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April 28, 1989

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April 20, 1989

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SSRL is operated by the Department of Energy, Office of Basic Energy Sciences, Division of Chemical Sciences. Support for research by SSRL staff is provided by that Office's Division of Material Science. The SSRL Biotechnology Program is supported by the NIH, Biomedical Resource Technology Program, Division of Research Resources.

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In this note, I calculate the response of various circuits for driving a kicker magnet of inductance L with a thyatron switch tube (drawn as a switch) and a pulse-forming network (PFN) made of coaxial cable with characteristic impedance Z_0 . The PFN is slowly charged to a voltage V_0 during the 100 ms between magnet pulses.

A. Uncompensated Circuit (from Heinz-Dieter Nuhn)

From transmission-line theory, the PFN voltage can be considered the

sum of a rightward-going wave, with voltage V_+ and current $I_+ = V_+/Z_o$, and a leftward-going wave, with V_- and $I_- = V_-/Z_o$. The voltage on a load Z is

$$V = V_+ + V_- \quad (1)$$

The current is

$$I = I_+ - I_- = \frac{V_+ - V_-}{Z_o} \quad (2)$$

with

$$V = IZ \quad (3)$$

Therefore, $\frac{Z}{Z_o}(V_+ - V_-) = V_+ + V_-$ and so

$$\frac{V_-}{V_+} = \frac{Z - Z_o}{Z + Z_o} \quad (4)$$

$$I = \frac{2V_+}{Z + Z_o} \quad (5)$$

Before the switch closes, $Z = \infty$ and $V_- = V_+ = V_o/2$. After the switch closes, the leftward-going wave V_- changes immediately near the switch. The change propagates to the left end of the cable, reflects back, and finally, after a delay of twice the cable length, the V_+ wave at the load changes from $V_o/2$. Hence, for times less than the cable's round-trip time, we can consider V_+ to be constant and solve for I and V_- .

$$\begin{aligned}
L \frac{dI}{dt} + RI &= V = V_+ + V_- = V_+ + (V_+ - Z_o I) \\
\therefore L \frac{dI}{dt} + (R + Z_o) I &= 2V_+ \quad (6)
\end{aligned}$$

For $V_+ = V_o/2$ and $I = 0$ at $t = 0$, the solution is:

$$I = \frac{V_o}{R + Z_o} (1 - e^{-\frac{R+Z_o}{L}t}) \quad (7)$$

$$\therefore V_- = V_+ - Z_o I = \frac{V_o}{2(R + Z_o)} (R - Z_o + 2Z_o e^{-\frac{R+Z_o}{L}t}) \quad (8)$$

$$\therefore V = V_+ + V_- = \frac{V_o}{R + Z_o} (R + Z_o e^{-\frac{R+Z_o}{L}t}) \quad (9)$$

At $t = 0$, the load sees a voltage V_o , the full charging voltage. The voltage decays to $\frac{R}{R+Z_o} V_o$ with a time constant $\frac{L}{R+Z_o}$. If $R = Z_o$, the voltage decays to $V_o/2$. If $R = 0$, then the voltage decays to zero, since the inductor has no voltage drop once a constant current V_o/Z_o flows through it.

After the reflection (8) returns to the switch, it becomes the new V_+ driving term in (6). We can use it to calculate the decay of the magnet current, with the initial condition $I = \frac{V_o}{R+Z_o}$ at this time; the result is

$$I = \frac{V_o}{(R + Z_o)^2} \left[R - Z_o + 2Z_o \left(1 + \frac{R + Z_o}{L} t \right) e^{-\frac{R+Z_o}{L}t} \right] \quad (10)$$

If $R < Z_o$, then the current will drop below zero, shutting the thyatron off.

B. Compensated Circuit (from Heinz-Dieter Nuhn)

The previous circuit can be changed to match the load ($Z = Z_o$), and thereby eliminate the reflection.

With the choices of resistance and capacitance shown, the PFN sees a load equivalent to a simple resistance of value Z_o :

$$\begin{aligned}
 Z &= (Z_o + Ls) \parallel \left(Z_o + \frac{1}{Cs}\right) = \frac{(Z_o + Ls)(Z_o Cs + 1)}{LCs^2 + 2Z_o Cs + 1} \\
 &= \frac{(Ls + Z_o)\left(\frac{L}{Z_o}s + 1\right)}{\left(\frac{L}{Z_o}s\right)^2 + 2\frac{L}{Z_o}s + 1} = Z_o
 \end{aligned} \tag{11}$$

Accordingly, we set $L = 0$ and $R = Z_o$ in (6) through (10), resulting in an instantaneous rise to half the charging voltage:

$$I = \frac{V_o}{2Z_o} \quad (12)$$

$$V_- = 0 \quad (13)$$

$$V = \frac{V_o}{2} \quad (14)$$

After one round-trip PFN delay, V and I drop instantaneously to zero.

The magnet current I_L rises like an LR circuit:

$$L \frac{dI_L}{dt} + Z_o I_L = V_+ = V \quad (15)$$

$$\therefore I_L = \frac{V_o}{2Z_o} (1 - e^{-\frac{Z_o}{L}t}) \quad (16)$$

The remainder of the total current I in (12) goes into the capacitor. Since there is no reflected pulse V_- , after one PFN round-trip time, V_+ goes to zero. I_L decays following (15) :

$$I_L = \frac{V_o}{2Z_o} e^{-\frac{Z_o}{L}t} \quad (17)$$

We see that the compensated circuit uses only half the charging voltage. The uncompensated circuit with $R = 0$ has similar speed. Its turn off briefly increases the current at first [see (10)], but shuts off reasonably quickly (time constant L/Z_o) since the current tries to reverse and the thyatron shuts off. The uncompensated circuit with $R = Z_o$ has half

the time constant for rise, but uses only half the voltage. Its fall time is not twice as fast, since (10) doesn't reverse sign and the thyatron doesn't suddenly shut off.

Next we consider two circuits from Lawrence Berkeley Laboratory. Both give faster rise and fall times, and also use the full charging voltage.

C. Capacitor-Driven Rise with Two Thyratrons (from Ferd Voelker)

The first version of the LBL circuit has a very flexible rise time and a good fall time, but requires two thyratrons. The capacitor's thyatron fires

first, letting the magnet current rise sinusoidally:

$$I_L = V_1 \sqrt{\frac{C}{L}} \sin \frac{t}{\sqrt{LC}} \quad (18)$$

The second thyatron fires when the sine reaches its peak:

$$t = \frac{\pi}{2} \sqrt{LC} \quad (19)$$

$$I_L = V_1 \sqrt{\frac{C}{L}} \quad (20)$$

We intend to choose V_o and Z_o to hold the current constant at this value. Eq. (19) gives the rise time (to 100% of the flat-top value; the rise is not exponential, and we no longer have to worry about whether we need 90% or 99% of the flat top for an acceptable kick).

We choose V_o and Z_o by noting that, with I_L constant, L has zero voltage across it. When a transmission line is shorted, it delivers a current V_o/Z_o , and so we choose

$$\frac{V_o}{Z_o} = V_1 \sqrt{\frac{C}{L}} \quad (21)$$

The PFN is used only to maintain the flat top. The rise is determined only by the choice of C . Note also that V_o and Z_o are not tightly constrained; only their ratio V_o/Z_o is specified.

Eq. (21) can also be derived by a more careful analysis, which will be useful later on. After both thyratrons fire,

$$V_+ + V_- = L \frac{dI_L}{dt} \quad (22)$$

$$C \frac{d}{dt} (V_+ + V_-) + I_L = I_+ - I_- = \frac{V_+ - V_-}{Z_o} = I \quad (23)$$

$$\therefore LC \frac{d^2 I_L}{dt^2} + I_L = \frac{V_+}{Z_o} - \frac{1}{Z_o} \left(L \frac{dI_L}{dt} - V_+ \right)$$

$$LC \frac{d^2 I_L}{dt^2} + \frac{L}{Z_o} \frac{dI_L}{dt} + I_L = \frac{2V_+}{Z_o} \quad (24)$$

With $V_+ = \frac{V_o}{2}$ and $I_L(t=0) = \frac{V_o}{Z_o}$, $I_L = \frac{V_o}{Z_o}$ is the solution, giving (21).

What happens at the end of the pulse? Because L is a short, eq. (4) shows that the reflected pulse $V_- = -V_+ = -\frac{V_o}{2}$. After this pulse makes a round trip through the PFN, the load suddenly sees a new rightward-moving wave,

$$V_+ = -V_o/2 \quad (25)$$

with the new initial conditions at this time:

$$I_L = \frac{V_o}{Z_o} \quad (26)$$

$$\frac{dI_L}{dt} = 0 \quad (27)$$

Following (24), the current I_L changes from $\frac{V_o}{Z_o}$ to $-\frac{V_o}{Z_o}$ like a damped sinusoid:

$$I_L = \frac{V_o}{Z_o} [(\cos \omega t + \frac{\alpha}{\omega} \sin \omega t) 2e^{-\alpha t} - 1] \quad (28)$$

$$\text{with } \omega = \sqrt{\frac{1}{LC} - \alpha^2} \quad (29)$$

$$\alpha = \frac{1}{2Z_o C} \quad (30)$$

and we now define $t = 0$ to be the time when the reflection returns to the load. From (23), we can also get the PFN and capacitor currents:

$$I_C = -\frac{V_o}{Z_o} \left(\cos \omega t - \frac{\alpha}{\omega} \sin \omega t \right) 2 e^{-\alpha t} \quad (31)$$

$$I = I_L + I_C = \frac{V_o}{Z_o} \frac{4\alpha}{\omega} \sin \omega t e^{-\alpha t} \quad (32)$$

Thyratrons conduct current only in one direction, and shut off soon after a reversal. From (31) we see that the capacitor's thyatron will shut off first at

$$t = \frac{1}{\omega} \arctan \left(\frac{\omega}{\alpha} \right) \quad (33)$$

After this time, the decay becomes exponential, following (6), until $I < 0$ and the PFN's thyatron shuts off. This is the dominant decay time, and so the fall time is roughly L/Z_o .

D. Capacitor-Driven Rise with One Thyatron (from George Gabor)

George Gabor simplified circuit C by removing one thyatron. The capacitor is charged together with the PFN.

When the thyatron fires, the capacitor quickly puts a lot of current into the magnet, improving the rise over the circuit of page 1. With the switches closed, this circuit is the same as circuit C, and so its equation is just (24), with the initial conditions at $t = 0$:

$$I_L = 0 \quad (34)$$

$$\frac{dI_L}{dt} = \frac{V_o}{L} \quad (35)$$

and with $V_+ = \frac{V_o}{2}$. The solution is a damped sinusoid plus a constant:

$$I_L = \frac{V_o}{Z_o} \left[1 - \left(\cos \omega t + \frac{\alpha - \frac{Z_o}{L}}{\omega} \sin \omega t \right) e^{-\alpha t} \right] \quad (36)$$

with ω and α given by (29) and (30).

In the figure, the capacitor has been chosen to give a bit of overshoot. By choosing critical damping,

$$\omega = 0 \quad (37)$$

$$\therefore \frac{1}{LC} = \alpha^2 = \frac{1}{4Z_o^2 C^2} \Rightarrow C = \frac{L}{4Z_o^2} \quad (38)$$

$$\therefore \alpha = \frac{2Z_o}{L} \quad (39)$$

$$\therefore I_L = \frac{V_o}{Z_o} \left[1 - \left(1 + \frac{\alpha}{2} t \right) e^{-\alpha t} \right] \quad (40)$$

we get the fastest rise possible without overshoot. Note that the time constant $\frac{1}{\alpha}$ is half as long as in (7), with $R = 0$ and no capacitor. A solution with overshoot is even faster. If the magnet current can be, for example, within $\pm 5\%$ of the ideal current, then we choose C for a 5% overshoot [by making C a little larger than (38)] to speed the rise. Note that we get this fast rise [compared to (7)] without adding a resistor R , so that we use the full available current V_o/Z_o . This means that with the same 20 kV we have discussed, we can now double Z_o to 25Ω and further increase the speed.

What about the fall time? I will present the general expression and give the rule for when the thyatron shuts off. Specific numbers require choosing the amount of overshoot we want. I will also give the expression and some typical numbers for critical damping, and, as we shall see, they are very good.

To calculate the fall, we first find the reflected wave V_- , as we have earlier:

$$\begin{aligned} V_- &= L \frac{dI_L}{dt} - V_+ \\ &= V_o \left[\left(\cos \omega t + \frac{\alpha}{\omega} \sin \omega t \right) e^{-\alpha t} - \frac{1}{2} \right] \end{aligned} \quad (41)$$

After one PFN round-trip delay, this signal becomes the driving wave V_+ . We use this in (24), with the initial conditions (26) and (27). We get:

$$I_L = \frac{V_o}{Z_o} \left\{ \left[\cos \omega t + \frac{\alpha}{\omega} \left(1 + \frac{Z_o \alpha}{L \omega^2} \right) \sin \omega t \right] 2e^{-\alpha t} + \left[-\frac{\alpha}{\omega} \cos \omega t + \sin \omega t \right] \frac{t}{LC\omega} e^{-\alpha t} - 1 \right\} \quad (42)$$

The thyatron shuts off when $I_L = 0$.

Again, for critical damping the solution is easier, although not the fastest possible. In this case [see (37) to (40)],

$$I_L = \frac{V_o}{Z_o} \left[(2 + 2\alpha t + \alpha^2 t^2 + \frac{1}{3} \alpha^3 t^3) e^{-\alpha t} - 1 \right] \quad (43)$$

For typical values - $L = 1.25 \mu H$ and $Z_o = 25 \Omega$ (for a 30-cm magnet) - and for critical damping, we get

$$C = 500 pF \quad (44)$$

$$\alpha = 4 \times 10^7 s^{-1} \Rightarrow \frac{1}{\alpha} = 25 ns \quad (45)$$

The rise and fall times are given in the tables:

Rise		Fall	
Time	% of Flat-Top	Time	% of Flat-Top
20 ns	37.1	20 ns	98.2
40	63.7	40	84.2
60	80.0	60	55.7
80	89.4	80	20.5
100	94.5	90	3.0
120	97.2	91	1.3
140	98.6	92	-0.3
160	99.3	The thyatron then shuts off within ≈ 20 ns.	

E. Two Magnets In Parallel (from Tom Hostetler)

Circuit D can be further improved by driving two adjacent magnet modules in parallel, connecting them to a common feedthrough between the two.

The resulting magnet inductance is cut in half. However, because the inductance of the thyatron and feedthrough is not changed, the total inductance is not halved. Using Heinz-Dieter Nuhn's calculation of 31 March 1989, but

omitting the resistors since they are not used in this circuit, we have

One coil	780 nH
Feedthrough assembly	150 nH
Thyratron assembly	220 nH
TOTAL	1150 nH

For two coils in parallel, we then get 760 nH, 66% of the original value.

To drive this circuit, we must double the current. This can be done without increasing the voltage by cutting Z_o in half [see (40) and (43)]. Cutting Z_o is simple: driven separately with $Z_o = 25\Omega$, each magnet would need two 50 Ω cables in parallel for its PFN. Instead, these four cables are connected together in parallel to form one 12.5 Ω PFN. No extra cable is used. Note that the thyratron (CX1585 from EEV) is rated for 5kA and can easily handle the 1.6 kA required. Also note that $C = L/(4Z_o^2)$ must increase to about 1200 pF.

The rise and fall times are proportional to $\frac{1}{\alpha} = \frac{L}{2Z_o}$ [see (40) and (43)]. If both L and Z_o were cut in half, the speed would not change. With L reduced to 66% and Z_o cut in half, the response times are 32% longer. Since the times tabulated in Section D are quite good, this may be an acceptable price to pay for cutting the number of thyratrons and feedthroughs in half.