background could be irreducible and therefore of great concern. For any future experiment it seems unlikely that the background due to $\mu^+\mu^-\gamma\gamma$ can be lowered. The signal to background ratio, assuming that only $\mu^+\mu^-\gamma\gamma$ will

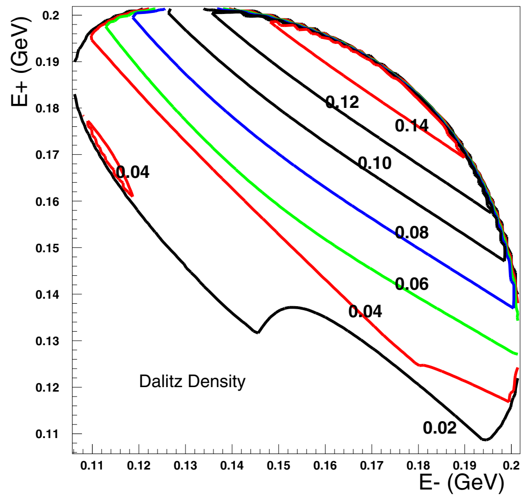

Figure 1: Dalitz decay distribution for $K_L \to \pi^0\mu^+\mu^-$. The units for the decay rate contours are arbitrary. The total calculated branching fraction was 6.6×10^{-12}. The form factors and the values of the parameters used are described in the text.

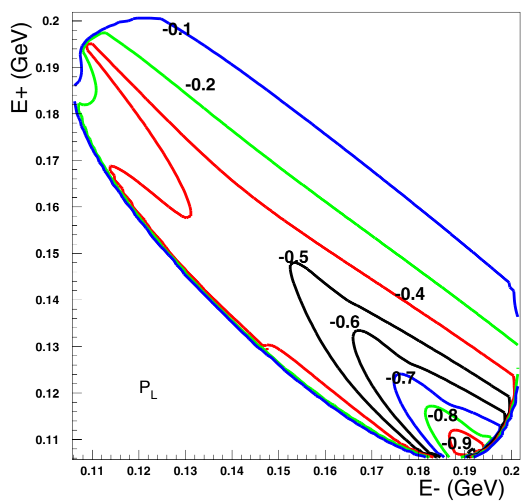

Figure 2: Longitudinal (P_L) polarization of μ^+ in $K_L \to \pi^0\mu^+\mu^-$ decay plotted as a function of μ^+ and μ^- energy in the kaon rest frame.

contribute in a future experiment, will be around 1/5, if the standard-model signal is taken as $B(K_L \to \pi^0\mu^+\mu^-) \sim 5 \times 10^{-12}$.[17]

Measuring the muon polarization asymmetries in $K_L \to \pi^0\mu^+\mu^-$, together with the branching ratio and the lepton energy asymmetry, could be a good way of defeating the intrinsic background from CP-conserving and indirect CP-violating amplitudes and the experimental background from $\mu^+\mu^-\gamma\gamma$. The large predicted asymmetries could be measured with sufficient statistics at new intense proton accelerators such as the Brookhaven National Laboratory AGS, the Fermilab main injector, or the Japanese Hadron Factory. An examination of the functional form of the form factors F_S and F_V is needed to see if the present form in terms of the parameters a_V and a_S is

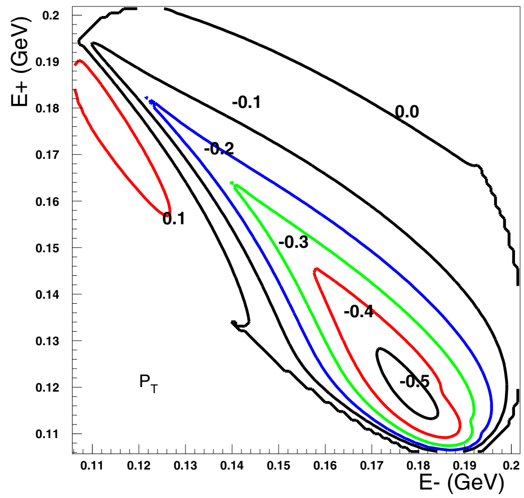

Figure 3: In-plane transverse (P_T) polarization of μ^+ in $K_L \to \pi^0\mu^+\mu^-$ decay plotted as a function of μ^+ and μ^- energy in the kaon rest frame.

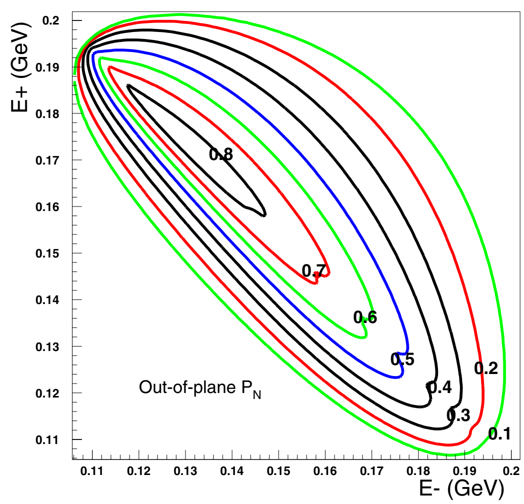

Figure 4: Out-of-plane (P_N) polarization of μ^+ in $K_L \to \pi^0\mu^+\mu^-$ decay plotted as a function of μ^+ and μ^- energy in the kaon rest frame.

adequate. Examination of the experimental technique to measure the different components of the polarization in the laboratory as well as the $\mu\mu\gamma\gamma$ background is needed to understand the possible sensitivity to the asymmetries.

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