SINGLE PASS COLLIDER MEMO

CN-69

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TITLE: SYNCHROTRON RADIATION IN THE COLLIDER ARCS

I. General

The synchrotron radiation levels in the Collider arcs are calculated with particular emphasis on radiation doses absorbed in silicon devices. The calculations are carried out with the following assumptions:

1) There will be 5×10^{10} particles per pulse (e^+ or e^-) in each arc, 180 pulses per second, giving an average current of $1.44 \mu\text{A}$ per arc;

2) The synchrotron radiation strikes a copper beam pipe (I.D.=14 mm, wall thickness = 1 mm) at an angle of 4 mradians. The fraction of energy that Compton scatters out of this vacuum pipe is calculated using a simple model which is developed from a more extensive calculation using the EGS¹ computer program.

3) The fraction of the solid angle through which the Compton scattered synchrotron radiation can escape without being absorbed in magnet iron or aluminum conductor is taken into account. At the same time, about 2% of each cell has no magnet iron (i.e., a gap between components). The total synchrotron radiation which escapes into the tunnel accounts for both, and is further increased to account for any gas-like "streaming" from the ends of the magnets.

4) The maximum electron energy will be between 50 and 70 GeV. Calculations are made for both energies, and 4000 operating hours per year.

5) The bending radius of the bend magnets is 292.8 m.

Table I summarizes the simple model calculations for doses absorbed by silicon devices inside the collider arc tunnels. The energy of the escaping synchrotron radiation (330 - 430 keV) is such that a reduction of a factor of 10 would be achieved by adding between 0.9 and 1.2 cm of lead for 50 GeV and 70 GeV energies respectively.

TABLE 1

E electron energy (GeV)	E_c critical energy (MeV)	P_T total synch. power. (kW)	E_{90} photon energy after scatter (keV)	P(tunnel) power into tunnel (W)	Annual dose to silicon (rad/yr)
50	0.947	2.72	332	151	5.0×10^4
70	2.60	10.4	427	437	1.5×10^5

II Calculations

A. General formulae and methods

The spectral distribution of synchrotron radiation has been adequately described by many authors, starting from the original derivation by Schwinger². We will use a form presented by Nelson et al³, since it can be calculated quite easily and rapidly using integration routines and functions currently available at SLAC. The spectrum per meter of bend is given by

$$\frac{dN}{dE_\gamma} = \frac{1.496 \times 10^{17}}{2\pi} \frac{I}{E^2} \frac{g(r)}{r} \quad (\text{MeV}^{-1}\text{-sec}^{-1}\text{-m}^{-1}) \quad (1)$$

where

$$\frac{g(r)}{r} = 0.4652 \int_0^\infty \frac{\cosh(5\alpha/3) \exp(-r \cosh \alpha) d\alpha}{\cosh \alpha} \quad (2)$$

and

$$r = E_\gamma / E_c \quad (3)$$

$$E_c = \text{critical energy (MeV)} = 2.218 \times 10^{-3} E^3/R \quad (4)$$

$$E = \text{beam energy (GeV),}$$

$$I = \text{average beam current } (\mu\text{A}),$$

$$R = \text{bend radius (m).}$$

The spectra corresponding to 50 and 70 GeV beams are shown in Figures 1 and 2 respectively (solid lines) for a bend radius of 292.8 meters. Over the photon energy interval corresponding to $r \in (0.1, 10.0)$, the spectrum is observed to change by about five orders of magnitude.

The total power radiated in a complete circle is given by

$$P_T = 88.46 \times 10^{-3} I E^4/R \quad (\text{Watts}) \quad (5)$$

The critical energy and total radiated power values corresponding to 50 and 70 GeV beams are given in Table 1 for $R = 292.8$ m and $I = 1.44 \mu\text{A}$.

The incident photons strike the inside surface of the vacuum pipe at a small grazing angle (about 4 mradians). We have used the EGS computer program to determine the fraction of the energy that scatters out of the beam pipe. To simplify the problem somewhat, we replaced the actual semi-infinite cylindrical geometry by two semi-infinite slabs as shown in Fig 3.

As noted above, the synchrotron spectrum changes drastically with energy, requiring special care to be exercised when sampling the incident photon energy in EGS. As in a previous calculation⁴, we elected to sample uniformly over the photon energy interval corresponding to $r \in (0.1, 10.0)$ and to assign a weight that is carried along with each particle as it is transported and scored. The results of this sampling are given as histograms in Figs 1 and 2 where agreement with the theoretical spectrum is clearly seen.

B. Results obtained with EGS

Table 2 gives the percentage of the energy that is deposited in each region for 50 and 70 GeV. The information is presented for each radiation component (e⁻, e⁺, γ).

TABLE 2
Percent Energy Deposition using EGS*

Type	1	2	Region 3	4	5	Total
a) 50 GeV						
e ⁻	0.09	11.05	0.0	41.25	0.15	52.54
γ	22.63	0.10	0.0	0.35	23.51	46.59
e ⁺	0.01	0.19	0.0	0.66	0.01	0.87
sum	22.73	11.34	0.0	42.26	23.67	100.0
b) 70 GeV						
e ⁻	0.72	13.93	0.0	39.66	1.46	55.77
γ	19.03	0.04	0.0	0.09	19.24	38.40
e ⁺	0.23	1.34	0.0	3.80	0.47	5.83
sum	19.98	15.31	0.0	43.55	21.16	100.0
	vac	cu	vac	cu	vac	

* (kinetic energy cutoff of 10 keV and 100 keV were used for e⁻ and e⁺ charged particles, respectively.)

We first note that most of the energy escapes (i.e., into region 5) in the form of photons for both energies. Slightly less energy escapes in the 70 GeV case because pair production becomes more important as the critical energy increases (from 0.947 to 2.6 MeV). This also accounts for the increase in energy deposition by e⁺ for the 70 GeV case (to 5.83% total).

C. Simple calculational model

In order to be able to make quick estimates of the energy escaping the vacuum pipe, or to estimate the energy actually deposited in the walls, an analytic model is needed. The model discussed here is based on the study of the EGS results (i.e., Table 2), and on a few simplifying assumptions. These are :

- a) For purposes of determining the fraction of energy which escapes, the energy is assumed all to be carried by photons.
- b) The physical processes are adequately described by a monoenergetic photon with energy, E_c (the critical energy).
- c) The photons all Compton scatter at 90° ; the energy of the photons after scattering is given by

$$E_{90} = E_c / (1 + E_c / 0.511) \quad (6)$$

- d) The fraction of energy carried by the photons after a single scatter is $\mu_s(E_c)/\mu(E_c)$, where $\mu_s(E_c)$ is the scattering cross section, and $\mu(E_c)$ is the total cross section, both evaluated at the critical energy, E_c .

- e) Half of the photons scatter outward through the wall (into region 5), and half scatter inward (into the vacuum region, and thence through the other wall into region 1). For either region 1 or 5, the fraction of energy is given by

$$f = 0.5 [\mu_s(E_c)/\mu(E_c)] \exp(-\mu_{en}(E_{90}) t) \quad (7)$$

where $\mu_{en}(E_{90})$ is the energy absorption cross section evaluated at E_{90} , and t is the pipe wall thickness in g-cm^{-2} .

- f) The charged particles are absorbed in either of the walls.

Table 3 gives data for the 50 and 70 GeV beam energies.

TABLE 3
(Values for copper; density = 8.93 g-cm^{-3})

E Beam Energy	E_c (MeV)	E_{90} (MeV)	$\mu_s(E_c)^6$ $\text{cm}^2\text{-g}^{-1}$	$\mu(E_c)^6$ $\text{cm}^2\text{-g}^{-1}$	$\mu_{en}(E_{90})^6$ $\text{cm}^2\text{-g}^{-1}$
50	0.947	0.332	0.0343	0.0609	0.0357
70	2.60	0.427	0.0162	0.0383	0.0309

The fraction of energy escaping into regions 1 or 5 is

$$\begin{aligned} f(50) &= 0.5 (0.0343/0.0609) \exp(-0.0357 \times 0.893) \\ &= 0.273 \end{aligned}$$

and

$$\begin{aligned} f(70) &= 0.5 (0.0162/0.0383) \exp(-0.0309 \times 0.893) \\ &= 0.206 \end{aligned}$$

These values compare reasonably well with the total values given by EGS in Table 2 for regions 1 and 5 (i.e., 0.2273 and 0.2367 for 50 GeV, and 0.1998 and 0.2116 for 70 GeV). In spite of the simplicity of the model, it is good enough for calculations of this type. It is obvious from Table 2, however, that there is more physics involved than has been included in the model (e.g., look at the charged particle components in the different regions).

D. Synchrotron Radiation Levels in the Collider Arcs

The collider transport line has a circumference-filling factor of 98%, made possible by threading 4 magnet cores on common coil conductors and "refrigerator tubing" vacuum chamber to form modules. Each module is composed of 2 cells similar to that shown in Fig. 4, with about 2% gaps between the magnets through which the synchrotron radiation can escape. The synchrotron power which actually reaches the tunnel from this source, then, is the total power, P_T , modified by twice the fraction given by Eq. 7 (because it escapes out both sides, i.e., into regions 1 and 5) and by the packing fraction; i.e.,

$$\begin{aligned} P(\text{gap}) &= (P_T)(2f)(0.02) \\ &= 0.04 (P_T)(f) \end{aligned}$$

Scattered synchrotron radiation can also escape through about a 30° angle formed by the vacuum pipe and pole face/aluminum conductor into the tunnel (30°/180° = 0.167). The synchrotron power which actually reaches the tunnel from this source, then, is the total power, P_T , modified by the fraction given by Eq. 7, by the azimuthal angle fraction, and by the magnet packing fraction; i.e.,

$$\begin{aligned} P(\text{magnet}) &= (P_T)(f)(0.167)(0.98) \\ &= 0.164 (P_T)(f) \end{aligned}$$

The total reaching the tunnel is the sum of the two terms; that is,

$$\begin{aligned} P(\text{tunnel}) &= (P_T)(f)[0.04 + 0.164] \\ &= 0.204 (P_T)(f) \end{aligned} \tag{8}$$

Values for $P(\text{tunnel})$ are given in Table 1 for 50 and 70 GeV respectively.

In estimating the dose to the devices in the tunnel, we have assumed that the power scattered into the tunnel, $P(\text{tunnel})$, is a uniform line source radiating into a torus of surface area, S , where (with a tunnel radius of 1.5 m)

$$\begin{aligned} S &= (2\pi R)(2\pi R(\text{tunnel})) \\ &= 2\pi(29280) 2\pi(150) \\ &= 1.73 \times 10^8 \text{ cm}^2. \end{aligned}$$

The energy absorption coefficients⁶ in silicon at E_{90} are $0.0295 \text{ cm}^2/\text{g}$ and $0.0297 \text{ cm}^2/\text{g}$. The dose rate is calculated from

$$dD/dt(\text{rad/sec}) = \frac{P[\text{tunnel}](W)}{S(\text{cm}^2)} \times \frac{\mu_{en}(\text{cm}^2/\text{g})}{\rho(\text{g/cm}^3)} \times \frac{10^5 \text{ g-rad}}{W\text{-sec}}$$

Assuming the machine runs for 4000 hours per year, the dose would be

$$D(50) = 3.7 \times 10^4 \text{ rads}$$

$$D(70) = 1.1 \times 10^5 \text{ rads}$$

To be conservative, and at the same time account for any gas-like "streaming" of synchrotron from the ends of the magnets, these values have been increased by 30 to 40%, giving

$$D(50) = 5.0 \times 10^4 \text{ rad/yr},$$

$$D(70) = 1.5 \times 10^5 \text{ rad/yr}.$$

These values are given in Table 1.

E. Reduction of scattered radiation by shielding

The EGS program was used to determine the effect of placing lead outside the vacuum pipe (i.e., in Region 5 of Fig 3 or Table 2), with the result that 2.03% and 2.58% of the total synchrotron power penetrates 1 cm of Pb (in the form of photons) for 50 and 70 GeV beams, respectively.

An analytic model, using μ_{en}/ρ for lead, evaluated at E_{90} , gives results that differ from the EGS values by as much as a factor of two (higher). The model was expected to be too simple (albeit conservative) because the cross section for lead changes dramatically in the energy range from 300 to 500 keV, and the model uses E_{90} for the photon energy. We were able to use E_c and E_{90} to evaluate cross sections in the case of copper earlier since the cross sections are varying only slowly, and the difference between using E_{90} and an actual spectrum will not be as important. A better analytic model would include averaging over the scattered spectrum.

For the SLC case using the EGS results, a factor of 10 reduction is obtained with 0.9 cm and 1.2 cm of Pb for 50 and 70 GeV beams respectively.

REFERENCES

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FIG.1 SYNCHROTRON SPECTRUM

FIG.2 SYNCHROTRON SPECTRUM

END VIEW

TOP VIEW

(NOTE: $\theta = 4$ MRADIANS & $U = \cos \theta$, $V = 0$, $W = \sin \theta$)

FIG 3

TYPICAL CELL

END VIEW

FIG 4