SINGLE PASS COLLIDER MEMO

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TITLE: REQUIREMENTS FOR RESISTIVE COATING ON CERAMIC TUBES IN KICKER MAGNETS

1. Requirements in General

The resistive coating on the inner surface of the alumina beam pipe in each of the kicker magnets in the Damping Ring serves to carry the image currents in the vacuum chamber wall smoothly through the kicker system and thus minimizes the higher-order mode losses in this region. This coating must be thick enough to contain the electromagnetic fields of the beam bunches but thin enough to allow the penetration of the kicker field in a time much less than 50 nanoseconds.

There is an optimum coating thickness with respect to the total heating produced by the two types of induced currents. The heating due to the image currents of the beam favors a low surface resistivity, and that due to the eddy currents induced by the pulsed kicker field favors a high resistivity. This report calculates the optimum resistivity and shows that the corresponding penetration time is long enough to ensure containment of the electromagnetic fields of the beam bunch but short enough to allow penetration of the kicker field.

The resistive coating must also be thick enough to be able to maintain its mechanical integrity in the presence of high electric fields and high current densities so that sparking does not occur.

2. Power Dissipation due to Image Currents

Consider the power dissipated in the resistive coating due to the images induced in the walls by the beam current. Assume that the coating is thinner than the skin depth at frequencies characteristic of the beam pulse ($\omega_b \sim c/\sigma_z = 3 \times 10^{10}(\text{cm/sec})/0.5(\text{cm}) = 6 \times 10^{10}\text{sec}^{-1}$), so that the current density is uniform throughout the layer. Assume that each beam pulse of charge q has a gaussian density distribution of rms longitudinal deviation σ_z and is centered as it transverse the tube of radius a ,

length L , and surface resistivity $R_{\square}$. For the case of 2 circulating beam bunches in a damping ring of circumference C_R , the average (over time) power P_i dissipated in the coating by the image currents is:

$$P_i(\text{watts}) = \frac{q^2 R_{\square} L c^2}{\sqrt{\pi} \sigma_z 2\pi a C_R} = I_b^2 R_{\square} \frac{C_R}{4 \sqrt{\pi} \sigma_z} = 146.5 R_{\square}$$

where I_b is the average beam current $= 2q \frac{C}{C_R} = 0.136$ Amp

$$q = e \times 5 \times 10^{10} \text{ per bunch} = 8 \times 10^{-9} \text{ coul/bunch}$$

$$C_R = 35.3 \text{ m}$$

$$R = \text{resistance of tube} = R_{\square} L / 2\pi a$$

$$R_{\square} = \text{surface resistivity (ohms/square)}$$

$$L = 40 \text{ cm}$$

$$a = 0.8 \text{ cm}$$

$$\sigma_z = 0.5 \text{ cm}$$

The corresponding power density p_i ($= P_i / 2\pi a L$, watts/cm²) is:

$$p_i = \frac{q^2 R_{\square} c^2}{\sqrt{\pi} \sigma_z (2\pi a)^2 C_R} = 0.73 R_{\square} (\text{w/cm}^2)$$

3. Power Dissipation due to Eddy Currents

When the kicker magnet is pulsed, eddy currents are induced in the resistive coating of the ceramic tube. Since the kicker has a dipole field, the angular distribution of the eddy currents varies approximately as $\cos \theta$. (Because of the presence of the ferrite and nearby conductors, some minor amounts of higher magnetic multipoles may also be induced. However, the higher order modes, in which the current distributions vary as $\cos n\theta$, should be relatively small to begin with and, furthermore, decay n -times faster than the dipole mode¹, so that they

¹ L. J. Laslett, "Decay of Image currents Induced in a Thin Conducting Circular Cylinder by a Line-Current Pair That Is Parallel to but not Necessarily Coincident with, the Cylinder Axis," LBL Internal Report ERAN 39, July, 1969.

can be safely ignored.) The dipole eddy-current distribution induced by step-function excitation decays exponentially with a characteristic time constant τ equal to $2\pi a/R_{\square}$ nanoseconds (with a in cm and $R_{\square}$ in ohms per square).

If the kicker field increases and decreases at a constant rate during the rise and fall times t_1 , which are long compared with the eddy-current decay time τ , the average (over time) eddy-current power density p_e is:

$$p_e = \frac{2 B_k^2 F 10^{-16} \cos^2 \theta}{t_1 R_{\square}} \quad (\text{watts/cm}^2)$$

where B_k is the peak magnetic field of the kicker (in gauss) and F is the repetition rate. For a kicker of 35 cm effective length and 8 milliradian deflection at a beam energy of 1.2 GeV, B_k is 914 gauss. Using $t_1 = 30$ ns and $F = 180$ pps, we get

$$p_e = 1.0 \frac{\cos^2 \theta}{R_{\square}} \quad (\text{watts/cm}^2)$$

4. Total Power Dissipation

The total average power density p_t at the worst point ($\theta = 0^0$) is

$$p_t(0^0) = p_i + p_e(0^0) = 0.73 R_{\square} + \frac{1.0}{R_{\square}}$$

This total power density at 0^0 is a minimum when p_i equals $p_e(0^0)$, from which condition the optimum resistivity is found to be

$$R_{\square \text{ opt}} = (1.0/.73)^{1/2} \cong 1.2 \text{ ohms/square}$$

If one retains the algebra, the general expression for the optimum resistivity is found to be

$$R_{\square \text{ opt}} = \frac{2\pi a B_k 10^{-8}}{qc} \left[\frac{2 \sigma_z C_R F \sqrt{\pi}}{t_1} \right]^{1/2}$$

With the optimum value of $R_{\square}$, the total power density at 0° in our case is 1.7 watts/cm². The entire image power is then $P_i = 176$ watts, and the entire eddy-current power P_e for an effective kicker length L_k of 35 cm is:

$$P_e = p_e(0^\circ) \frac{2\pi a}{2} L_k = 73 \text{ watts,}$$

the sum of the image-plus-eddy-current power being about 250 watts.

5. Temperature Rise

At equilibrium the heat conduction through the alumina must equal the total power density, which is a maximum of 1.7 watts/cm² in the case of optimum resistivity:

$$p_t = K \overline{\Delta T} / \Delta r$$

where K is the heat conductivity of alumina (about 0.3 w/cm-degree at 100°C),

$\overline{\Delta T}$ is the average temperature rise across the ceramic wall thickness Δr (0.3 cm).

We thereby obtain:

$$\overline{\Delta T} = 1.7 \frac{(w/cm^2) 0.3(cm)}{0.3(w/^\circ cm)} = 1.7^\circ C$$

Thus the interior of the ceramic will be less than 2° warmer than the outer surface. Presumably the total power dissipation of 250 watts can be handled by forced air cooling.

The instantaneous temperature rise ΔT_i in the coating can be estimated by neglecting the heat conduction between the coating and the ceramic during the kicker pulse. The energy density Q deposited at 0° during one kicker pulse is equated to the instantaneous temperature rise ΔT_i associated with the heat capacity c_p of the coating (assumed here to be stainless steel):

$$Q = \frac{p_e(0^\circ)}{F} = c_p \rho_c t_c \Delta T_i$$

where ρ_c is the density of the coating and t_c is its thickness. Using $\rho_c = 8.0 \text{ gm/cm}^3$, $c_p = 0.11 \text{ cal/gm deg}$, $t_c = 5.8 \times 10^{-5} \text{ cm}$ (from section 6 of this report), $p_e(0^0) = 0.85 \text{ w/cm}^2$, $F = 180 \text{ pps}$, we find $\Delta T_i = 22^0\text{C}$, which is acceptably low.

6. Coating Thickness

Consider the thickness t_c of the coating with optimum resistivity, 1.2 ohms/square,

$$t_c = \rho / R_{\square}$$

where ρ is the bulk resistivity. For large t we want a relatively large ρ .

Take $\rho = 70 \times 10^{-6} \text{ ohm-cm}$, corresponding to stainless steel, or its equivalent.

Then $t_c = 5.8 \times 10^{-5} \text{ cm}$ (0.58 micron, or $5800\text{\AA}$).

Let us check that this thickness t_c is indeed small with respect to the skin depth δ at frequencies characteristic of the beam bunch. Use

$$\delta(\text{cm}) \approx 5.0 \times 10^3 \sqrt{\frac{\rho}{\mu f}}$$

where μ is the relative permeability and f the frequency. For a relativistic electron beam bunch of length $2\sigma_z$, the characteristic frequency is about $c/4\sigma_z$, or $1.5 \times 10^{10} \text{ Hz}$ for $\sigma_z = 0.5 \text{ cm}$. For a stainless-steel coating then, $\delta = 3.4 \times 10^{-4} \text{ cm}$, which is about 6 times greater than the thickness of the 1.2 ohms/square coating. Therefore, the assumption of uniform image current density across the layer is justified.

If the coating material is ferro-magnetic (like kovar, e.g.), the skin depth is reduced by the factor $\mu^{1/2}$, which typically might be $(10^3)^{1/2}$ or 32. As an example, consider an iron-like case: $\rho = 10 \times 10^{-6} \text{ ohm-cm}$, and $\mu = 10^3$. Then the coating thickness for 1.2 ohms/square is $8.3 \times 10^{-6} \text{ cm}$, and the skin depth at $1.5 \times 10^{10} \text{ Hz}$ is $4.1 \times 10^{-6} \text{ cm}$, in which case the effective surface resistivity for the image currents is about $2 \times 1.2 \text{ ohms/square}$. In such a case we would have to modify the method of calculating the optimum surface resistivity accordingly.

7. Penetration Time

The time constant τ for the decay of a dipole pattern of eddy-currents in a tube of radius a (cm) with a coating of surface resistivity $R_{\square}$ (ohms/square) is:

$$\tau = \frac{2\pi a}{R} 10^{-9} \text{ (sec)} = 4.2 \text{ ns}$$

for the case $a = 0.8 \text{ cm}$, $R_{\square} = 1.2 \text{ ohms/square}$

For a step-function external field of strength B_0 applied at t_0 , the interior field B_i is zero at t_0 and grows as the eddy currents exponentially decay with the time constant τ .

$$B_i(t) = B_0 \left(1 - e^{-\frac{t-t_0}{\tau}} \right)$$

Since an applied field of arbitrary time dependence $B(t)$ can be considered as a sum of step functions of amplitude $(dB(t)/dt)dt$, the time dependence of the interior field can be evaluated by the faltung integral:

$$B_i(t) = B(t) - \int_0^t \frac{dB(t')}{dt'} e^{-\frac{t-t'}{\tau}} dt'$$

It is of interest to evaluate this expression for two simple waveforms--a trapezoid and a half-sinusoid--because the real kicker waveform being developed by Fatin Bulos and Bill Tomlin is intermediate between them.

For the case that $B(t)$ is trapezoidal waveform,

$$B_i(t) = \begin{cases} \frac{B_0}{t_1} [t - \tau(1 - e^{-t/\tau})] & , 0 \leq t \leq t_1 \\ \frac{B_0}{t_1} [t_1 - \tau^{-\frac{t}{\tau}} (e^{t_1/\tau} - 1)] & , t_1 \leq t \leq t_2 \\ \frac{B_0}{T_2} [t_3 - t - \frac{\tau T_2}{t_1} e^{-t/\tau} (e^{t_1/\tau} - 1) \\ \quad + \tau(1 - e^{-(t-t_2)/\tau})] & , t_2 \leq t \leq t_3 \\ B_0 \frac{\tau}{t_1} e^{-t/\tau} [1 - e^{t_1/\tau} + \frac{t_1}{T_2} (e^{t_3/\tau} - e^{t_2/\tau})] & , t \geq t_3 \end{cases}$$

This expression is plotted in Figure 1 for the case $t_1 = 30$ ns, $t_2 = 60$ ns, $T_2 = 30$ ns, and several values of τ .

For the case that $B(t)$ is a half-sinusoid waveform beginning at $t=0$:

$$B(t) = \begin{cases} 0 & , t \leq 0 \\ B_0 \sin \omega t & , 0 \leq \omega t \leq \pi \\ 0 & , \omega t \geq \pi \end{cases}$$

the interior field is given by:

$$B_i(t) = \begin{cases} \frac{B_0}{1 + \omega^2 \tau^2} \left[\sin \omega t - \omega \tau \cos \omega t + \omega \tau e^{-t/\tau} \right] & , 0 \leq \omega t \leq \pi \\ \frac{B_0 \omega \tau}{1 + \omega^2 \tau^2} e^{-t/\tau} \left[1 + e^{\pi/\omega \tau} \right] & , \omega t \geq \pi \end{cases}$$

This expression is plotted in Figure 1 for a sinusoid of period 180 ns and for the same values of τ as in the trapezoidal case.

For $t \gg \tau$ this waveform is a sinusoid of amplitude $B_0 / \sqrt{1 + \omega^2 \tau^2}$ delayed in phase by the angle $\omega \tau$ from that of the applied field. For our optimum case ($R_{\square} = 1.2$ ohms/square, $\tau = 4.2$ ns), the peak interior field is only 1% less than the peak applied field and is delayed in phase by $\omega \tau = 0.15$ radian, or 4 nanoseconds

Figure 1. Penetration of two representative kicker magnetic field patterns into a cylindrical resistive chamber in which the decay of eddy currents is characterized by the time constant τ . For a circular cylinder of radius a (cm) having a thin resistive coating of surface