Neutrino Oscillations with Two Δm^2 Scales

Irina Mocioiu and Robert Shrock
C. N. Yang Institute for Theoretical Physics,
State University of New York

An approximation that is often used in fits to reactor and atmospheric neutrino data and in some studies of future neutrino oscillation experiments is to assume one dominant scale, $\Delta m^2_{\rm atm} \sim 3 \times 10^{-3}$ eV^2. Here we investigate the corrections to this approximation arising from $\Delta m^2_{\rm sol}$, assuming the large mixing angle solution. We show that for values of $\sin^2(2\theta_{13})$ in the range of interest for long-baseline neutrino oscillation experiments terms involving $\Delta m^2_{\rm sol}$ can be comparable to the terms involving $\Delta m^2_{\rm atm}$. Accordingly, we emphasize the importance of performing a full three-flavor, two-Δm^2 analysis of the data on $\nu_\mu \to \nu_e$, $\nu_e \to \nu_\mu$, $\bar\nu_e \to \bar\nu_\mu$ oscillations.

1. Introduction

There is increasingly strong evidence for neutrino oscillations, and thus neutrino masses and lepton mixing. All solar neutrino experiments that have reported results (Homestake, Kamiokande, SuperKamiokande, SAGE, GALLEX/GNO and SNO) show a significant deficit in the neutrino fluxes coming from the Sun [1]. This deficit can be explained by oscillations of the ν_e's into other weak eigenstate(s). The currently favored region of parameters to fit this data is the large mixing angle solution (LMA) with $\tan^2\theta_{12} \sim 0.4$ and $\Delta m^2_{\rm sol} \lesssim 5 \times 10^{-5}$ eV^2 [1, 2]. Solutions yielding lower-likelihood fits to the data include the LOW solution with $\Delta m^2_{\rm sol} \sim 10^{-7}$ eV^2 and essentially maximal mixing and the small mixing angle solution (SMA), with $\tan^2\theta_{12} \sim 4 \times 10^{-4}$ and $\Delta m^2_{\rm sol} \lesssim 5 \times 10^{-6}$ eV^2 Another piece of evidence for neutrino oscillations is the atmospheric neutrino anomaly, observed by Kamiokande, IMB, Soudan, SuperKamiokande with the highest statistics, and MACRO [3]. The SuperK experiment has fit its data by the hypothesis of $\nu_\mu \to \nu_\tau$ oscillations with $\Delta m^2_{\rm atm} \sim 3 \times 10^{-3}$ eV^2 and maximal mixing, $\sin^2 2\theta_{\rm atm} = 1$. The possibility of $\nu_\mu \to \nu_s$ oscillations involving light electroweak-singlet ("sterile") neutrinos has been disfavored by SuperK, and the possibility that $\nu_\mu \to \nu_e$ oscillations might play a dominant role in the atmospheric neutrino data has been excluded both by SuperK and, for the above value of $\Delta m^2_{\rm atm}$, by the Chooz and Palo Verde reactor antineutrino experiments. The K2K long-baseline neutrino experiment between KEK and Kamioka has also reported results [4] which are consistent with the SuperK fit to its atmospheric neutrino data. The LSND experiment has reported evidence for $\bar\nu_\mu \to \bar\nu_e$ and $\nu_\mu \to \nu_e$ oscillations with $\Delta m^2_{\rm LSND} \sim 0.1 - 1$ eV^2 and a range of possible mixing angles. This result is not confirmed, but also not completely ruled out, by a similar experiment, KARMEN. The solar and atmospheric data can be fit in the context of three-flavor neutrino oscillations; global fits include [5]. We shall work within this context of three-flavor neutrino mixing. The fact that these inferred values of neutrino mass squared differences satisfy the hierarchy $|\Delta m^2_{\rm sol}| << |\Delta m^2_{\rm atm}|$ has led to using the approximation where one neglects $\Delta m^2_{\rm sol}$ compared with $\Delta m^2_{\rm atm}$ for most analyses of atmospheric neutrino data. For certain neutrino oscillation transitions, such as $\nu_\mu \to \nu_\tau$, this is a good approximation. It is worthwhile, however, to have a quantitative evaluation of the corrections to this approximation and a determination of the ranges of parameters where these corrections could become significant. This is relevant to planning for both neutrino factory and conventional beam experiments [6]-[8]. For sufficiently small values of the lepton mixing angle θ_{13} (defined below in eq. (1)), e.g., $\sin^2 2\theta_{13} \sim 10^{-2}$, and sufficiently large values of $\Delta m^2_{\rm sol}$, e.g., $\Delta m^2_{\rm sol} \sim 10^{-4}$ eV^2, this approximation is not reliable for certain oscillation channels such as $\nu_\mu \to \nu_e$. Here we are referring to CP-conserving quantities; the one-Δm^2 approximation is, of course, not used for calculating CP-violating quantities since, since the CP violating effects disappear in this limit. Since the values of $\sin^2(2\theta_{13})$ and $\Delta m^2_{\rm sol}$ for which the one-Δm^2 approximation breaks down are in the range of interest for future experimental searches for $\nu_\mu \to \nu_e$ via both conventional neutrino beams generated by pion decay and via neutrino beams from neutrino "factories" based on muon storage rings, this complicates the analysis of the sensitivity and data analysis from these experiments.

2. Neutrino Oscillations on Long Distances

In the framework of three active neutrinos, the unitary transformation relating the mass eigenstates ν_i, $i = 1, 2, 3$, to the weak eigenstates ν_a is given by $\nu_a = \sum_{i=1}^{3} U_{ai}\nu_i$ where the lepton mixing matrix is

$$U = R_{23}KR_{13}K^*R_{12}K' = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K' \quad (1)$$

Here R_{ij} is the rotation matrix in the ij subspace, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, $K = \mathrm{diag}(e^{-i\delta}, 1, 1)$ and $K' = \mathrm{diag}(1, e^{i\delta_1}, e^{i\delta_2})$ involves further possible phases (due to Majorana mass terms) that do not contribute to neutrino oscillations (as can be seen from the invariance of the quantity $K_{ab,ij}$ below under neutrino field rephasings). One can take $\theta_{ij} \in [0, \pi/2]$ with $\delta \in [0, 2\pi)$.

In vacuum, the probability that a weak neutrino eigenstate ν_a becomes ν_b after propagating a distance L (assuming that $E >> m(\nu_i)$ and the propagation of the mass eigenstates is coherent) is

$$\begin{aligned} P(\nu_a \to \nu_b) &= \delta_{ab} - 4\sum_{i>j=1}^{3} Re(K_{ab,ij})\sin^2\phi_{ij} \\ &\quad +4\sum_{i>j=1}^{3} \mathrm{Im}(K_{ab,ij})\sin\phi_{ij}\cos\phi_{ij} \end{aligned} \quad (2)$$

where $K_{ab,ij} = U_{ai}U_{bi}^*U_{aj}^*U_{bj}$, $\Delta m_{ij}^2 = m(\nu_i)^2 - m(\nu_j)^2$ and $\phi_{ij} = \Delta m_{ij}^2 L/4E$

In matter the evolution of the weak eigenstates is given by:

$$i\frac{d}{dx}\nu = \left(\frac{1}{2E}UM^2U^\dagger + V\right)\nu \quad (3)$$

where

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (4)$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \; V = \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

We show here results for the case of large pathlengths and the $\nu_\mu \to \nu_e$ transition relevant to the existing data on atmospheric oscillations. For simplicity, we take the CP-violating phase equal to zero here, but it is straightforward to include it.

These results can also be applied to data on $\nu_e \to \nu_\mu$ that might become available with a possible future neutrino factory. In Figure 1 we plot $P(\nu_\mu \to \nu_e)$ as a function of L/E for $\sin^2 2\theta_{23} = 1$, $\Delta m_{32}^2 = 3\times10^{-3}$ eV^2, $\sin^2 2\theta_{13} = 0.04$, $\sin^2 2\theta_{12} = 0.8$, and $\Delta m_{21}^2 = 2\times10^{-4}$ eV^2, the upper end of the LMA region. The higher-frequency oscillations are driven by the terms involving $\sin^2\phi_{32}$ while the lower-frequency oscillation is driven by the terms involving Δm_{21}^2. The one-Δm^2 approximation is shown as the dashed curve; of course, this lacks the low-frequency oscillation component. One sees that the full calculation differs strikingly from the result of the one-Δm^2 approximation. Even for the best-fit LMA solution, the effect of Δm_{21}^2 can be large for large pathlengths, and this would affect the $\nu_\mu \leftrightarrow \nu_e$ oscillations in atmospheric neutrino data, as shown in Figure 2, for which we take the central values of $\sin^2 2\theta_{21}$ and Δm_{21}^2 in the LMA fit, and other parameters the same as in the previous figure. Note that for the dominant $\nu_\mu \to \nu_\tau$ transition in the atmospheric neutrinos, Δm_{21}^2 effects are not so important; this is clear from the fact that this transition does not directly involve ν_e. We next show, in Figure 3, the result of integrating (3) in the full three-flavor mixing scenario and using the actual density profile of the Earth. For this

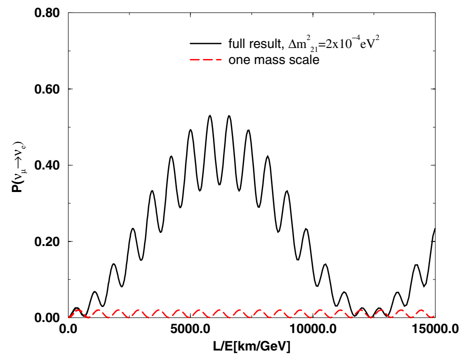

Figure 1: Plot of $P(\nu_\mu \to \nu_e)$ as a function of L/E for $\sin^2\theta_{23} = 1$, $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{12} = 0.8$, and $\Delta m^2_{21} = 2 \times 10^{-4}$ eV^2, and $\sin^2 2\theta_{13} = 0.04$.

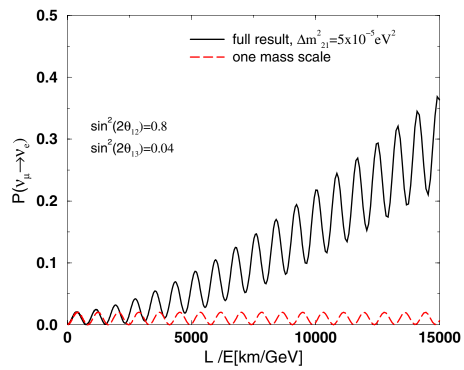

Figure 2: Plot of $P(\nu_\mu \to \nu_e)$ as a function of L/E for $\sin^2\theta_{23} = 1$, $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2, central LMA values $\sin^2 2\theta_{12} = 0.8$ and $\Delta m^2_{21} = 5 \times 10^{-5}$ eV^2, and $\sin^2 2\theta_{13} = 0.04$.

figure we use $\sin^2 2\theta_{23} = 1$, $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2, $\sin^2(2\theta_{12}) = 0.8$, $\Delta m^2_{21} = 5 \times 10^{-4}$ eV^2, and $\sin^2(2\theta_{13}) = 0.04$. As expected, the Δm^2_{21} corrections are big for low energies and large distances. For the choice of a large distance, $L = 10^4$ km (shown in Figure 3), we observe a very significant difference between the full calculation and the one-Δm^2 approximation. This shows again (as do the recent illustrative studies in Ref. [9]), that it is useful to carry out a more complete analysis of the SuperK and other atmospheric neutrino data with not just three-flavor oscillations, but also two Δm^2 values included. Although the SuperK fit to its data shows that the $\nu_\mu \leftrightarrow \nu_e$ oscillations make a small contribution, it is important to include this contribution correctly, and the one-Δm^2 approximation is not, in general, reliable for this transition. Presently, there is a very extended planning effort for experiments based on either very high intensity conventional neutrino beams or beams from muon storage rings. These will probe the $\nu_e - \nu_\mu$ transition with very high sensitivity. In this case the one-Δm^2 approximation used in many planning studies may well be inadequate, and one should use a more general theoretical framework. If Δm^2_{21} and $\sin^2 2\theta_{21}$ are at the upper end of the LMA region, then the one-Δm^2 approximation can break down. As a numerical example, one can consider the parameter set $\sin^2 2\theta_{12} = 0.8$, $\Delta m^2_{21} = 2 \times 10^{-4}$ eV^2, $\sin^2 2\theta_{13} = 0.01$, $\delta = \pi/6$, with the usual SuperK values $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2 and $\sin^2 2\theta_{23} = 1$. Further, take the JHF-SuperK pathlength $L = 295$ km and narrow-band-beam energy $E = 0.7$ GeV. Then, if one were to evaluate the $\nu_\mu \to \nu_e$ oscillation probability using the one-Δm^2 approximation, one would obtain $P(\nu_\mu \to \nu_e) = 5.0 \times 10^{-3}$. However, correctly including the contribution from the term involving $\sin^2\phi_{21}$, one gets an oscillation probability that is more than twice as large as the one predicted by the one-Δm^2 approximation: $P(\nu_\mu \to \nu_e) = 1.4 \times \times 10^{-2}$ This clearly shows that for experimentally allowed input parameters involving the LMA solar fit, and in particular, for a value of $\sin^2 2\theta_{13}$ that can be probed by the JHF-SuperK experiment and others that could achieve comparable sensitivity, the one-Δm^2 approximation may not be valid. Thus, it is important that the KamLAND experiment will test the LMA and anticipates that, after about three

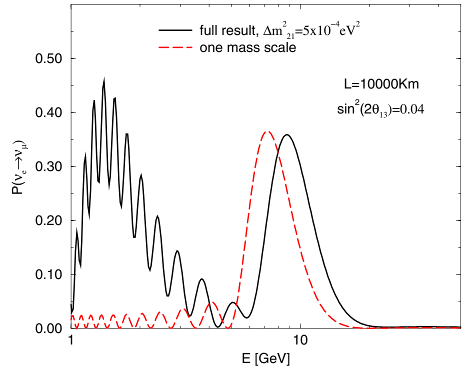

Figure 3: Plot of $P(\nu_\mu \to \nu_e)$ as a function of E for $\sin^2\theta_{23} = 1$, $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2, and $L = 10^4$ km, with other input values as shown. This calculation takes account of the full density profile of the earth.

years of running, it will be sensitive to the level $\Delta m^2_{\rm sol} \lesssim 10^{-5}$ eV^2. If, indeed, the LMA parameter set is confirmed by KamLAND, then it may well be necessary to take into account three-flavor oscillations involving two independent Δm^2 values in the data analysis for the JHF-SuperK experiment and other $\nu_\mu \to \nu_e$ neutrino oscillation experiments that will achieve similar sensitivity. This point is thus certainly also true for long-baseline experiments with a neutrino factory measuring $\nu_e \to \nu_\mu$, $\bar\nu_e \to \bar\nu_\mu$ oscillations, since they anticipate sensitivity to values of $\sin^2 2\theta_{13}$ that are substantially smaller than the level to which the JHF-SuperK collaboration will be sensitive, and as one decreases θ_{13} with other parameters held fixed, the $\sin^2\phi_{21}$ corrections to the one-Δm^2 approximation become relatively more important.

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