PEP-X: RF Issues and Initial Estimates

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Abstract

This note addresses some very fundamental issues related to the estimation of useful RF systems for the base PEP-X design. Its purpose is to estimate the effectiveness of the existing PEP-II technologies and implementations, recommend alternatives, and determine areas where additional research is necessary. The PEP-X characteristics used in this report are those from the nominal design as defined in [1].

The analysis includes an overview of the proposed PEP-X RF system, and an estimate of longitudinal motion and instabilities, with potential impacts for a synchrotron light source. It examines the options and difficulties in re-using the existing normal-conducting PEP-II RF and LLRF systems.

The baseline PEP-X design has identified the short Touschek-scattering limited lifetime of the beam as a serious concern. The RF system based on the design characteristics defined in [1], lead to an unacceptable bunch length and lifetime for the PEP-X purpose. The PEP-X design includes a third harmonic RF system to increase the bunch length from 2.5 mm to 5 mm. This note estimates the bunch length and studies the necessary operational characteristics for a normal or super-conducting implementation of these harmonic systems.

The presence of an ion clearing gap on the beam train will introduce a significant voltage modulation to the RF accelerating voltage, effectively leading to a substantial variation in bunch length over a turn (up to a factor of 2 based on initial estimates). This issue is of great concern to the authors. Contrary to the rest of the subjects studied in this note, the work on this issue is not complete and feasible configurations have not been determined. Different approaches are mentioned in Section 5 and the beginning of their analysis is shown. More work is necessary in this crucial area to complete models for the analysis and to study the alternative solutions.

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1 Introduction

This note addresses the main issues related with the accelerating Radio Frequency (RF) system and the longitudinal beam dynamics for the proposed PEP-X light source. The analysis and the parameter estimation for those systems are based on the accelerator parameters defined in [1]. The first design option considered for the PEP-X RF system is to reuse the high power RF components from the PEP-II facility. Based on this design option, we analyze different scenarios bounded by the device limits (maximum cavity voltage, maximum klystron power and maximum reflected power) to conceptually pre-design the fundamental RF system. Even if the PEP-II technology is not implemented, the necessary framework is available to study any possible alternative RF devices.

The design parameters for the light source [1], in combination with the RF system parameters, lead to a bunch length of 2.5 mm and a very short Touschek-scattering lifetime for the purposes of PEP-X. In order to double the bunch length and achieve the design goal of 5 mm, it has been proposed to include a second RF system operating at three times the RF frequency. There are many options for the implementation of this third-harmonic RF system: normal or super-conducting cavities, passive or active RF stations etc. We try to address these options in this work. Again, this analysis is based on reusing the PEP-II cavities for the fundamental RF system, but can be easily adjusted for other implementations.

The interaction of the beam longitudinal dynamics with the fundamental and high-order mode (HOM) impedances of both RF systems is addressed to estimate the necessary stabilizing feedback systems. The coupled-bunch beam dynamics affected by the HOM cavity impedance are estimated based on the data measured/calculated during the design of the PEP-II cavities. The coupled-bunch beam dynamics defined by the fundamental and third-harmonic RF system impedance are estimated using the RF systems characteristics for a system based on the PEP-II normal-conducting cavities. This analysis shows that the longitudinal modal beam dynamics are more damped than the PEP-II modal dynamics, mainly because of the presence of wigglers in the machine, which significantly reduce the momentum compaction factor. This fact reduces the requirements for the feedback systems. The available feedback configurations are briefly discussed in this work.

All the beam patterns proposed for PEP-X operation include a minimum ion clearing gap of 2 % of the revolution period. The non-continuous bunch distribution around the ring induces voltage transients per turn in the accelerating voltage in the main RF cavities, as well as modulations on the voltage within the double RF system cavities¹. The periodic accelerating

¹The double RF system is composed of the fundamental and third-harmonic RF sys-

voltage variation induces a change in the synchrotron frequency and length of each individual bunch. There are different options to improve the RF system to reject this perturbation. For normal-conducting cavities, feedback loops around the RF station are able to reduce the impedance that the station presents to the beam, rejecting the transient perturbation due to the beam pattern. To be effective with that solution, the klystron powering the station has to be able to respond during the transient within its dynamic range. This solution sets the operation point of the klystron to a very low power so that it has enough headroom to respond to the transient. The RF system designed for PEP-II did not use this concept and biased the klystron near the maximum power of the device. In that machine, the accelerating voltage variation along the turn modulated the individual bunch length and position. For PEP-II, the bunch length uniformity along the turn was not so critical as it is for PEP-X.

The PEP-II RF system implementation leads to non-optimal bunch length variation (up to a factor of 2 over a turn) for PEP-X. This fact motivated the investigation of alternative implementations of the RF system other than the reuse of the PEP-II cavities. RF systems with higher stored energy (for example super-conducting cavities) have the potential of giving better implementations of the double RF system to achieve a uniform bunch length of 5 mm along the turn. Preliminary studies of a super-conducting fundamental RF system are presented, considering the operation point of the RF station, the multi-bunch stability and the bunch length uniformity per turn in presence of ion clearing gap. Alternative solutions are also discussed. This area would benefit most from additional research.

The beam is perturbed longitudinally by low frequency noise mainly coupled through the RF system. Our experience with PEP-II has shown that the klystron power supply ripple was a major source of perturbation on the longitudinal beam motion. For the PEP-X case this issue is more critical, since the spectrum of this low frequency perturbation is over the synchrotron frequency of the beam, increasing the coupling between the perturbation and the longitudinal beam motion. Solutions to reduce this beam motion are proposed based on feedback systems around the klystron in the RF stations. Further work will be necessary to fully quantify the effectiveness of each system on the PEP-X performance.

The note is structured as follows. Section 2 presents estimates of the number of required RF cavities, and their possible configurations with 1.2 MW Klystrons. Impacts from known power limits (Klystron power, reflected power, cavity voltage etc.) in the existing RF windows and some operational concerns are included. The synchrotron frequency and the bunch length are also estimated.

tems.

The base design considers the use of harmonic RF cavities to lengthen the bunches (to improve Touschek-scattering limited lifetimes). In Section 3 we discuss different options in defining the harmonic cavity characteristics, for a normal or super-conducting implementation.

In Section 4 we estimate the longitudinal dynamics of the known cavity HOMs and the cavity fundamental, using the performance of the existing PEP-II impedance-controlled LLRF topology as a metric. Issues of beam stability arising from fundamental cavity and HOM impedances are discussed via modal growth and damping rate estimates.

Section 5 studies the important issue of the bunch length modulation due to the ion clearing gap. The variation in bunch length and tune around the filling pattern, is estimated for passive or active implementations of the harmonic RF system, and is shown to be unacceptable for the PEP-X purposes. Alternative approaches are mentioned. The potential impact of RF system driven noise in the longitudinal plane is also studied.

As the PEP-X design matures, the methodology of this initial study will be expanded to look in greater detail and sophistication. At the time of writing of this note, funding and support to expand this analysis is not available. Should there be such support in the future, we conclude in Section 6 with some suggestions on useful directions for more detailed study of these issues.

2 Operational Design Considerations

Operational limits in the RF system are mainly imposed by the maximum klystron power, the maximum reflected power per cavity, the maximum power transferred through the window and maximum voltage per cavity. The combination of those limits define the minimum number of stations in the ring for a given beam current.

2.1 Cavity Voltage

The designed maximum voltage $V_{C_{MAX}}$ for PEP-II cavities is equal to 1 MV. At SSRL cavities are operating with an availability of almost 100% at $V_{C_{MAX}} = 800$ kV. During PEP-II operation at high beam currents, this voltage $V_{C_{MAX}}$ was limited to 500 – 550 kV to avoid RF system aborts and beam loss due to arcs in the window and the waveguide. The minimum voltage $V_{C_{MIN}}$ per cavity for a given beam current is usually defined by the limits imposed by the klystron power, the reflected power, etc. It is important to notice that for reliable operation the cavities can operate within a voltage range ($V_{C_{MIN}}, V_{C_{MAX}}$).

For operational flexibility of the machine, it may be important to be able to park one RF station in case of major failure in that particular station. This option minimizes the down time of the whole machine, allowing the diagnosis and repair of the RF station either while the machine is operating or during a scheduled maintenance time. One necessary condition for this option is that the nominal operational gap voltage should be achievable with one parked RF station. Then

$$V_{GAP} = N_{st}N_C V_{C_{MIN}} = (N_{st} - 1)N_C V_{C_{MAX}} \quad (1)$$

where N_{st} is the number of stations and N_C is the number of cavities per station. This equation defines a necessary condition for the minimum number of RF stations.

$$\begin{aligned} \text{For } N_C = 2, V_{C_{MAX}} = 800 \text{ kV}, V_{C_{MIN}} = 600 \text{ kV} &\Rightarrow N_{st_{min}} = 4 \\ \text{For } N_C = 2, V_{C_{MAX}} = 700 \text{ kV}, V_{C_{MIN}} = 600 \text{ kV} &\Rightarrow N_{st_{min}} = 8 \\ \text{For } N_C = 2, V_{C_{MAX}} = 700 \text{ kV}, V_{C_{MIN}} = 500 \text{ kV} &\Rightarrow N_{st_{min}} = 4 \\ \text{For } N_C = 2, V_{C_{MAX}} = 600 \text{ kV}, V_{C_{MIN}} = 500 \text{ kV} &\Rightarrow N_{st_{min}} = 6. \end{aligned}$$

2.2 Klystron Power

The operating klystron power P_{kly} has an operative limit from the maximum klystron power $P_{kly_{LIM}}$. The number of stations N_{st} and the gap voltage V_{GAP} for different beam currents, and the RF guide-cavity couplings β can be evaluated assuming that $P_{kly_{LIM}}$ limits the operation of the RF stations.

The klystron power limit $P_{kly_{LIM}}$ is defined mainly by the klystron tube characteristics and the RF station operation point. Then

$$P_{kly_{LIM}} \geq P_{kly} = N_C \frac{(1 + \beta)^2}{8R\beta} (V_C + V_{br} \sin(\phi_b))^2 \quad (2)$$

with $R = R_a/2$: wall cavity resistance, V_C : Cavity Voltage, $V_{br} = 2I_b R / (1 + \beta)$ and $\sin(\phi_b) = \frac{P_{Bloss}}{I_b V_{GAP}}$ with $P_{Bloss} = C_{HOM} I_b^2 + U_{rad} I_b$. Assuming that all the cavities operate at the same voltage and deliver the same power to the beam,

$$V_{br} \sin(\phi_b) = \frac{2I_b R}{1 + \beta} \frac{P_{Bloss}}{I_b V_{GAP}} \simeq \frac{2R P_{Bloss}}{(1 + \beta) N_C N_{st} V_C} \quad (3)$$

From (2) and (3)

$$\frac{P_{kly_{LIM}} 8R\beta}{N_C (1 + \beta)^2} \geq (V_C + \frac{2R P_{Bloss}}{(1 + \beta) N_C N_{st} V_C})^2 \quad (4)$$

Equation (4) defines the number of stations N_{st} and the gap voltage as a function of the beam current for different parameters β, V_C . Assuming $P_{kly_{LIM}} = 900$ kW we can now derive the minimum number of stations and the maximum gap voltages achievable for different parameters and operative conditions. For example, Fig. 1 shows the estimated parameters for $\beta = 3.9$. This scenario and some additional examples are shown in Appendix A, Figs. 24-26.

2.3 Cavity Reflected Power

Setting the limit for the reflected power per cavity $P_{refl_{LIM}} = 100$ kW, the number of stations N_{st} and the gap voltage V_{GAP} for different beam currents and RF guide-cavity coupling coefficients β can be evaluated.

The reflected power per cavity is

$$P_{refl_{LIM}} \geq P_{C_{refl}} = \frac{(1 - \beta)^2 P_C}{4\beta} + \frac{(1 - \beta) P_b}{2\beta} + \frac{P_b^2}{4\beta P_C} \quad (5)$$

with $P_C = V_C^2 / R_a = V_C^2 / (2R)$: power dissipated in the wall cavity resistance, V_C : Cavity Voltage and $P_b \simeq \frac{P_{Bloss}}{N_{st} N_c}$, assuming that all the cavities operate at the same voltage and deliver the same power to the beam. Then

$$\begin{aligned} (P_{refl_{LIM}} - \frac{(1 - \beta)^2 P_C}{4\beta}) 4\beta &\geq 2(1 - \beta) P_b + \frac{P_b^2}{P_C} \\ (P_{refl_{LIM}} - \frac{(1 - \beta)^2 P_C}{4\beta}) \frac{4\beta}{R_a} &\geq 2(1 - \beta) \frac{V_C}{R_a} (\frac{P_{Bloss}}{N_{st} N_c V_C}) + (\frac{P_{Bloss}}{N_{st} N_c V_C})^2 \end{aligned} \quad (6)$$

Figure 1: Minimum number of stations and Gap Voltages for $P_{klyLIM} = 900$ kW and $\beta = 3.9$

Defining $x = \frac{P_b}{N_{st}N_cV_C}$ as the unknown in (6) and solving the 2^{nd} order equation, the number of stations and the gap voltage can be analyzed as a function of the beam current for different parameters β , V_C assuming $P_{reflLIM} = 100$ kW per cavity. The cases with $\beta = 3.9$, $\beta = 7.5$, as well as the one for $V_C = 600$ kV are depicted in Appendix A in Figs. 27-29.

2.4 Preliminary Conclusions for Currents $I_b = 1.4 - 1.6$ A

For beam currents around 1.5 A and with the existing RF systems (coupling $\beta = 3.9$), Fig. 1 shows that is necessary to implement a minimum of 8 stations. The gap voltage range is between 8 – 13 MV operating the cavities at $V_C = 500 - 800$ kV, respectively. This limit is imposed by the maximum klystron power $P_{klyLIM} = 900$ kW. Comparing Fig. 1 with Fig. 27, it is possible to observe that the limit imposed by the cavity reflected power $P_{reflLIM} = 100$ kW is lower than the one defined by klystron power limit for that same beam current range. Table 1 presents choices that optimize the

gap voltage for $I_b = 1.5$ A and 8 RF stations (minimizing V_{GAP}).

Table 1: 8 RF station configuration - $I_b = 1.5$ A

N_{st}	V_{Cmin}	V_{Cmax}	V_{GAP}	P_{kly}	P_{refl}	P_{cav}
8	495 kV	-	7.92 MV	900 kW	98 kW	32.8 kW
8	500 kV	-	8.0 MV	894 kW	94 kW	33.5 kW
7	-	571 kV	8.0 MV	983 kW	128.5 kW	43.8 kW

From Table 1 we can observe that in the case of one parked RF station, the system cannot operate at the gap voltage $V_{GAP} = 8$ MV. Both the P_{kly} and the P_{refl} limits are exceeded. Even if the cavity voltage is increased to $V_C = 800$ kV, the klystron power limit is exceeded as shown in Table 2.

Table 2: 8 RF station configuration - $I_b = 1.5$ A

N_{st}	V_{Cmin}	V_{Cmax}	V_{GAP}	P_{kly}	P_{refl}	P_{cav}
8	500 kV	-	8.0 MV	894 kW	94 kW	33.5 kW
7	-	800 kV	11.2 MV	921 kW	55 kW	85.8 kW

If the design goal includes the option of nominal operation with one parked station, a minimum of $N_{st} = 9$ stations are necessary for the RF configuration to operate at $I_b = 1.5$ A. Table 3 describes this condition.

Table 3: 8-9 RF station configuration - $I_b = 1.5$ A

N_{st}	V_{Cmin}	V_{Cmax}	V_{GAP}	P_{kly}	P_{refl}	P_{cav}
9	450 kV	-	8.1 MV	820 kW	99.3 kW	27.14 kW
8	-	500 kV	8.0 MV	894 kW	94 kW	33.5 kW

Around this configuration ($N_{st} = 8 - 9$), it is possible to define another operational condition by trading P_{kly} and P_{refl} limits for V_{Cmax} and V_{Cmin} limits. In Table 4, the station configurations are defined for a total gap voltage $V_{GAP} = 10$ MV, still assuming that $\beta = 3.9$.

Table 4: 8-9 RF station configuration - $I_b = 1.5$ A

N_{st}	V_{Cmin}	V_{Cmax}	V_{GAP}	P_{kly}	P_{refl}	P_{cav}
9	555 kV	-	10 MV	733 kW	41.56 kW	41.37 kW
8	-	625 kV	10 MV	811.7 kW	34.3 kW	52.36 kW

From Fig. 26, for the same beam current range, note that if the coupling factor β is increased, the number of RF stations can be reduced to 7. Fig. 26 depicts an example for $\beta = 7.5$, with 7 stations and $V_C = 600 - 700$ kV,

where the gap voltage can be adjusted between 8 – 10 MV. Similarly to the case $\beta = 3.9$, the minimum number of stations and minimum gap voltage is limited by the maximum klystron power and not by the reflected power per cavity (Figs. 28 and 29).

2.5 Estimation of Synchrotron Frequency and bunch length for $I_b = 1.5$ A

Using the PEP-X nominal design, assuming $\alpha = 4.72 \cdot 10^{-5}$, $\omega_{RF} = 2\pi 476$ MHz, $\omega_0 = 8.5610^5$ rad/sec, $E_o = 4.5$ GeV and $\sigma_E/E_o = 1.12 \cdot 10^{-3}$, the synchrotron frequency and bunch length can be estimated for different gap voltages. This estimation excludes ion clearing gap transient modulation effects and the third-harmonic cavity bunch lengthening.

For $V_{GAP} = 13$ MV, $\Rightarrow f_s = 1.1654$ kHz, $\sigma_z = 2.166$ mm.

For $V_{GAP} = 10$ MV, $\Rightarrow f_s = 1.0089$ kHz, $\sigma_z = 2.502$ mm.

For $V_{GAP} = 8$ MV, $\Rightarrow f_s = 885.2562$ Hz, $\sigma_z = 2.851$ mm.

3 Double RF System - Third-Harmonic Cavities

The bunch length obtained with a single RF system operating at a nominal voltage of $V_{GAP} = 10$ MV and $f_{RF} = 476$ MHz is $\sigma_z = 2.5018$ mm. In order to double σ_z , an additional RF system operating at $3f_{RF}$ is being analyzed. In this note we call *Double RF system* the fundamental RF system at f_{RF} together with the harmonic RF system at $3f_{RF}$. The operation conditions of the harmonic RF system and its performance are analyzed and initially described in this section. Since the beam pattern exhibits an ion clearing gap, the overall gap voltage V_{GAP} around the revolution period is not constant, affecting the synchrotron frequency of the oscillation of each individual bunch and its bunch length σ_{z_i} . Details of these effects and their dependence on the design parameters of the system are still under analysis, and are briefly described in Section 5.1. For the present section though, gap transient effects are not included.

3.1 Accelerating Voltage in a Double RF System

The dynamics for the centroid of the i^{th} individual bunch in the PEP-X ring can be written by

$$\frac{d^2\tau_i(t)}{dt^2} + 2d_r \frac{d\tau_i(t)}{dt} + \frac{\alpha(e v_{GAP}(t_i) - U_o)}{E_o T_o} = 0 \quad (7)$$

where $\tau_i(t)$ is the time deviation of the i^{th} bunch with respect to the RF reference, $v_{GAP}(t)$ is the instantaneous total accelerating voltage, t_i is the time when the i^{th} bunch is accelerated by $v_{GAP}(t)$, d_r is the damping rate, $U_o = P_{Bloss}/I_b$ is the energy lost per turn including the losses by radiation, HOM in cavities, etc., α is the momentum compaction factor, E_o is the beam energy and T_o is the revolution period. The time deviation $\tau_i(t)$ can be expressed as an angle using $\phi_{B_i}(t) = \omega_{RF}\tau_i(t)$. The instantaneous accelerating voltage is the total voltage contribution of all the cavities that are part of the double RF system. $v_{GAP}(t)$ is given by

$$\begin{aligned} v_{GAP}(t) &= \sum_{j=1}^{N_{st}} V_{1j}(t) \sin(\omega_{RF}t + \delta_j(t)) + \sum_{k=1}^{M_{st}} V_{3k}(t) \sin(3\omega_{RF}t + \delta_k(t)) \\ &= V_1(t) \sin(\omega_{RF}t + \theta_1(t)) + V_3(t) \sin(3\omega_{RF}t + \theta_3(t)) \end{aligned} \quad (8)$$

where N_{st} , M_{st} are the number of stations of the fundamental and third-harmonic RF systems, respectively, and $V_{1j}(t)$, $V_{3k}(t)$, $\delta_j(t)$, $\delta_k(t)$ are slow time variant signals modulating in amplitude and phase the fundamental and third-harmonic RF accelerating voltages. Fig. 2 depicts the resulting

Figure 2: Resulting Accelerating voltages for the double RF system

harmonic accelerating voltages for the system and shows the variables defined in (8). In steady-state, since $\frac{d^2\tau_i(t)}{dt^2} = \frac{d\tau_i(t)}{dt} = 0$, $\forall i$, the individual bunches are positioned around the ring at an angle ϕ_{B_i} with respect to the RF reference. The individual position of the bunches are defined by the energy balance

$$e v_{GAP}(t_i) - U_o = 0 \quad \forall i = 1, \dots, N_{bunch}$$

If the bunch train presents an ion clearing gap, the variables $V_{1_j}(t)$, $V_{3_k}(t)$, $\delta_j(t)$, $\delta_k(t)$ are T_o - periodic in steady state, then

$$\begin{aligned} V_1(t)\sin(\omega_{RF}\tau_i + \theta_1(t)) + V_3(t)\sin(3\omega_{RF}\tau_i + \theta_3(t)) - U_o/e = \\ V_1(t)\sin(\phi_{B_i} + \theta_1(t)) + V_3(t)\sin(3\phi_{B_i} + \theta_3(t)) - U_o/e = 0 \end{aligned} \quad (9)$$

for $t \in [0, T_o)$, $\forall i = 1, \dots, N_{bunch}$,

Assuming Gaussian longitudinal shape for the bunch, the standard deviation which corresponds to the bunch length σ_z is given by

$$\sigma_z = \frac{\alpha C}{\omega_s} \left(\frac{\sigma_E}{E_o} \right) \quad (10)$$

where σ_E/E_o is the energy spread and ω_s the synchrotron angular frequency. For accelerating voltages which increase the bunch length, the longitudinal focusing effect is weaker and the bunch shape deviates from the Gaussian form. Still that expression gives a first order approximation to evaluate the ratio between σ_z and ω_s even when the longitudinal bunch shape is not

Gaussian due to the weaker focusing. The synchrotron angular frequency for the i^{th} bunch is given by

$$\omega_{s_i}^2 = \frac{e \alpha}{E_o T_o} \left. \frac{dv_{GAP}(t)}{dt} \right|_{t=t_i} \quad (11)$$

Using (8), (10), and (11), the relationship between the components of the accelerating RF voltages and the i^{th} bunch length is

$$V_1(t) \cos(\phi_{B_i} + \theta_1(t)) + 3 V_3(t) \cos(3\phi_{B_i} + \theta_3(t)) = \frac{E_o T_o \alpha}{e \omega_{RF} \sigma_{z_i}^2} \left(\frac{\sigma_E}{E_o} \right)^2 \quad (12)$$

For a given set of accelerator parameters α, E_o, T_o, U_o , etc., and time variant accelerating voltages, Eqns. (9) and (12) define a set that allows the analysis of the relation between the individual bunch position ϕ_i and the bunch length σ_{z_i} .

3.2 Nominal Case - No ion clearing gap

For the analysis of the nominal case where the beam train has no gap, we assume that the voltages and phases $V_1(t), V_3(t), \theta_1(t), \theta_3(t)$ remain constant for all $t \in [0, T_o)$. In this case, Eqns. (9) and (12) can be simplified to

$$V_1 \sin(\zeta_1) + V_3 \sin(\zeta_3) = U_o/e = K_U \quad (13)$$

$$V_1 \cos(\zeta_1) + 3 V_3 \cos(\zeta_3) = \frac{E_o T_o \alpha}{e \omega_{RF} \sigma_z^2} \left(\frac{\sigma_E}{E_o} \right)^2 = K_\sigma \quad (14)$$

Given a set of accelerator parameters and defining the desired bunch length σ_Z , this set of equations defines the constraints for the double RF system. Assuming that V_1 was previously defined by other considerations, this is a set of 2 equations with 3 unknowns, leading to several solutions.

3.2.1 Option 1: ζ_1 defined

One option to set the third-harmonic RF system is to first define the fundamental RF system and then set the third-harmonic cavity so that the bunch position with respect to the original RF system is unaffected. Then, from Eqns. (9) and (13), for $V_3 = 0$

$$V_1 \sin(\zeta_1) = U_o/e = K_U \Rightarrow \phi_{B_i} = \arcsin\left(\frac{K_U}{V_1}\right) - \theta_1 \text{ for } i = 1, \dots, N_{bunch} \quad (15)$$

With this value, from (13) and (14), we obtain

$$V_3^2 = (K_U - V_1 \sin(\zeta_1))^2 + \left(\frac{(K_\sigma - V_1 \cos(\zeta_1))}{3} \right)^2 \quad (16)$$

$$\tan(\zeta_3) = \frac{3(K_U - V_1 \sin(\zeta_1))}{(K_\sigma - V_1 \cos(\zeta_1))} \quad (17)$$

Table 5 summarizes the design for the PEP-X case with $\sigma_z = 5$ mm. It is assumed in these calculations that U_o increases 10% when the third-harmonic RF system is included into the ring. Additionally, the fundamental voltage $v_1(t)$ is assumed to be in phase with the RF reference ($\theta_1 = 0^\circ$).

Table 5: Double RF system - $V_1 = 10$ MV - OP#1: ϕ_B fixed

ϕ_B	$K_U = U_o/e$	σ_Z	K_σ	V_3	ζ_3	θ_3
160.1°	3.4 MV	2.5 mm	9.41 MV	0 MV	—	—
160.1°	3.5 MV	5 mm	2.35 MV	2.35 MV	2.53°	242.21°

3.2.2 Option 2: Minimum V_3

In order to reduce the auxiliary third-harmonic RF system, one option is to minimize the required third-harmonic voltage to achieve the desired bunch length σ_Z . Eqn. (16) can be used to minimize V_3 with respect to the independent variable ζ_1 . The design for minimum third-harmonic RF system voltage is presented in Table 6, assuming $V_1 = 10$ MV, $\theta_1 = 0^\circ$ and $\sigma_z = 5$ mm.

Table 6: Double RF system - $V_1 = 10$ MV - OP#2: Minimum V_3

V_3	ϕ_B	$K_U = U_o/e$	σ_Z	K_σ	ζ_3	θ_3
2.317 MV	157.52°	3.5 MV	5 mm	2.35 MV	-7.85°	239.57°

3.2.3 Option 3: Max/Min cavity impedance angle ($\phi_{Z_3} \simeq \pm 90^\circ$)

This case is further analyzed in Subsection 3.3. Let us assume that the third-harmonic RF system is passive and the cavities are detuned so that the cavity impedance angle $\phi_{Z_3} \simeq \pm 90^\circ$. Since the auxiliary voltage $v_3(t) = V_3 \sin(3\omega_{RF}t + \theta_3)$ is induced by the interaction between the beam current and the third-harmonic cavity impedance, the angles ζ_1 and ζ_3 are related by (Eqns. (22), (23) in Subsection 3.3).

$$\zeta_3 = 3\omega_{RF}t + \theta_3 = 3\zeta_1 + 3\pi/2 - 3\zeta_1 + \phi_{Z_3} = 3\pi/2 + \phi_{Z_3} = \pi \text{ or } 2\pi$$

Table 7: Double RF system - $V_1 = 10$ MV - OP#3:Max/Min ϕ_{Z_3}

ϕ_B	$K_U = U_o/e$	ϕ_Z	σ_{Z_3}	K_σ	V_3	ζ_3
159.47°	3.5 MV	$+90^\circ$	5 mm	2.35 MV	2.337 MV	0°
159.47°	3.5 MV	-90°	1.9 mm	16.37 MV	2.337 MV	180°

Then, from (13) and (14), $V_1 \sin(\zeta_1) = K_u$ and $V_1 \cos(\zeta_1) \pm 3V_3 = K_\sigma$. Table 7 summarizes this option.

It is interesting to observe how the magnitude of V_3 changes for different synchronous angles ϕ_1 ($\theta_1 = 0^\circ$). Fig. 3 depicts that variation and shows the magnitude for the cases described in Tables 5, 6 and 7.

Figure 3: Voltage magnitude V_3 of third-harmonic RF system for different synchronous angles ϕ_B and a bunch length $\sigma_z = 5$ mm. 'x': Solution for $\phi_B = 160.1^\circ$. 'o': Solution for V_3 minimum. '*': Solution for $\phi_{Z_3} = \pm 90^\circ$.

3.3 Implementation: Passive Case

For a third-harmonic RF system implemented without active RF power, the accelerating voltage $v_3(t)$ is induced mainly by the interaction between the third-harmonic beam current and the auxiliary cavity impedance. For a fully loaded ring, assuming a point-charge beam, the current can be described by

$$i_b(t) = \sum_{n_b=-\infty}^{\infty} \delta\left(t - \frac{\phi_b}{\omega_{RF}} - \frac{2\pi}{\omega_{RF}} n_b\right) \quad (18)$$

where $\delta(t)$ is the Dirac-impulse. This current can be developed in Fourier series as $i_b(t) = \sum_{k=-\infty}^{\infty} C_k e^{i\omega_o k t}$, where the Fourier coefficients are

$$C_k = \frac{1}{T_{RF}} e^{-ik\phi_b} \quad \text{for } k = -\infty, \dots, \infty \quad (19)$$

For a Gaussian bunch distribution with $\sigma_Z = 5$ mm, the Fourier coefficients C_k are well approximated by (19) for $k \ll c/(\sigma_Z \omega_{RF}) = 20$. It is important to observe that the relative phase of the harmonic components of the beam is given by $-k\phi_b$. The component at fundamental RF frequency has a relative phase $-\phi_b$, while the third-harmonic component phase is $-3\phi_b$ with respect to the same reference. This relationship between harmonic components in the current sets the ratio between the angles ζ_1 and ζ_3 . Fig. 4 depicts the relation between the current and voltage for the fundamental frequency and the third-harmonic components. In each diagram, the phase relationship among voltage and current components is valid $\forall t$, but the relative phase relationship between the diagrams at ω_{RF} and $3\omega_{RF}$ is only valid for $t = 0, T_{RF}, \dots, kT_{RF}, \dots, \infty$.

For the fundamental frequency, the main cavities are detuned so that the principal RF system is Robinson stable and the klystron power delivered to the RF cavities is minimum (Klystron current phasor I_K in phase with cavity voltage phasor V_1). On the right plot, the vector diagram describes the locus for the voltage V_3 induced in the third-harmonic cavity by the harmonic phasor I_{B_3} . This locus is defined by the Min/Max values of the cavity impedance angle $\phi_{Z_3} = \pm 90^\circ$. From this plot, the phase θ_3 of the voltage V_3 can be adjusted within an 180° range $[\xi, \eta] = [180^\circ - 3\phi_B, 360^\circ - 3\phi_B]$ by detuning the third-harmonic cavity.

Figure 4: Phasor Diagram for principal voltages and currents in the double RF System. *left: Fundamental frequency, right: Third-harmonic.*

For the synthesis of the third-harmonic cavity we assume a high-Q cavity whose fundamental impedance is given by

$$Z_3(\Delta\omega) \simeq \frac{R_3}{1 + i2Q_3 \frac{\Delta\omega}{\omega_{RF}}} \quad (20)$$

where $\Delta\omega = 3\omega_{RF} - \omega_3$ and ω_3 is the resonant frequency of the third-harmonic cavity. The impedance angle ϕ_{Z_3} defined in Fig. 4 is given by

$$\tan(\phi_{Z_3}) = -\frac{2Q_3\Delta\omega}{3\omega_{RF}} \quad (21)$$

The relationship among ϕ_{Z_3} , θ_3 and ζ_3 can be analyzed based on Fig. 4. From the third-harmonic diagram, the phase θ_3 is

$$\theta_3 = \frac{3\pi}{2} - 3\phi_b + \phi_{Z_3}, \quad (22)$$

then $\zeta_3 = 3\omega_{RF}t + \theta_3$ for $\omega_{RF}t = \phi_b$ is

$$\zeta_3 = \frac{3\pi}{2} + \phi_{Z_3}, \quad (23)$$

Substituting those equations in (13) and (14), we get

$$V_1 \sin(\zeta_1) - V_3 \cos(\phi_{Z_3}) = K_U \quad (24)$$

$$V_1 \cos(\zeta_1) + 3 V_3 \sin(\phi_{Z_3}) = K_\sigma \quad (25)$$

From those equations, the third-harmonic voltage (Eqn. 16) and the impedance angle ϕ_{Z_3} can be calculated as a function of the synchronous angle $\zeta_1 = \phi_b$. The solutions are

$$\begin{aligned} V_3^2 &= (K_U - V_1 \sin(\phi_b))^2 + \left(\frac{K_\sigma - V_1 \cos(\phi_b)}{3} \right)^2 \\ \tan(\phi_{Z_3}) &= \frac{V_1 \cos(\phi_b) - K_\sigma}{3(V_1 \sin(\phi_b) - K_U)} \end{aligned}$$

Fig. 5 depicts the voltage V_3 and the phase ϕ_{Z_3} as a function of the synchronous angle ϕ_b . The passive implementation of the auxiliary third-harmonic RF system restricts the impedance angle to $\phi_{Z_3} \in (-90^\circ, +90^\circ)$. The solution described previously as 'option 1' is only possible with an active third-harmonic RF system. The 'option 3' defines one of the bounds for the passive implementation.

The voltage V_3 is defined by $V_3 = I_3 Z_3 N_3$, where N_3 is the number of cavities of the third-Harmonic RF system. From (20) and (21), the total shunt

Figure 5: The magnitude of the voltage V_3 of the third-harmonic RF system for different synchronous angles ϕ_B and Impedance angle ϕ_{Z_3} of the third-harmonic RF cavity. 'x': Solution for $\phi_B = 160.1^\circ$. 'o': Solution for V_3 minimum. '*': Solution for $\phi_{Z_3} = \pm 90^\circ$.

cavity resistance $R_3 N_3$ can be expressed as a function of the synchronous angle ϕ_b for a given beam current.

$$\begin{aligned}
 R_3 N_3 &= \frac{V_3(\phi_b)}{I_3} \sqrt{1 + \left(\frac{2Q_3 \Delta\omega}{3\omega_{RF}} \right)^2} \\
 &= \frac{V_3(\phi_b)}{I_3} \sqrt{1 + (\tan(\phi_{Z_3}(\phi_b)))^2} \\
 &= \frac{(K_U - V_1 \sin(\phi_b))^2 + \left(\frac{(K_\sigma - V_1 \cos(\phi_b))}{3} \right)^2}{I_3 (K_U - V_1 \sin(\phi_b))}
 \end{aligned} \tag{26}$$

Fig. 6 depicts the magnitude of the total shunt resistance of the third-harmonic cavities as a function of the synchronous angle ϕ_b to achieve $\sigma_Z = 5mm$ for different beam currents I_b . From the plots depicted in Figs. 5 and 6 it is possible to define the parameters for the third-harmonic cavities operating at $I_b = 1.5$ A for the case 'Option 2: Minimum V_3 ' previously analyzed. Table 8 summarizes that design.

Figure 6: Total Shunt Resistance $N_3 R_3$ of the third-harmonic RF cavities for different synchronous angles ϕ_B for $I_b = 1.5$ A, $I_b = 1.0$ A and $I_b = 0.5$ A.

Table 8: third-Harmonic RF system - Passive - OP#3:Minimum V_3

V_3	ϕ_B	ϕ_{Z_3}	$R_3 N_3$
2.317 MV	157.52°	82.14°	5.65 MΩ

3.3.1 Examples of passive third-Harmonic RF system designs

Table 9 shows the principal characteristics of the cavities installed in other accelerator facilities to increase the bunch length. The table includes normal-conducting and super-conducting designs.

Table 9: Summary of Passive third-Harmonic RF system - Worldwide designs

Facility	Type	R/Q	Q_3	V_{3MAX}	V_{3OP}	I_b
Bessy	NC	62 Ω	13.9K	-	60 KV	
ALS	NC	80.4 Ω	21 K	-	125 KV	
Elettra-SLS	SC	44.2 Ω	1 10 ⁸ – 2 10 ⁸	1 MV	600 KV	

Let us consider a nominal design based on a super-conducting cavity with $R/Q = 40$ Ω and $V_{3OP} = 500$ KV. From Fig. 5, to achieve a bunch length $\sigma_Z = 5$ mm with $I_b = 1.5$ A, the minimum voltage necessary is $V_3 = 2.337$ MV. This sets the minimum number of SC cavities to $N_3 = 5$. If $Q_3 = 1$ 10⁸, then $R_3 = 4$ GΩ and $\phi_b \simeq 159.47^\circ$. The detuning of these cavities can be

calculated by

$$V_3 \simeq \frac{I_3 N_3 R_3}{2Q_3 \Delta\omega / 3\omega_{RF}} \Rightarrow \Delta\omega \simeq \frac{3I_3 N_3 \omega_{RF} R/Q}{2V_3} = 1.15 \text{ Mrad/sec.}$$

and the third-Harmonic cavity will be tuned 183.3 KHz below $3f_{RF}$.

If the nominal design is based on a normal-conducting cavity with $R/Q = 100 \Omega$ and $V_{3OP} = 125 \text{ KV}$, the minimum number of NC cavities is $N_3 = 19$. Assuming $Q_3 = 20 \text{ K}$, then $R_3 = 2 \text{ M}\Omega$, $R_3 N_3 = 38 \text{ M}\Omega$ and, from Fig. 6 for $Ib = 1.5 \text{ A}$ and Fig. 5, $\phi_b = 158.6^\circ$ and $V_3 = 2.32 \text{ MV}$, respectively. In this design the third-Harmonic cavity will be tuned 1.75 MHz below $3f_{RF}$. Table 10 summarizes these designs.

Table 10: Examples of Passive third-Harmonic RF system for PEP-X

Type	N_3	V_3	V_3/cav	R/Q	Q_3	$\Delta\omega/2\pi$	ϕ_{Z_3}
SC	5	2.337 MV	467 kV	40 Ω	1 10^8	183.3 kHz	+90°
NC	19	2.32 MV	122 kV	100 Ω	20 10^3	1.75 MHz	+88.8°

In order to evaluate the operation of the passive third-Harmonic RF system at different beam currents two different scenarios are considered. One option is to set the cavity detuning to the fixed magnitude calculated above (for $Ib = 1.5 \text{ A}$, $\sigma_Z = 5 \text{ mm}$) and evaluate how the bunch width changes with current. The other option is to set and keep the voltage V_3 constant for all beam currents by adjusting the detuning of the cavities. In the first case the bunch length will change from $\sigma_Z = 2.5 \text{ mm}$ to $\sigma_Z = 5 \text{ mm}$ while the beam current increases from $I_b = 0 \text{ A}$ to $I_b = 1.5 \text{ A}$. In the second case, the bunch length will change slightly when the beam current ramps from zero to 1.5 A. Figs. 7 and 8 depict the bunch variation and the detuning for both designs, assuming an even-fill (no gap).

3.4 Implementation: Active Case

Work in progress.

Figure 7: Bunch length variation σ_Z as function of the beam current for fixed detuning in the third-Harmonic cavities (even fill).

4 Longitudinal Stability

In this section the stability of HOM driven and cavity fundamental driven coupled-bunch longitudinal modes is estimated. The study is conducted assuming a double RF system to increase σ_Z from 2.5mm to 5mm.

4.1 HOM driven coupled-bunch longitudinal instabilities

The small amplitude longitudinal oscillation in modal domain around the synchronous position for mode l is described by:

$$\ddot{\tau}_l + 2d_r\dot{\tau}_l + \omega_s^2\tau_l = 2j\omega_s\lambda_l\tau_l \quad (27)$$

where τ_l is the time displacement of the l^{th} -beam mode with respect to the synchronous position, ω_s is the synchrotron angular frequency, d_r is the damping rate, and λ_l is a term proportional to the effective longitudinal impedance and the beam current.

$$\lambda_l \simeq \frac{\pi e \alpha I_0 f_{RF}^2}{E_o h \omega_s} Z^{||eff}(l\omega_o + \omega_s) \quad (28)$$

for small tune shifts, where e is the charge of an electron, α is the momentum compaction factor, I_0 is the average beam current, h is the harmonic number, and $Z^{||eff}(\omega)$ is the effective ring longitudinal impedance.

Figure 8: Bunch length variation σ_Z and detuning $\Delta\omega/2\pi$ as function of the beam current for $V_3 = \text{constant}$ and even fill.

Assuming that $d_r \ll \omega_s$, Eqn. 27 becomes

$$2j\omega_s(\dot{\tau}_l + (d_r - j\omega_s)\tau_l) \approx 2j\omega_s\lambda_l\tau_l \quad (29)$$

so that

$$\dot{\tau}_l + (d_r - j\omega_s)\tau_l = \lambda_l\tau_l \quad (30)$$

Eqns (28)-(30) are valid if for each mode $\omega_l = \omega_s + \text{Im}(\lambda_l) \simeq \omega_s$. Under this assumption, the growth rate of mode l is $\sigma_l = -d_r + \text{Re}(\lambda_l)$. If the growth rate of any mode is positive, that mode is unstable and must be controlled via some feedback mechanism (or the driving impedance must be reduced, or the current reduced).

To evaluate Eqn. 28, we determine the effective impedance as follows:

$$Z^{\text{eff}}(l\omega_o + \omega_s) = \frac{1}{2\pi f_{RF}} \sum_{p=-\infty}^{p=+\infty} (pN_{\text{bunch}}\omega_o + l\omega_o + \omega_s) Z^{\parallel}(pN_{\text{bunch}}\omega_o + l\omega_o + \omega_s) \quad (31)$$

where $Z^{\parallel}(\omega)$ is the total ring impedance and N_{bunch} is the number of bunches. Essentially, the effective impedance for mode l is a scaled sum of the impedances

at the upper synchrotron sideband of the l_{th} revolution harmonic for all integer multiples of f_{RF} . To determine the HOM driven instabilities, we estimate the effective impedance for the even-fill eigenmodes (the cases with uniform equal charge filling patterns).

During the PEP-II design, Bob Rimmer measured the center frequencies and quality factors for different HOM resonances in a prototype cavity, as shown in Table 11 [2]. Using this data, simulations of the PEP-II cavity during development provided estimates of the total impedance from the HOM

Table 11: Cavity Resonances

f_r (GHz)	R/Q (Ω)	Q
0.758	44.6	18
1.009	0.43	128
1.283	6.70	259
1.295	10.3	222
1.595	2.43	300
1.710	0.44	320
1.820	0.13	543
1.898	0.17	2588
2.121	1.82	338
2.160	0.053	119
2.265	0.064	1975

resonances. These estimates result in the HOM impedance magnitude per cavity shown in Fig. 9.

To provide a reference point, the growth rates for both rings in PEP-II are briefly evaluated, and compared to the PEP-X scenario. Assuming the LER design configuration with 4 cavities (rather than the 8 of the final run) and the design parameters $\alpha = 1.23 \cdot 10^{-3}$, $I_o = 2.25 A$, $E_o = 3.1$ GeV, $h = 3492$, and $\omega_s = 2\pi 4 \cdot 10^3$, Fig. 10 shows the absolute value of $Re(\lambda_l)$ for each mode, considering a 'by-2' (every other bucket filled) beam filling pattern for PEP-II. Similarly, for the HER design configuration of 20 cavities (28 in the end of PEP-II) and $\alpha = 2.41 \cdot 10^{-3}$, $I_o = 1$ A, $E_o = 9$ GeV, $h = 3492$, and $\omega_s = 2\pi 6 \cdot 10^3$, Fig. 11 depicts the absolute value of $Re(\lambda_l)$ for each mode. Effectively, the filling pattern determines the frequency band into which the HOM impedances are aliased. In these figures, we are plotting the absolute value of $Re(\lambda_l)$ (red for positive and blue for negative) and comparing them with d_r (horizontal line) to determine the HOM coupled-bunch stability. Unstable modes are those for which the red $Re(\lambda_l)$ is greater than d_r . In PEP-II unstable modes were present in a region around mode $l = 800$ that corresponds to a frequency of 108 MHz. This instability was assessed using a wide-band bunch-by-bunch longitudinal feedback system.

Figure 9: PEP-II estimated HOM impedance per cavity.

Figure 10: PEP-II LER $|Re(\lambda_l)|$ for $I_o = 3A$ 'by-2' fill

Figure 11: PEP-II HER $|Re(\lambda_l)|$ for $I_o = 2A$ 'by-2' fill

During PEP-II runs, growth rates for modes around $l = 800$ were measured at different beam currents, giving results very close to the one depicted in the figures.

Adapting the HOM impedance information to the PEP-X, it is possible to analyze the effects of the HOM impedances in the coupled-bunch stability. The total number of cavities estimated for PEP-X is $N_C N_{st} = 16$ (PEP-II type cavities) and the beam filling pattern is 'by-1' (every bucket filled). The cavity impedance of the third-harmonic RF system implemented by either normal-conducting (NC) or super-conducting (SC) cavities (Subsection 3.3.1) is included in the calculations of the total impedance. It should be noted that

only the fundamental resonance of the third-harmonic cavities is included, since there are no data available for other resonances. This omission has a negligible effect for the super-conducting case, but it could be noticeable for the normal-conducting case, depending on the final cavity parameters. The resonant frequency of the third-harmonic cavities is at about 1.43 GHz and the third-harmonic cavity parameters used in these calculations are the optimal parameters presented previously in Table 10. Figs. 12 and 13 depict

Figure 12: Total fundamental cavity HOM impedance and third-harmonic normal-conducting cavity impedance for PEP-X.

Figure 13: Total fundamental cavity HOM impedance and third-harmonic super-conducting cavity impedance for PEP-X.

the HOM impedance for the PEP-X case of 16 cavities taking into account the implementation of the third-harmonic system using 19 normal-conducting or 5 super-conducting third-harmonic cavities, respectively.

The estimate of $Re(\lambda_l)$ for the PEP-X case is depicted in Figs. 14 and 15, based on the impedance calculated above and the PEP-X parameters from [1]: $\omega_{RF} = 2\pi 476$ MHz; $\omega_0 = 8.56 \cdot 10^5$ rad/sec; $E_o = 4.5$ GeV, and $\sigma_E/E_o = 1.12 \cdot 10^{-3}$. These estimates also assume a synchrotron frequency of about 500 Hz due to the third-harmonic cavities. From Figs. 14, 15 it is not expected that the HOM driven instabilities will be greater than the radiation damping for PEP-X. The reason for this improvement is two-fold. One obvious difference is the transition from the 'by-2' to the 'by-1' filling pattern. The PEP-X nominal filling pattern (every bucket filled) results in 3492 normal modes, spanning a 238 MHz bandwidth. In comparison, the PEP-II 'by-2' filling pattern results in 1/2 the number of normal modes (1746), with a much narrower span of 119 MHz for the folded impedances. As a result, the most unstable modes for the PEP-II case were in a band around mode 800, whereas in the PEP-X case this same impedance becomes a damped mode, while three bands of potentially unstable HOM driven modes at 60 MHz, 160 MHz and 225 MHz can be now seen. Furthermore, while

Figure 14: PEP-X estimated $|Re(\lambda_l)|$, assuming 8 RF stations with 16 fundamental cavities, 19 normal-conducting third-harmonic cavities, $I_o = 1.5$ A and 'by-1' filling pattern.

the growth rates are inversely proportional to the synchrotron frequency, the greatly reduced momentum compaction and increased radiation damping due to the wigglers, act to keep all the modes stable for a beam current of 1.5 A. The wigglers reduce the momentum compaction factor to $\alpha = 4.72 \cdot 10^{-5}$ for PEP-X, lower than $\alpha = 2.41 \cdot 10^{-3}$ in PEP-II HER and $1.23 \cdot 10^{-3}$ in PEP-II LER.

On the other hand, one can see from Figs. 14, 15 that the third-harmonic cavities alias on top of the fundamental impedance. With their resonance aliasing at 477.75 MHz or 476.18 MHz for the normal-conducting or superconducting implementation, they affect a range of modes (wider range for the normal-conducting case) around mode 13 or mode 1 respectively. Since this impedance is added on the fundamental - but is not reduced by the feedback loops - further development of our beam models and simulations are necessary to determine the effect of the third-harmonic cavities on the fundamental-driven modes.

Figure 15: PEP-X estimated $|Re(\lambda_l)|$, assuming 8 RF stations with 16 fundamental cavities, 5 super-conducting third-harmonic cavities, $I_o = 1.5$ A and 'by-1' filling pattern.

4.2 Coupled-bunch longitudinal instabilities driven by the cavity fundamental and third-harmonic system impedance.

Our initial simulations of the cavity fundamental driven beam modes suggest that that low mode coupled-bunch instabilities driven by the cavity fundamental will force damping and RF feedback system requirements quite differently than those developed for PEP-II. In our analysis we are assuming that the RF systems employed will be consistent in achieved effective impedance with the systems used for PEP-II, even though they will likely not use the narrowband comb loop of the PEP-II design. It is also important to note that these studies do not include the effect of the harmonic cavity impedance on the cavity fundamental driven modes (the harmonic cavity impedance aliases right on top of the fundamental as described earlier). Our beam models and simulation are being updated to include the effect of the harmonic cavity.

For our initial studies we are using the macro-bunch and RF system simulation tools developed to study the PEP-II dynamics [3]-[5]. In this study we model the low mode dynamics of the nominal PEP-X design, and we use the existing PEP-II Direct loop topology (but not a comb loop) and implementation for the RF system model. The narrowband comb loop is not particularly helpful in the PEP-X case, and the very low synchrotron frequencies, and large tune variation around the turn for the harmonic cavity

cases are not consistent with the existing comb filter implementation.

From these initial studies (further investigation will follow), we expect the fastest growing beam mode driven by the de-tuned cavity fundamental to be approximately 0.4 ms^{-1} , as can be seen in Fig. 16.

Figure 16: PEP-X estimated growth rates with a direct feedback loop.

To determine stability, this growth rate has to be compared with the maximum achievable damping from the chosen feedback system. The accelerator parameters for PEP-X are much different than those in PEP-II, so the damping rates achieved in the past cannot be necessarily implemented. In PEP-II two active feedback systems were employed. The Longitudinal Feedback (LFB) was a wideband system that suppressed all modes. With consistent timing, it achieved damping rates in excess of 0.5 ms^{-1} . The current implementation of the LFB is close to a possible design for PEP-X. The second system was the Low Group Delay Woofer (LGDW), a narrowband design that achieved much higher damping rates (up to 3 ms^{-1}) for the modes near the cavity fundamental. Due to the significantly lower synchrotron frequency though, the PEP-X scenario is beyond the current capabilities of the LGDW. A different control algorithm would have to be implemented (possibly with the existing hardware), with a smaller number of taps in the designed filter. In our next steps, we would like to evaluate the effectiveness of different algorithms and their susceptibility to noise.

5 Beam Characteristics and Performance

In this section important longitudinal characteristics of the beam for PEP-X are analyzed. The most important characteristic is the bunch length modulation and the uniformity of σ_Z along the train in presence of an ion clearing gap in the beam pattern. Second, the longitudinal beam motion is analyzed and mechanisms to attenuate the driven motion due to the RF station noise are suggested.

5.1 Modulation of bunch length due to the ion clearing gap

The ion clearing gap is of great importance for PEP-X, since it will modulate the voltage of the RF cavities and **cause significant tune shifts and bunch length variations around a turn**. The extent of these shifts depends on the interaction between the energy stored in the cavity, the energy deposited by the RF system, and the energy received by the beam. This interaction leads to modulation of the cavity voltage and consequently of the bunch length. A possible solution is to increase the amount of stored energy in the cavity. This can be achieved by operation at a different gap voltage or use of super-conducting cavities or energy storing cavities. Other solutions reduce the beam current, increase the klystron power or make more sophisticated use of the existing klystron power. A more detailed description of the gap transient effect for the nominal PEP-X design follows in Section 5.1.1. Some of the possible approaches to reduce the bunch length modulation will be briefly analyzed in Section 5.1.2:

- Use of additional klystron power to directly reduce the bunch length modulation through additional fundamental cavities/RF stations.
- A novel “Gap Feedforward” compensation technique, to use some klystron power for modulation rejection.
- An active third-harmonic RF system, to store energy in the harmonic cavities which would reduce the bunch length modulation.
- Energy-storage cavities coupled to normal-conducting RF cavities (KEK B ARES-cavity energy storage technique)
- A super-conducting fundamental RF system, to increase the cavity stored energy.

5.1.1 Normal-conducting fundamental cavity case

Using normal conducting cavities, we propose an RF station topology similar to the one used in PEP-II. The PEP-II LLRF design incorporates direct feedback, which is a prompt wideband feedback loop comparing the cavity vector voltage to a reference. The block diagram for a PEP-II RF station is depicted in Fig. 17.

Figure 17: Block diagram of PEP-II RF station.

In PEP-II the ion clearing gap in the filling patterns modulated the beam current to such an extent that the direct loop would drive the klystron to saturation in the attempt to regulate the cavity voltage during this modulation. To avoid saturation effects, the LLRF system implements a “Gap FeedForward” function, which is a feedback system that drives a correction transient signal into the direct loop regulator to cancel out the gap error signal (Gap Loop) [6], [7]. The overall effect of this technique is that there is a current-dependent steady-state gap transient, which modulates the RF voltage around the turn, but the klystron itself is not saturated during the turn and can still provide small signal modulations to achieve impedance control.

In PEP-II this steady-state gap transient was significant in that the angle ϕ_{B_i} of each bunch varied around the turn, and matching of HER and LER transients was important if the collision point was to be controlled. For PEP-X, this ion clearing gap has a very significant impact on the operating

point and voltage of the passive harmonic cavities. Because of this modulation, each bunch in the turn sees a different harmonic voltage, and there are significant tune shifts and bunch length variations around a turn.

Our preliminary study of the bunch length variation over a turn is based on a beam-cavity IQ coupled model from the existing literature [8], which imperfectly approximated the effective cavity impedance due to the RF feedback. The beam-cavity interaction analyzed in [8] is based on a lumped element circuit equivalent of the fundamental RF resonance, as shown in Fig. 18. Transfer functions between the cavity voltage V_c and each of the

Figure 18: Schematic of the RF cavity model with two input currents and feedback loops.

total current I_T , the beam current I_B , and the generator current I_G can then be derived. The time-invariant nature of the beam dynamics and the feedback loops though do not allow us to completely estimate the stability of the system based on this frequency analysis alone. To simplify the problem, the gap transient is calculated assuming that the the coupled bunch modes are stable.

The existing models include the following approximations. First, they neglect second order effects of the accelerating voltage on the beam phase. Therefore, they are only true for deviations of approximately less than 10° from the synchronous position, where the voltage is approximately linear. Second, the models do not take into consideration the fact that the “Gap Feedforward” loop is acting in a bandwidth different than the beam dy-

namics, essentially presenting a time-varying impedance to the beam. This assumption essentially nulls the effect of the direct feedback. The models also do not include variations in the beam dynamics. Developing a formalism that can incorporate the time-varying behavior of the “Gap Feedforward” Loop is essential for the development of more accurate models.

Fig. 19 shows the bunch length variation over a turn for PEP-X using the simplified beam-cavity IQ coupled model described above. This limited

Figure 19: Bunch length variation over a turn calculated using the beam-cavity IQ coupled model.

accuracy initial model shows the substantial variation of bunch length due to the gap voltage (up to a factor of 2), with a minimum length of 3.7 mm. This estimate assumes the base PEP-X design based on the PEP-II RF systems and a passive third-harmonic cavity as defined in Sections 2, 3.

The complexity of this issue was historically illustrated in the commissioning of the third-harmonic cavities at the ALS. The initial expectation was that these harmonic cavities would uniformly lengthen the bunch over a turn and through Landau damping mechanisms the beam would be longitudinally stable for all HOM modes. In fact operation proved to be very difficult, only roughly 1/3 of the turn received any bunch lengthening and ultimately 3 out of 5 third-harmonic cavities installed were eventually parked due to the complex effect on gap transients [9]-[11]. The results from ALS on the effect of two or three harmonic cavities on the bunch length and beam offset are shown in Fig. 20 from [11].

For PEP-X more work is necessary to develop the necessary models to accurately describe the beam motion due to the gap transient, but these results suggest that it is not possible to achieve the necessary bunch length for

Figure 20: Longitudinal beam offset and bunch length along the bunch train for two or three harmonic cavities at the ALS.

the required Touschek-scattering limited lifetimes. Alternative approaches have been identified, and an initial analysis has begun for some cases.

5.1.2 Alternative Approaches

- Several extra RF systems could be used, above the base minimum needed calculated in Section 2. For this purpose the additional klystron RF power is used to directly reduce the beam current modulation from the gap transient (as employed in the LHC). This technique – even though it is energy consuming – does not require the infrastructure overhead related with a super-conducting system. Its disadvantages include the higher cost from power consumption as well as the added complexity from the additional impedance presented to the beam and

the reliability impact from the increased number of RF stations.

- The “Gap Feedforward” in the LLRF systems could implement a different compensation technique than used in PEP-II. Such a technique would modulate the “Gap Feedforward” correction signal in such a way that there will still be some gain available at the revolution harmonics. As was described earlier this would be an intermediate approach between the brute force implementation of extra klystron power and the PEP-II technique in which no klystron power was allocated to the steady state RF transient modulation in the cavity. The advantage of this intermediate compensation implementation would be its very low cost. On the other hand, such a system has not been developed or studied before, so there are a lot of unknowns related to it. While this technique would probably reduce the bunch length modulation over the turn through the use of some klystron power, it cannot cancel it completely. Instead it may be applicable to change the rms variation and min/max structure of the bunch length over a turn to a more acceptable level.
- Using powered harmonic RF systems could also reduce the bunch length modulation. This design would also have higher operational costs, and also higher complexity, which would require further studies to adapt the operational optimization routines. In this case the power stored in the harmonic cavities would have to be great enough to compensate for the modulation in the main RF system. The harmonic cavities become also accelerating cavities and the gap voltage levels or the number of harmonic cavities might not be economically feasible.
- Another possible method to improve the modulation of the accelerating voltage and bunch length is to use an RF system design developed and utilized by the KEKB accelerator (ARES), shown in Figs. 21, 22. This technique, pioneered by Shintake [12]-[16], uses a normal conducting RF accelerating cavity coupled to a normal-conducting energy storage cavity. The energy storage cavity improves the transient response of the combined system to the gap transient beam loading perturbation, so that the magnitude of the accelerating voltage modulation is reduced. This approach avoids the cryogenic infrastructure complexity and operational issues of superconducting RF cavities, however it adds extra complexity to the design of the normal-conducting RF system, adds extra concerns with potential HOMs in the energy storage cavity coupling to the beam through the combined system. It is necessary to develop this design approach more completely with some initial analysis to begin to estimate the attractiveness, or unattractiveness, of this

Figure 21: Accelerating RF system for KEK B [12].

potential technique for PEPX.

- A super-conducting system has higher stored energy in the cavity. As a result, we expect that the bunch length would be more uniform over a turn with this implementation. The obvious disadvantage is that the infrastructure for a super-conducting system is not presently available. Therefore, the cost to design and build such a system would be much higher. We briefly discuss below the use of super-conducting cavities for the fundamental RF system, considering the operation point of the RF station and the multi-bunch stability.

5.1.3 Super-conducting fundamental cavity case

Assuming a super-conducting cavity similar to the one used at CERN LHC, the main parameters for the SC cavities are $R/Q = 45$, $V_{C_{max}} = 2 \text{ MV}$, $I_b \simeq 1.5 \text{ A}$. The coupling factor between the cavity and the waveguide is variable for the CERN set-up, with a loaded quality factor in the range $Q_L = 60\text{k} - 180\text{k}$ and the maximum klystron power of 300 W.

Figure 22: A photograph of the first high-power ARES cavity [13].

Taking as base for our calculations the main parameters of that cavity, different configurations of the RF station using 1 and 2 cavities per station are shown in Table 12, assuming the coupling between the cavity and the waveguide can be optimized for minimum reflected power at nominal beam current $I_b = 1.5A$.

Table 12: Operative conditions for the super-conducting case of the PEP-X RF system, $V_{GAP} = 10$ MV, $I_b = 1.5$ A, $P_{C_{refl}} \simeq 0$.

N_{st}	N_C	V_C	P_{kly}	Q_L
5	1	2.0 MV	1021 KW	43.57 K
6	1	1.66 MV	851 KW	36.31 K
7	1	1.42 MV	729 KW	31.12 K
6	2	833 KV	851 KW	18 K

From these results, the RF configuration using one cavity per klystron is the only solution for the given combination of V_C and P_{kly} . If the limit of $P = 500$ kW is set as the maximum power transferred to the cavity, the option with two cavities, six RF stations is the nominal for this design. If we assume the system is designed with headroom to allow for a parked RF

Table 13: Operative conditions for the super-conducting case of the PEP-X RF system, $V_{GAP} = 10$ MV, $I_b = 1.5$ A

<i>Condition</i>	N_C/N_{st}	V_C	P_{kly}	Q_L	$P_{C_{refl}}$
Nominal	1/6	1.66 MV	851 KW	36.31 K	~ 0
Parked	1/5	2.0 MV	1030 KW	36.31 K	8.3 KW
Nominal	1/7	1.42 MV	729 KW	31.12 K	~ 0
Parked	1/6	1.66 MV	856.2 KW	31.12 K	5 KW
Nominal	2/6	833 KV	851 KW	18 K	~ 0
Parked	2/5	1.0 MV	1.03 MW	18 K	4.18 KW
Nominal	2/7	715 KV	729.6 KW	15.5 K	~ 0
Parked	2/6	833 KV	856 KW	15.5 K	2.48 KW

station when it is not operative, the possible system designs are summarized in Table 13.

Results on the bunch length variation around a turn are not yet available for this implementation of the RF system, but we expect a significant reduction over the normal-conducting case due to the higher stored energy (high R/Q ratio).

From this analysis it is obvious that the gap transient effect on the bunch length uniformity over a turn is a significant issue for PEP-X. Even though there are a few possible solutions, additional work is necessary to first evaluate and select the optimal approach, and then proceed with its design.

5.2 RF station noise

While the beam may be free of HOM instability as shown in Subsection 4.1. it will not be necessarily stable longitudinally. We must estimate the amount of driven motion at the synchrotron frequency from noise mechanisms in the RF systems, master oscillator, etc. as well as the natural energy spread from quantum fluctuations. The synchrotron frequency for PEP-X is much lower than the 4 kHz in the LER and the 6 kHz in the HER, and it is in a band where the RF station noise is significant. As it was shown earlier for $V_{GAP} = 13$ MV, $\Rightarrow f_s = 1.1654$ kHz, for $V_{GAP} = 10$ MV, $\Rightarrow f_s = 1.0089$ kHz, and for $V_{GAP} = 8$ MV, $\Rightarrow f_s = 885.2562$ Hz.

The significant reduction of the HOM and cavity fundamental driven coupled-bunch instabilities for PEP-X in comparison with PEP-II is largely attributed to the significantly lower momentum compaction factor α . The new lattice and the use of wigglers reduces α by almost two orders of magnitude. This reduction helps mitigate the effect of the lower synchrotron frequency on the impedance driven growth rates.

On the other hand, the reduced synchrotron frequency is a cause of concern due to the RF station noise. In Fig. 23 one can see noise measurements for the LER during PEP-II operations. The two “noisy” bands correspond to

Figure 23: PEP-II LER noise measurements

the beam (around the synchrotron frequency of 4 kHz) and the RF station (around 720 Hz). Since the estimated synchrotron frequency for PEP-X is at about 1 kHz (or 500 Hz with a third harmonic cavity present), the effect of the station noise will be significant in exciting noise driven beam motion within the synchrotron resonance bandwidth. This is not “unstable” motion, instead the beam is simply excited from the noise power in the RF system and has an energy spectral distribution determined by the beam resonance and the spectrum of RF station noise.

In PEP-II this noise was dominated by the RF Klystron high voltage power supply noise, and while the direct loop has some effect in reducing the cavity voltage variations the residual noise may still be of concern for PEP-X. As stated in [1], the longitudinal phase stability requirement has to be a small fraction of 1° or equivalently the time-of-arrival jitter has to be less than 1 ps. Using data from PEP-II, we can estimate the expected longitudinal phase with respect to the synchronous particle $\Delta\phi$, and the time-of-arrival jitter $\Delta\tau$ due to the RF station noise.

Through a small signal analysis of the accelerating voltage, we model the longitudinal phase as

$$\Delta\phi = \Delta\phi_{RF} + a(t) \tan \phi_b \quad (32)$$

where $\Delta\phi_{RF}$ is the phase noise induced by the RF station, ϕ_b is the angle between the cavity voltage and the beam current, and $a(t)$ is the amplitude

modulation of the gap voltage. We can approximate $a(t)$ by $\Delta V_G/V_{GAP}$ where ΔV_G is the effect of the station noise on the gap voltage. With this model and data from PEP-II, we can estimate $\Delta\phi$ for a PEP-X scenario to 0.1° or equivalently 0.59 ps, assuming the RF station will present similar noise characteristics.

Even though this time-of-arrival jitter is smaller than the stability requirement set in [1], it only includes the effect of the RF station noise on the beam. Further effects in the transverse plane could possibly increase the jitter. It is also very important to note that this is an estimate based on PEP-II data. As mentioned earlier and shown in Fig. 23, the RF station noise is centered around 720 Hz, so the effects on a PEP-X beam with synchrotron frequency at 500 Hz or 1 kHz, would be more pronounced, in which case this impact of RF driven noise must be better estimated and studied. A Klystron Linearizer, a technical direction from the PEP-II effort, might be applicable to control this high voltage power supply noise [17], [18].

6 Future Directions

This initial study has only touched on several important RF system concerns for PEP-X. The single largest unknown at this time is the operational impact of passive harmonic cavities, with the variation in bunch length, variation in synchrotron tune around a turn as fundamental concerns to mitigate. To better understand the options to achieve the desired improvement in bunch length and lifetime, we propose studies to explore options to deal with Gap transient effects and the impact on bunch length. In particular, we think it is important to explore the value of using more RF stations than estimated from power limits, and potentially use the extra klystron power to reduce the gap transients (of course, besides the extra operational energy cost the extra cavities add extra impedances, too). We propose to study the RF power necessary to reduce gap transients, and study the operational benefits (particularly tuning flexibility) of multiple $3f_{RF}$ passive cavities. We think it important to study active $3f_{RF}$ systems and estimate possible benefits, complexity and required power. The Shintake cavity approach and the superconducting configuration also need further evaluation.

The difficulties controlling the fundamental-driven low mode instability should be studied via the dynamics simulation, and potential control schemes explored.

Finally, we think it is very important to estimate the impact of noise-driven longitudinal motion on beam size and emittance in a dispersive region. While some of the historic PEP-II noise data is an excellent start to estimate these effects, it would be of interest to study the operating RF systems at SPEAR-III, and particularly measure the noise spectrum in the RF system in the very low frequency band of concern for the PEP-X application. SPEAR-II is also an interesting test-bed to explore methods to reduce the RF system noise, such as the use of the Klystron linearizer approach, the development of a system similar to the LHC polar loops, or other control techniques.

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A Operational Design Considerations - Supporting Material

Figure 24: Minimum number of stations and Gap Voltages for $P_{kly_{LIM}} = 900$ kW and $\beta = 3.9$

Figure 25: Minimum number of stations and Gap Voltages for $P_{kly_{LIM}} = 900$ kW and $\beta = 7.5$

Figure 26: Minimum number of stations and Gap Voltages for $P_{klyLIM} = 900$ kW and $V_C = 600$ kV

Figure 27: Minimum number of stations and gap voltages for $P_{refLIM} = 100$ kW and $\beta = 3.9$

Figure 28: Minimum number of stations and gap voltages for $P_{refLIM} = 100$ kW and $\beta = 7.5$

Figure 29: Minimum number of stations and gap voltages for $P_{refLIM} = 100$ kW and $V_C = 600$ kV