Incorporating Multiplicative Systematic Errors in Branching Ratio Limits ^{*}

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Abstract

The issue of including multiplicative systematics in the setting of branching ratio limits is discussed. In particular, a method for including them when the signal extraction is done with a multi-dimensional fit, rather than a “cut-and-count” technique is presented.

If a search for a rare decay finds no significant excess of events over that expected from background, the usual procedure is to set a 90% confidence upper limit on the branching ratio for that decay. In doing so, one must take account of the multiplicative systematic uncertainties, such as signal efficiency. A common, although questionable, procedure is to inflate the limit obtained by $1 + \sigma_\epsilon$, where σ_ϵ is the relative multiplicative uncertainty.

Cousins and Highland [1] give a prescription for including multiplicative systematic errors in limits for “cut and count” type analyses, where no events are observed in the signal box. They note that if systematics are ignored, the probability of seeing zero events is:

$$P_0(B) = e^{-BS},$$

where S is the sensitivity ($S \equiv \epsilon N_{BB}$). The 90% confidence limit (L) is then calculated by

$$0.1 = \int_L^\infty e^{-BS} dB.$$

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Doing the integral, they find $L = \ln 10/S = 2.3/S$, and the familiar limit $B > 2.3/S$. To include multiplicative systematics, they convolve $P_0(B)$ with the the Probability Distribution Function of the sensitivity, which is assumed to be Gaussian with central value, $\hat{S}$ and error σ_S :

$$P_0(B) \propto \int_0^\infty e^{-BS} e^{-(S-\hat{S})^2/2\sigma_S^2} dS.$$

Doing this integral, they find that the limit becomes

$$B > 2.3(1 + 2.3(\sigma_S/\hat{S})^2)/\hat{S}$$

With a 10% relative error on S , the limit only increases by 2.3%.

Many analyses, however, do not use a “cut and count” signal extraction, but rather a Maximum Likelihood Fit. The output of such an analysis is a central value of the branching ratio ($\hat{B}$) and a statistical error (σ_B). The likelihood function is then:

$$p(B) \propto e^{(B-\hat{B})^2/2\sigma_B^2}$$

The extension of the above prescription for including multiplicative systematics to this case is:

$$p(B) \propto \int_0^1 e^{-(SB/\hat{S}-\hat{B})^2/2\sigma_B^2} e^{-(S-\hat{S})^2/2\sigma_S^2} d\epsilon.$$

Doing this integral (for simplicity, we assume $\sigma_S \ll \hat{S}$), we find:

$$p(B) \propto \frac{1}{\sqrt{\frac{B^2}{2\sigma_B^2} + \frac{\hat{S}^2}{2\sigma_S^2}}} e^{-(B-\hat{B})^2/2(\frac{B^2\sigma_S^2}{\hat{S}^2} + \sigma_B^2)}$$

We can then integrate this PDF to get the confidence limits.

Figure 1 shows the shape of this PDF for various values of $\sigma_\epsilon \equiv \sigma_S/\hat{S}$. For small values of σ_ϵ , the curve is roughly Gaussian. However, as σ_ϵ becomes large, the curve develops a high-side tail. The presence of this tail degrades the confidence limits.

References

- [1] R. Cousins and V. Highland, Nucl. Instrum. Methods A.**320**, 331-335 (1992)

Figure 1: Example plots of the effect of including multiplicative systematic errors as a function of σ_ϵ , the relative error on the sensitivity.