A Precision Measurement of the Spin Structure of the Proton at SLAC

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A PRECISION MEASUREMENT OF THE SPIN STRUCTURE OF THE PROTON AT SLAC

A DISSERTATION
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Abstract

E143 at SLAC Endstation A performed deep-inelastic scattering measurements of polarized electrons from polarized protons and deuterons within cryogenic $^{15}\text{NH}_3$ and $^{15}\text{ND}_3$, respectively. Data were taken at incident energies of 29.1, 16.2 and 9.7 GeV, and covered the kinematical range $x > 0.03$ and $0.3 < Q^2 < 12 \text{ (GeV/c)}^2$. The scattered electrons were detected by two spectrometers at angles of 4.5° and 7.0° . From these data, the spin-dependent structure functions $g_1^p(x, Q^2)$ and $g_1^d(x, Q^2)$ were determined. This dissertation describes the experiment, with emphasis on the results on the proton spin structure. The integral of g_1 over the range $0 < x < 1$ was found to be $\Gamma_1^p = 0.130 \pm 0.003 \pm 0.008$ for the proton and $\Gamma_1^d = 0.044 \pm 0.003 \pm 0.004$ for the deuteron. Both values are in agreement with world data, and violate the Ellis-Jaffe sum rule by more than 3 standard deviations. The neutron structure function was obtained by combining proton and deuteron results, giving $\Gamma_1^n = -0.035 \pm 0.007 \pm 0.010$. From this the integral $\Gamma_1^p - \Gamma_1^n$ followed, yielding $0.165 \pm 0.009 \pm 0.016$ at $Q^2 = 3 \text{ (GeV/c)}^2$, in agreement with the Bjorken sum rule to within one standard deviation. The Q^2 -dependence of the ratio g_1/F_1 was determined to be small for $Q^2 > 1 \text{ (GeV/c)}^2$, validating the assumption of no Q^2 -dependence used in obtaining the integrals. A small rise with increasing Q^2 was seen in the ratio for $Q^2 < 1 \text{ (GeV/c)}^2$, however. The total quark contribution to the spin was found to be $\Delta q = 0.28 \pm 0.09$ for the proton, and $\Delta q = 0.32 \pm 0.05$ for the deuteron. Furthermore, a large negative spin contribution from the strange sea quarks was measured for both nucleons, giving $\Delta s = -0.10 \pm 0.03$ and $\Delta s = -0.09 \pm 0.02$ for the proton and deuteron respectively, in contradiction to the Ellis-Jaffe assumption of no polarized strange sea.

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Chapter 1

Introduction

The elements that form the world in which we live, such as carbon, hydrogen, iron, and so forth, are forms of atoms. Atoms, once thought of as the most basic building blocks in the universe, are now known to be formed from smaller particles of matter, namely electrons surrounding a massive nucleus. Furthermore, the nucleus is in turn made up other particles: the nucleons. Nucleons can be either positively-charged protons, or neutral particles called neutrons. Attempting to understand the nucleon is an interesting pursuit in itself, but it is essential to understanding the structure of matter and the world in which we live.

Stanford and the Stanford Linear Accelerator Center (SLAC) has a rich history in the study of nucleons. In the 1950's, elastic and quasi-elastic scattering experiments [1] performed on Stanford campus in the High Energy Physics Laboratory (HEPL) indicated that the nucleon has a finite size, of the order of 10^{-13} cm. These studies resulted in a Nobel prize for Robert Hofstadter in 1961. In the mid-1960's, various experiments [2] showed that the nucleon

cross-section fell with increasing momentum transfer, suggesting that the nucleon might be a composite particle. Then, in 1969, some of the first deep-inelastic scattering (DIS) results from SLAC [3] showed a cross-section larger than expected, with only a weak dependence on the momentum transfer (Q^2). This behavior, known as scaling, was interpreted to imply that the nucleon consists of point-like charged particles, and was the first evidence of the existence of constituent particles in the nucleon. The measurement resulted in a Nobel prize in 1990 for the leaders of the experiment: Professor Richard Taylor of SLAC, and Professors Jerome Friedman and Henry Kendall of the Massachusetts Institute of Technology (MIT).

Today it is believed that nucleons are composed of what are called **valence quarks**, **sea quarks**, and **gluons**. In particular, three valence quarks of *up* and *down* flavor determine net quantum numbers of the nucleons such as charge and baryon number. Gluons are massless vector bosons, and are the mediators of the strong force, responsible for holding the quarks within the nucleon together, as well as the nucleons within the nuclei. Sea quarks are

virtual pairs of quarks and anti-quarks created and destroyed by interactions with the gluon field, and are predominantly of *up*, *down*, and *strange* flavor.

A number of experiments using unpolarized deep-inelastic scattering (DIS) have been performed at SLAC and elsewhere since the discovery of nucleon substructure. Such experiments have studied the structure of the proton and neutron by probing the number density of partons within the nucleon, leading to nucleon form factors and unpolarized structure functions. However, there has been great interest in studying the *spin structure* of the nucleons, which indicates how the total spin-1/2 quantum number of the proton and neutron is distributed amongst the nucleon constituents.

The simplest model for nucleon spin, which is contained in what's known as the *naive Quark-Parton Model (QPM)*, combines only the spin-1/2 valence quarks to form a state with total spin of 1/2. In this simplistic picture, the SU(6) model of baryons describes the magnetic moments of the nucleons well.

In 1966, James Bjorken used current algebra as well as isospin symmetry to derive a sum rule [4] relating the spin structure functions of the proton and neutron to the well known vector-axial vector coupling constant in neutron beta decay. In addition, in 1974, John Ellis and Robert Jaffe [5] derived two more sum rules for the proton and neutron alone, relating them to the hyperon decay constants F and D. Here Ellis and Jaffe assumed SU(3) flavor symmetry, as well as the naive QPM, which contains no contribution to nucleon spin from polarized strange sea quarks.

Experimental evidence confirming or ruling out these sum rules had to wait until 1976, however, when SLAC had obtained a polarized electron beam, and when methods of polarizing nucleon targets had been achieved. Pioneering measurements were made by a SLAC-Yale collaboration in 1976 [6] and again in 1983 [7] at SLAC on the spin structure of the proton. Yet the results were not statistically precise enough to confirm or refute the theories of nucleon

structure at the time, and were consistent with both the Bjorken and Ellis-Jaffe sum rules to within experimental error.

Nothing more was done experimentally on the study of nucleon spin structure until 1988, when the European Muon Collaboration (EMC) at CERN published results from scattering polarized muons from polarized protons in frozen butanol [8]. This experiment found that the quark actually carries only a small fraction of the proton spin, and found also that the strange sea has a significant negative polarization with respect to the proton spin, in disagreement with the Ellis-Jaffe sum rule and its assumptions. These results were so surprising that a so-called *spin crisis* developed, and many theoretical papers were written in an attempt to explain them. However, in 1993 the Spin Muon Collaboration (SMC) at CERN and the E142 collaboration at SLAC published measurements on the deuteron and neutron spin structure functions gathered from data taken with muons scattering from frozen butanol, and with electrons scattering from gaseous ^3He , respectively. The SMC concluded that the EMC results were correct [9], as did E142 [10], and that there was really no spin crisis after all. The problem had been with the assumption of the naive quark parton model, and the exclusion of spin contributions from the other constituents of the nucleon: the sea quarks and gluons.

Since then, other experiments on the proton, neutron, and deuteron at CERN [9, 11, 12, 13, 14], SLAC [10, 15] [16, 17, 18, 19, 20, 21, 22] [23, 24, 25] [26, 27, 28, 29], and HERMES at DESY [30] have all furthered our understanding of nucleon spin structure, the QPM, and QCD— the theory of the strong force. This thesis details the second of the SLAC experiments, **SLAC E143** [21], and will emphasize in particular the results on **the spin structure of the proton**.

E143 was performed at SLAC a year after E142, and investigated the structure of the proton and deuteron via polarized electron scattering from polarized

protons and deuterons in $^{15}\text{NH}_3$ and $^{15}\text{ND}_3$, respectively. E143 is an international collaboration of over 80 physicists from 17 institutions. The experiment was performed at SLAC Endstation A, the site of the 1969 SLAC-MIT experiment that found first evidence of quarks. It was a high-precision measurement, more accurate than the previous SMC and E142 experiments. In addition, the data were taken at three different incident energies, providing information on the evolution of the spin structure functions with momentum transfer Q^2 .

Chapter 2

Theoretical Background

2.1 Electron - Nucleon Scattering

The scattering of electrons off of nucleons has been a powerful method of studying the inner structure of the nucleon for the past 30 years. To first order, the effective probe of nucleon structure is a single virtual photon exchanged between the incoming electron and the nucleon [31]. Figure 2.1 shows the first-order Feynman diagram for electron-nucleon scattering. The photon four-vector momentum transfer is $q = k - k'$, where k is the momentum of the incoming electron, k' is the momentum of the scattered electron, and $q^2 = -Q^2$ with $Q^2 > 0$. The resolving power of the virtual photon as a probe is proportional to the wavelength $\hbar/Q$, with smaller wavelengths allowing greater resolution. As a result, the extent to which the nucleon structure can be determined increases with Q^2 , which is dependent upon the energy E of the incoming electron, the energy E' of the electron after scattering, and the scattering angle θ via $Q^2 = 4EE' \sin^2 \theta/2$. In the laboratory frame, the energy of the virtual photon is $\nu = E - E'$, the difference in energy of the incoming and scattered electron.

Figure 2.1: First order Feynman diagram of electron-nucleon scattering.

At low momentum transfer, where $Q^2 \approx 0.01 \text{ GeV}^2$, the resolution is poor, and when the photon collides, the entire **nucleus** will either recoil as a whole (coherent scattering) with $\nu = Q^2/2M_A$, where M_A is the nuclear mass, or it will jump into an excited state. Here one can define the variable $x_A = Q^2/2M_A\nu$ using notation from reference [31], so that such *elastic* scattering from the nucleus occurs at $x_A = 1$. As the energy increases, the virtual photon can penetrate the nucleus, and the interaction becomes predominantly scattering from **nucleons** inside of the nucleus. This *quasi-elastic* scattering peaks at $x_A = M/M_A \simeq 1/A$. This is the regime containing the nucleon resonances Δ and N^* , which are smeared out by the Fermi motion of the nucleons.

As Q^2 continues to increase to above 0.5 GeV^2 , nucleon resonances and form factors decrease and we enter the region where the **nucleon structure** is being probed. Since nucleons are known to contain up and down quarks, (uud) for the proton and (udd) for the neutron, then by analogy with the nucleons revealing themselves at $x_A \simeq 1/A$, we might expect quark structure to present itself at $x = 1/3$. Experimentally, however, as Q^2 increases toward $x = 0$ there is a smooth continuum, which is now believed to be due to a *sea* of quark-antiquark pairs, and can include the strange (s) flavor. This scattering is in the **deep-inelastic** (DIS) region, where the nucleons actually break up and lose

their former identity in the debris. However, the debris does not necessarily pose a problem for studying nucleon structure. DIS experiments in which *only* the scattered electron is detected without identifying other particles, a so-called “*inclusive reaction*” [$e + N \rightarrow e + X$] [32], have been very successful.

2.1.1 Scaling and the Quark Parton Model

The structure functions most typically used [31] to characterize the features of the scattering of leptons off of nucleons are known as W_1 and νW_2 (described below). In 1968 [33], Bjorken used current algebra to argue that in the limit of $\nu, Q^2 \rightarrow \infty$ with the ratio $Q^2/\nu \sim x$ fixed,

$$MW_1(x, Q^2) \equiv F_1(x, Q^2) \rightarrow F_1(x) \quad (2.1)$$

$$\nu W_2(x, Q^2) \equiv F_2(x, Q^2) \rightarrow F_2(x) \quad (2.2)$$

where $F_{1,2}$ are the unpolarized nucleon structure functions most commonly referred to. This was known as *Bjorken scaling*. It implied that in the scaling limit (high Q^2, ν), the nucleon structure functions no longer depend on ν and Q^2 independently, but rather on the single variable $x = Q^2/2M\nu$, also referred to as *Bjorken x* , or x_{BJ} . This is the same x_A referred to previously in the discussion of elastic scattering, and will be referred to simply as x or x_{BJ} interchangeably.

Then, in 1969 [34], Callan and Gross found that the currents coupling to the virtual photon consisted of spin 1/2 particles (current quarks), and they used this fact to relate the two scaling structure functions

$$F_2(x) = 2xF_1(x), \quad (2.3)$$

from which it followed that the cross-section σ_L for longitudinally polarized photons must vanish (ie- the partons cannot absorb them).

The first DIS Experiments (ie- large Q^2) at SLAC in the late 1960s of electrons from proton and deuteron targets had interesting results. First, they showed that the electrons scattered at large angles about thirty times more abundantly than expected [35] for scattering off the nucleon. Furthermore, other experiments at SLAC [3] [36] demonstrated the predicted *scaling* of the structure functions.

It was Feynman [37] who in 1969 developed the picture of the *quark parton model* (QPM) to understand the data and Bjorken scaling. The proton was no longer considered a point particle but rather a collection of *partons* from which the virtual photon elastically scattered. These hard, point-like partons inside the proton explained the dramatic angles seen in the scattering experiments. Furthermore, the DIS cross-section should be the incoherent sum of these individual elastic photon-parton cross-sections. In the high proton momentum limit ($P \rightarrow \infty$) the Bjorken x variable was simply the fraction of the proton's momentum carried by the struck parton. This meant that during scattering, an incident virtual photon having a particular value of Bjorken x could only be absorbed if the struck free quark carries the same fraction x of the nucleon momentum. Therefore the absorption cross-section was proportional to the probability $q_i(x)$ of the photon finding a type i parton with momentum xP in the proton.

Scaling could be explained within the QPM as follows: in the limit of $Q^2, \nu \rightarrow \infty$, during the short interval in which the hard scattering took place, the quarks were thought to behave as if they were free particles with no interactions between them. This was known as *asymptotic freedom*, and could account for the scaling effect.

It wasn't long before non-Abelian gauge theories were developed [38] that possessed asymptotic freedom, and the resulting field theory Quantum Chromodynamics (QCD) became the established theory for strong interactions in general. At the same time, an experiment making a precise measurement

of nucleon structure indicated that the charged partons carried only half of the total nucleon momentum, and that neutral partons carried the other half. Feynman's charged partons were later determined to be *quarks*, the point-like constituents of the nucleons, and the neutral partons were *gluons*, the mediators of strong interactions.

In practice, the QPM with scaling is only an approximation of perturbative QCD (pQCD). In the naive QPM, or in the scaling limit at high Q^2 , the quarks behave like free particles, and are the only partons interacting with the photon probe. In QCD, however, quarks can interact by exchanging gluons, and gluons themselves interact as well, thus modifying the structure functions based on the Q^2 of the interaction.

2.2 Deep Inelastic Scattering Kinematics

The *inclusive reaction* [32] discussed in section 2.1 in which DIS experiments simply measure the scattered electron is of the form:

$$e + N \rightarrow e' + X \quad (2.4)$$

where N is the nucleon, e, e' the incident and scattered electrons respectively, and X is any final particles other than the detected electron. Refer to Figure 2.2.

The important kinematical quantities for the first order reaction shown here in the lab frame are the energy of the incoming (outgoing) electron $E(E')$ electron, the scattering angle with respect to the angle of incidence θ , and the four-momentum transfer squared $q^2 = (k - k')^2$. In calculations and in experiment, however, it is common to use the quantity $Q^2 \equiv -q^2 \geq 0$ where

Figure 2.2: The inclusive reaction of electrons scattered off of nucleons (N) in DIS. The X is all the final state particles that are not to be detected. The four-momenta are given in the laboratory frame.

$$Q^2 \approx 2EE'[1 - \cos\theta] = 4EE' \sin^2 \frac{\theta}{2} \quad (2.5)$$

as well as the “Bjorken x ” variable discussed prior, defined as

$$x = \frac{Q^2}{2M\nu} \quad (2.6)$$

where M is the nucleon mass and $\nu = E - E'$ as previously defined. The variables x and Q^2 are Lorentz-invariant.

While the virtual photon-electron interaction is a simple point-like interaction and is well understood, the photon-nucleon interaction is more complex and is represented by a circle in Figure 2.2. If the photon wavelength is smaller than the nucleon, the photon is able to resolve the individual quarks inside of it. The four-momentum of the nucleon is here represented by p . Then the invariant mass W of the undetected fragments after the scattering, labeled X , is given by

$$W^2 = (p + q)^2 = M^2 - Q^2 + 2M\nu = M^2 + Q^2 \left(\frac{1}{x} - 1 \right). \quad (2.7)$$

In the DIS regime, the invariant mass W must be greater than any resonance in the nucleon. The threshold between the resonance region and the deep-inelastic region is not well defined, but it is usually taken to be at about $W^2 = 4$.

2.3 General Formalism for Lepton-Nucleon Deep Inelastic Scattering

Here the formalism for lepton-nucleon deep inelastic scattering (DIS) will be discussed, closely following the conventions of Anselmino, Efremov, and Leader [39]. The kinematic variables are the same as those described in Section 2.2 but with the addition now of spin four-vectors. The lepton mass will be denoted as m , while $k(k')$ is the initial (final) lepton four-momentum and $s(s')$ its covariant spin four-vector. The nucleon has mass M and four-momentum $P = (M, \mathbf{0})$ as before, and spin four-vector S . For the lepton, $s \cdot k = 0$ ($s' \cdot k' = 0$) by convention, while $s \cdot s = -1$ ($s' \cdot s' = -1$). Also, α is the fine structure constant.

For the single photon exchange process, the differential cross-section for detecting the scattered lepton within the solid angle $d\Omega$ with energy in the range $(E', E' + dE')$ is

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu} \quad (2.8)$$

with the leptonic tensor

$$L_{\mu\nu}(k, s; k', s') = [\bar{u}(k', s') \gamma_\mu u(k, s)]^* [\bar{u}(k', s') \gamma_\nu u(k, s)] \quad (2.9)$$

$$\begin{aligned} &= L_{\mu\nu}^{(S)}(k; k') + iL_{\mu\nu}^{(A)}(k, s; k') \\ &+ L_{\mu\nu}^{'(S)}(k, s; k', s') + iL_{\mu\nu}^{'(A)}(k; k', s') \end{aligned} \quad (2.10)$$

where (S) and (A) stand for symmetric and antisymmetric part respectively under μ and ν interchange. The four terms are defined as

$$L_{\mu\nu}^{(S)}(k; k') = k_\mu k'_\nu - g_{\mu\nu}(k \cdot k' - m^2) \quad (2.11)$$

$$L_{\mu\nu}^{(A)}(k, s; k') = m\epsilon_{\mu\nu\alpha\beta} s^\alpha (k - k')^\beta \quad (2.12)$$

$$L_{\mu\nu}'^{(S)}(k, s; k', s') = (k \cdot s')(k'_\mu s_\nu + s_\mu k'_\nu - g_{\mu\nu} k' \cdot s) \quad (2.13)$$

$$\begin{aligned} & - (k \cdot k' - m^2)(s_\mu s'_\nu + s'_\mu s_\nu - g_{\mu\nu} s \cdot s') \\ & + (k' \cdot s)(s'_\mu k_\nu + k_\mu s'_\nu) - (s \cdot s')(k_\mu k'_\nu + k'_\mu k_\nu) \\ L_{\mu\nu}'^{(A)}(k; k', s') & = m\epsilon_{\mu\nu\alpha\beta} s'^\alpha (k - k')^\beta. \end{aligned} \quad (2.14)$$

Summing Equation 2.10 over s' gives $L_{\mu\nu} = 2L_{\mu\nu}^{(S)} + 2iL_{\mu\nu}^{(A)}$. Averaging over s as well gives the usual *unpolarized* leptonic tensor, $2L_{\mu\nu}^{(S)}$.

In Equation 2.8 the hadronic tensor $W_{\mu\nu}$ can also be split into symmetric and antisymmetric tensors. These can be then expressed in terms of four structure functions as follows:

$$W_{\mu\nu}(q; P, S) = W_{\mu\nu}^{(S)}(q; P) + iW_{\mu\nu}^{(A)}(q; P, S) \quad (2.15)$$

where

$$\frac{1}{2M}W_{\mu\nu}^{(S)}(q; P) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(P \cdot q, q^2) \quad (2.16)$$

$$\begin{aligned} & + \left(P_\mu - \frac{P \cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P \cdot q}{q^2}q_\nu\right)\frac{W_2(P \cdot q, q^2)}{M^2} \\ \frac{1}{2M}W_{\mu\nu}^{(A)}(q; P, S) & = \epsilon_{\mu\nu\alpha\beta}q^\alpha\{MS^\beta G_1(P \cdot q, q^2)\} \\ & + [(P \cdot q)S^\beta - (S \cdot q)P^\beta]\frac{G_2(P \cdot q, q^2)}{M}. \end{aligned} \quad (2.17)$$

Here W_1 and W_2 are the structure functions for unpolarized scattering and G_1 and G_2 are the structure functions for polarized scattering, all of which describe the photon-nucleon interaction, and depend upon the actual structure of the nucleon.

Combining Equations 2.8, 2.9, 2.15, the differential cross-section becomes

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} [L_{\mu\nu}^{(S)} W^{\mu\nu(S)} + L_{\mu\nu}'^{(S)} W^{\mu\nu(S)} - L_{\mu\nu}^{(A)} W^{\mu\nu(A)} - L_{\mu\nu}'^{(A)} W^{\mu\nu(A)}]. \quad (2.18)$$

Averaging over the outgoing electron spin s' as we would in an experiment, we obtain

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} [L_{\mu\nu}^{(S)} W^{\mu\nu(S)} - L_{\mu\nu}^{(A)} W^{\mu\nu(A)}]. \quad (2.19)$$

Each term in the brackets is a measurable quantity that is a function of either the spin-averaged structure functions W_1 and W_2 (the term containing the symmetric component $W_{\mu\nu}^{(S)}$) or the spin-dependent structure functions G_1 and G_2 (the term containing $W_{\mu\nu}^{(A)}$). For instance, the usual unpolarized cross-section is

$$\begin{aligned} \frac{d^2\sigma}{d\Omega dE'}(k, P; k') &= \frac{1}{4} \sum_{s, s', S} \frac{d^2\sigma}{d\Omega dE'}(k, s, P, S; k', s') \\ &= \frac{\alpha^2}{2Mq^4} \frac{E'}{E} 2L_{\mu\nu}^{(S)} W^{\mu\nu(S)} \end{aligned} \quad (2.20)$$

whereas cross-sections differences with opposite target spins are

$$\begin{aligned} \sum_{s'} \left[\frac{d^2\sigma}{d\Omega dE'}(k, s, P, -S; k', s') - \frac{d^2\sigma}{d\Omega dE'}(k, s, P, S; k', s') \right] = \\ \frac{\alpha^2}{2Mq^4} \frac{E'}{E} 4L_{\mu\nu}^{(A)} W^{\mu\nu(A)}. \end{aligned} \quad (2.21)$$

Because we are describing the process $eN \rightarrow eX$ where a single photon is exchanged, the structure functions here describe the *neutral current* reaction. It is possible to exchange a Z^0 boson in the neutral current sector as well but it is highly suppressed and so is neglected here. *Charged-current* processes where a $W^\pm$ is exchanged, such as $e^-N \rightarrow \nu X$, $\nu N \rightarrow e^-X$, and $\bar{\nu}N \rightarrow e^+X$, have different structure functions to describe them.

2.3.1 Unpolarized Scattering

The unpolarized (spin-averaged) cross-section for DIS of leptons on nucleons in the laboratory frame can be written as

$$\frac{d^2\sigma^{unp}}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4} \left[2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right], \quad (2.22)$$

where the lepton mass has been neglected and θ is the scattering angle of the lepton. Measuring the cross-section gives information about the structure functions $W_1(P \cdot q, Q^2)$ and $W_2(P \cdot q, Q^2)$. In the DIS regime (the Bjorken limit) when

$$-q^2 = Q^2 \rightarrow \infty \quad \nu = E - E' \quad x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu} \approx \text{fixed} \quad (2.23)$$

the structure functions approximately scale with fixed x . In addition, Equations 2.1, 2.2 hold, so that the commonly used F_1 and F_2 vary only very slowly with Q^2 at each particular X . Then F_1 and F_2 are related by

$$F_1 \equiv F_2 \frac{1 + \gamma^2}{2x(1 + R)} \quad (2.24)$$

where $R = \sigma_{1/2}^L / \sigma^T$ is the ratio of longitudinal to transverse *virtual scattering* (virtual scattering is described in Section 2.4), and

$$\gamma^2 \equiv \frac{4M^2 x^2}{Q^2} = \frac{Q^2}{\nu^2} = \frac{2Mx}{\nu}. \quad (2.25)$$

In the Bjorken limit ($Q^2, \nu \rightarrow \infty$), R and γ^2 go to zero, and we obtain $F_1 = F_2/2x$, the ‘‘Gross-Callan relation’’.

2.3.2 Polarized Scattering

When the lepton beam and the nucleon target are both polarized, the cross section difference can be expressed in terms of the structure functions G_1 and G_2 . If both lepton and nucleon are polarized longitudinally, then we have

$$\frac{d^2\sigma^{\downarrow\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{Q^2 E} \left[(E + E' \cos \theta) M G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2) \right]. \quad (2.26)$$

Here the first cross-section (σ) superscript indicates the beam helicity and the second indicates the target helicity, so that $\sigma^{\uparrow\uparrow}$ ($\sigma^{\downarrow\uparrow}$) indicates that the beam and target helicities are oriented parallel (antiparallel) to each other.

For the transverse orientation we have [40]

$$\frac{d^2\sigma^{\uparrow\leftarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\leftarrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{Q^2 E} E' \sin \theta \cos \phi \left[M G_1(\nu, Q^2) + 2 E G_2(\nu, Q^2) \right]. \quad (2.27)$$

The azimuthal angle ϕ is defined to be between the plane defined by the incoming and outgoing lepton directions $\hat{k}$ and $\hat{k}'$, and the plane defined by $\hat{k}$ and the target polarization $\hat{S}$. For E143 the experimental setup was such that kinematically the angle ϕ was either 0° or 180° , so that $\cos \phi = \pm 1$ and we could maximize the cross-section difference. For this discussion, I have chosen $\cos \phi = 1$.

Typically in experiments absolute differential cross-sections are difficult to measure because many factors such as detector acceptances and efficiencies are hard to know precisely. To avoid this, it is more common to measure the differences in cross-sections in the form of asymmetries such as

$$A_{\parallel} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}} \quad A_{\perp} = \frac{\sigma^{\downarrow\leftarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\downarrow\leftarrow} + \sigma^{\uparrow\leftarrow}}. \quad (2.28)$$

Here common factors in both the numerator and denominator, such as detector acceptance, cancel. Furthermore, since the cross-sections are proportional to

the rates– or the number of scattered particles detected per number of incoming particle– it is simply a matter of counting. For E143 in particular we were interested in measuring the number of scattered electrons of each helicity, N_L and N_R , where L(left-handed) implies the electrons are anti-aligned with the nucleon target and R(right-handed) implies the electrons are aligned with the target. The asymmetries are then expressed as

$$A_{\parallel}(A_{\perp}) \approx \frac{1}{f P_b P_t} \frac{N_L - N_R}{N_L + N_R}. \quad (2.29)$$

The correction factors in the denominator are the dilution factor f , the beam polarization P_b , and the target polarization P_t , all of which need to be factored in as types of inefficiencies. In addition, nuclear and radiative corrections need to be applied to obtain the final values for $A_{\parallel}$ and $A_{\perp}$. Each of these corrections will be discussed in detail in Chapter 5.

In the Bjorken limit (large Q^2 and ν), the spin structure functions approximately scale as well, depending mainly on x :

$$g_1(x, Q^2) \equiv M^2 \nu G_1(\nu, Q^2) \xrightarrow{\text{lim Bj}} g_1(x), \quad (2.30)$$

$$g_2(x, Q^2) \equiv M^2 \nu G_2(\nu, Q^2) \xrightarrow{\text{lim Bj}} g_2(x). \quad (2.31)$$

These structure functions can thus be obtained by the experimentally measured asymmetries $A_{\parallel}$ and $A_{\perp}$ via the expressions:

$$g_1(x, Q^2) = \frac{F_1(x, Q^2)}{D'} \left[A_{\parallel} + \tan \frac{\theta}{2} A_{\perp} \right] \quad (2.32)$$

$$g_2(x, Q^2) = \frac{F_1(x, Q^2)}{D'} \frac{y}{2 \sin \theta} \left[\frac{E + E' \cos \theta}{E'} A_{\perp} - \sin \theta A_{\parallel} \right] \quad (2.33)$$

where D' is the kinematical expression

$$D' = \frac{(1 - \epsilon)(2 - y)}{y[1 + \epsilon R(x, Q^2)]} \quad (2.34)$$

with y and ϵ defined as

$$y = \frac{E - E'}{E} = \frac{\nu}{E} \qquad \frac{1}{\epsilon} = 1 + 2 \left[1 + \frac{\nu^2}{Q^2} \right] \tan^2 \frac{\theta}{2}. \quad (2.35)$$

Here ϵ is the longitudinal polarization of the virtual photon, so that for $\theta = 0, \epsilon = 1$. For real photons $Q^2 = 0$ always and the polarization is always transverse, so $\epsilon = 0$.

For E143, as for all such experiments done so far, the scattering angle θ is quite small. For small θ , the factor $\tan^2 \frac{\theta}{2}$ in Equation 2.32 is often negligible, so that g_1 mainly depends on $A_{\parallel}$. Likewise, $\sin \theta$ is small so that g_2 mainly depends on $A_{\perp}$. Therefore, g_1 and g_2 are referred to as the **longitudinal and transverse spin-structure functions**, respectively. Typically experiments report results by combining Equation 2.24 and Equation 2.32 to obtain

$$g_1 \approx \frac{A_{\parallel}}{D'} \frac{F_2(x)}{2x[1 + R(x)]} \quad (2.36)$$

where both $\tan \theta/2 \approx 0$, and $\gamma^2 \approx 0$ in the Bjorken limit (see Equation 2.25) have been assumed. For E143, however, we measured $A_{\perp}$ with very good statistical precision, enough to report the g_1 result both neglecting and including the contribution from $A_{\perp}$. Note that knowledge of the unpolarized structure function F_1 , or equivalently F_2 and R , is needed. Fortunately, accurate data have been gathered recently by CERN [41] and SLAC [42] for both F_2 and R .

2.4 Virtual Photon-Nucleon Interaction

The neutral current interaction of the lepton with the nucleon is mediated primarily by a virtual photon. The photon-nucleon interaction depends upon the photon polarization as well as that of the nucleon. The cross-section of this interaction is connected to the imaginary part of the forward virtual

photon-nucleon Compton scattering amplitude $\gamma^* N \rightarrow \gamma^* N$. This gives four *independent* helicity amplitudes of the form $M_{ab;cd}$, with a, b, c and d values for the helicities of the photon and nucleon initial and final states, respectively. The states are [43]

$$M_{1,\frac{1}{2},1,\frac{1}{2}} \quad M_{1,-\frac{1}{2},1,-\frac{1}{2}} \quad M_{0,\frac{1}{2},0,\frac{1}{2}} \quad M_{1,\frac{1}{2},0,-\frac{1}{2}}. \quad (2.37)$$

All other possible combinations of initial and final photon and nucleon helicities are related to the above by time reversal:

$$M_{a,b,c,d} = M_{c,d,a,b} \quad (2.38)$$

and parity transformation:

$$M_{a,b,c,d} = M_{-a,-b,-c,-d}. \quad (2.39)$$

The four independent amplitudes can be written in terms of both spin-dependent structure functions and $\gamma^* N$ cross-sections:

$$\sigma_{\frac{1}{2}}^T = \frac{4\pi^2\alpha}{K} M_{1,\frac{1}{2},1,\frac{1}{2}} = \frac{4\pi^2\alpha}{MK} \left(F_1 + g_1 - \frac{2Mx}{\nu} g_2 \right) \quad (2.40)$$

$$\sigma_{\frac{3}{2}}^T = \frac{4\pi^2\alpha}{K} M_{1,-\frac{1}{2},1,-\frac{1}{2}} = \frac{4\pi^2\alpha}{MK} \left(F_1 - g_1 - \frac{2Mx}{\nu} g_2 \right) \quad (2.41)$$

$$\sigma_{\frac{1}{2}}^L = \frac{4\pi^2\alpha}{K} M_{0,\frac{1}{2},0,\frac{1}{2}} = \frac{4\pi^2\alpha}{MK} \left(\frac{F_2}{\nu} \left(1 + \frac{\nu^2}{Q^2} \right) - \frac{1}{M} F_1 \right) \quad (2.42)$$

$$\sigma_{\frac{1}{2}}^{TL} = \frac{4\pi^2\alpha}{K} M_{1,\frac{1}{2},0,-\frac{1}{2}} = \frac{4\pi^2\alpha}{MK} \frac{Q}{\nu} (g_1 + g_2) \quad (2.43)$$

where $K = \nu - Q^2/2M$ is the virtual photon flux.

The virtual photon-absorption cross-sections above have superscripts referring to the initial and final photon polarization being longitudinal L or transverse T . The superscript TL is for the case when the photon polarization direction changes during the interaction. The subscripts refer to the total spin

of the photon-nucleon system, J_z . The *total* cross-sections for the transversely and longitudinally polarized photons are, respectively,

$$\sigma^T \equiv \frac{1}{2}(\sigma_{1/2}^T + \sigma_{3/2}^T) = \frac{4\pi^2\alpha}{MK}F_1 \quad \text{and} \quad \sigma^L = \sigma_{1/2}^L. \quad (2.44)$$

In addition, the function $R(x, Q^2)$ is the ratio of longitudinal to transverse cross-section:

$$R \equiv \frac{\sigma^L}{\sigma^T} = \frac{F_2}{F_1} \frac{(1 + \gamma^2)}{2x} - 1. \quad (2.45)$$

Recall that $\gamma^2 = 2Mx/\nu$.

One can now use the virtual photon-absorption cross-sections to create the “virtual photon-absorption asymmetries”:

$$A_1 \equiv \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} \quad \text{and} \quad A_2 \equiv \frac{\sigma_{1/2}^{TL}}{\sigma_T}. \quad (2.46)$$

Using the expressions 2.40- 2.43 for the cross-sections, these asymmetries become

$$A_1 = \frac{1}{F_1}(g_1 - \gamma^2 g_2) \quad \text{and} \quad A_2 = \frac{1}{F_1}\gamma(g_1 + g_2). \quad (2.47)$$

In terms of the measured asymmetries $A_{\parallel}$ and $A_{\perp}$, the photon-absorption asymmetries become:

$$\begin{aligned} A_1 &= \frac{1}{D'} \left[A_{\parallel} (1 + \gamma^2 y/2) - A_{\perp} \frac{\gamma^2 y}{2 \tan(\theta/2)} \right], \\ A_2 &= \frac{\gamma(2 - y)}{2D'} \left[A_{\parallel} + A_{\perp} \frac{y(1 + \gamma^2 y/2)}{(1 - y) \sin \theta} \right]. \end{aligned} \quad (2.48)$$

Inverting the above relations gives expressions for the spin structure functions g_1 and g_2 in terms of the virtual photon-absorption asymmetries:

$$g_1 = \frac{F_2}{2x(1 + R)}(A_1 + \gamma A_2), \quad (2.49)$$

$$g_2 = \frac{F_2}{2x(1+R)}(A_2/\gamma - A_1).$$

Many times the relationships between $A_{\parallel}$, $A_{\perp}$, A_1 , and A_2 are expressed in terms of different kinematical variables, defined by:

$$D = \frac{1 - E'\epsilon/E}{1 + \epsilon R}; \quad \eta = \frac{\epsilon\sqrt{Q^2}}{E - E'\epsilon}; \quad d = D\sqrt{\frac{2\epsilon}{1 + \epsilon}}; \quad \zeta = \eta\frac{1 + \epsilon}{2\epsilon}. \quad (2.50)$$

Here the factors D and d are referred to as the *depolarization factors* and correct for the fact that the helicity of the virtual photon can have both longitudinal and transverse components. The various relations then take the following forms:

$$A_1 = \frac{A_{\parallel}}{D(1 + \eta\zeta)} - \frac{\eta A_{\perp}}{d(1 + \eta\zeta)} \quad \text{and} \quad A_2 = \frac{\zeta A_{\parallel}}{D(1 + \eta\zeta)} + \frac{A_{\perp}}{d(1 + \eta\zeta)} \quad (2.51)$$

with the inverse relations of the forms:

$$A_{\parallel} = D(A_1 + \eta A_2) \quad \text{and} \quad A_{\perp} = d(A_2 - \zeta A_1). \quad (2.52)$$

Note that the magnitude of the asymmetry A_1 can, of course, never exceed 1. The magnitude of A_2 is also bound by the “positivity constraint”.

2.4.1 A_2 Positivity Constraint

The virtual photon-absorption cross-section A_2 is related to the cross-sections σ^T and $\sigma_{1/2}^{TL}$ by Equation 2.46. In forward Compton scattering, the $\sigma_{1/2}^{TL}$ cross-section represents the probability of the incoming transversely-polarized photon flipping its spin during the interaction and becoming longitudinally-polarized. The probability for this to occur is expressed via the transverse amplitude A_T and the longitudinal amplitude A_L . Likewise, the probability for the initial and final photons being transversely polarized can be expressed

in terms of A_T only. Since the probability is the square of the amplitudes, the asymmetry A_2 in Equation 2.46 can then be written as

$$A_2 = \frac{|\langle A_L | A_T \rangle|^2}{|\langle A_T | A_T \rangle|^2} e^{i\phi} \quad (2.53)$$

where $e^{i\phi}$ is the phase difference between the cross-sections. The magnitude of A_2 is then

$$|A_2| = \frac{|\langle A_L | A_T \rangle|^2}{|\langle A_T | A_T \rangle|^2}. \quad (2.54)$$

Then, using the Schwartz inequality [44] one can rewrite the magnitude as

$$|A_2| \leq \frac{|\langle A_L | A_L \rangle| \cdot |\langle A_T | A_T \rangle|}{|\langle A_T | A_T \rangle|^2} = \frac{|\langle A_L | A_L \rangle|}{|\langle A_T | A_T \rangle|} \quad (2.55)$$

Referring to Equation 2.45 we obtain the “positivity constraint” for A_2 :

$$|A_2(x, Q^2)| \leq \sqrt{R(x, Q^2)}. \quad (2.56)$$

2.5 Structure Functions and the QPM

In the Quark Parton Model, for which scaling is valid, structure functions can be viewed as distributions of quarks with a given momentum fraction x . $q_i(x)$ is defined to be the probability to find the quark of flavor i having momentum fraction in the range x to $x + dx$. Then the unpolarized structure functions F_1 and F_2 can be interpreted as

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x) \quad \text{and} \quad F_2(x) = x \sum_i e_i^2 q_i(x). \quad (2.57)$$

Recall that $F_2 = 2xF_1$. Experimental measurements of the unpolarized structure function F_2 first showed that in the scaling regime, only 50% of the momentum of the nucleon is actually carried by the quarks, while the rest is

carried by the other “parton” within the nucleon— the gluon.

The spin structure functions g_1 can also be interpreted in terms of quark distributions in the scaling region. Let $q_i^\uparrow$ represent the number of quarks of flavor i with helicity parallel to the nucleon helicity, and let $q_i^\downarrow$ represent quarks with flavor i with helicity antiparallel to the nucleon helicity. Then

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^\uparrow(x) - q_i^\downarrow(x)]. \quad (2.58)$$

When integrated over all x , g_1 is directly related to the net spin carried by each quark flavor.

The parton model interpretation of the transverse case is not as straightforward, however. In the infinite momentum frame, the quarks’ spins will be oriented parallel or antiparallel to the direction of motion, so by helicity conservation a quark can only absorb transversely polarized photons. Thus the cross-section $\sigma_{1/2}^L = 0$. From Equation 2.45 we see that in this case $R(x, Q^2) = 0$, so that from the positivity constraint, $A_2(x, Q^2) = 0$. Similarly, since in the scaling limit there is no transverse momentum, and since the g_2 structure function is directly related to the transverse momentum distribution of the quarks, we expect that

$$g_2(x) = 0. \quad (2.59)$$

However, under experimental conditions we have finite Q^2 , bringing the quark masses slightly off-shell. Hence, a non-zero value for g_2 will be measured, due to Fermi motion and QCD effects sensitive to quark masses. By giving the quarks transverse momentum one can obtain [39]

$$g_2(x) = \frac{1}{2} \sum_i e_i^2 \left(\frac{m_i}{xM} - 1 \right) (q_i^\uparrow(x) - q_i^\downarrow(x)) \quad (2.60)$$

where M is the total mass of the nucleon. In this situation g_2 is extremely

sensitive to the off-shell quark masses m_i , making it difficult to make any parton model prediction. When the quark interactions are neglected, the quarks are back on-shell and have mass $m_i = xM$, (M being the nucleon mass) so that the result in Equation 2.59 is obtained once more.

2.6 The Structure Function g_2

For most purposes the structure function g_2 can be approximated by the model

$$g_2^{WW}(x, Q^2) \equiv -g_1(x, Q^2) + \int_x^1 \frac{g_1(x, Q^2)}{y} dy \quad (2.61)$$

where the WW superscript refers to the authors Wandzura and Wilczek who introduced the model [45]. $g_2^{WW} \approx g_2$ is a twist-2 term (leading twist), implying that it scales at high Q^2 up to QCD correction terms $O(\alpha_s)$. Higher twist terms contribute to g_2 as well, but they are small enough that when the transverse asymmetry $A_\perp$ has not been measured or is not known well, $g_2 = g_2^{WW}$ is often used. For the E143 results, although we measured $A_\perp$, the assumption $A_\perp = 0$ is normally used for the extraction of g_1 .

In the derivation of g_2^{WW} , the kinematic factor $\gamma^2 = 4M^2x^2/Q^2$ was assumed to be zero. For E143 kinematics, however, γ^2 is not so small, so Equation 2.61 was re-derived [46], but the difference in value was found to be negligible.

2.7 Extracting the Neutron

Since the deuteron consists of a proton and a neutron, it is natural to think of the deuteron structure function as essentially a combination of the proton and neutron structure functions. As measured by experiment, the deuteron

magnetic moment $\mu_d = 0.857$ [47] is not very different than the combined magnetic moment of the proton and neutron $\mu_p + \mu_n = 2.793 - 1.913 = 0.880$. This indicates that the proton and neutron within the deuteron are typically in the ground state, and together have orbital angular momentum $\mathbf{L} = 0$ [48]. The small difference between the deuteron magnetic moment and that of the proton and neutron combined arises from the fact that while the deuteron is mainly in the ground state, it is also to some degree in the D -state with $\mathbf{L} = 2$. This fact also gives it a small quadrupole moment.

The deuteron has been measured to have total orbital angular momentum $\mathbf{J} = 1$. In general, the total orbital angular momentum $\mathbf{J}$ is given by the addition of the orbital angular momentum $\mathbf{L}$ and the total spin $\mathbf{S}$: $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The operator $\mathbf{J}^2$ has eigenvalue $J(J + 1)$. Since the deuteron has $\mathbf{J} = 1$, its projection onto the z -axis $\mathbf{J}_z$ has eigenvalues $J_z = 1, 0, -1$. These eigenvalues were used in polarizing the deuteron targets. The deuterons were polarized by aligning J_z along one direction and then measuring the expectation value of $\mathbf{J}_z$, the polarization, using NMR techniques (see Section 3.4.3). However, since the deep-inelastic scattering is dependent upon the spin S_z , it is necessary to determine the spin orientation for a given J_z .

The deuteron spin operator $\mathbf{S}$ consists of the two spin- $\frac{1}{2}$ operators from the proton and neutron, which form the eigenstates $S = 0$ with $S_z = 0$ and $S = 1$ with $S_z = 1, 0, -1$. In the ground state (S -state) we have $L = 0$, $L_z = 0$. Yet for the D -state of the deuteron, we have $L = 2$, so that $L_z = 2, 1, 0, -1, -2$. Then adding any two values of $\mathbf{S}$ and $\mathbf{L}$ gives the selection rules for $\mathbf{J}$:

$$|S - L| \leq J \leq S + L \quad (2.62)$$

$$J_z = S_z + L_z. \quad (2.63)$$

Requiring that $J = 1$ as is measured for the deuteron then indicates that if the proton and neutron are in the S -state, where $L = 0$, the spin $\mathbf{S}$ is necessarily $S = 1$ with $J_z = S_z$. In the D -state, however, where $L = 2$, the spin is still

Table 2.1: For the D -state where $L = 2$, the projection J_z of the eigenvector J can be obtained from various different combinations of L_z and S_z , as shown in this table. The last two columns show the amplitude coefficients and the associated probabilities for each combination.

Clebsch-Gordan				
J_z	L_z	S_z	Coefficients	Probability
-1	0	-1	$\sqrt{1/10}$	0.10
-1	-1	0	$-\sqrt{3/10}$	0.30
-1	-2	+1	$\sqrt{3/5}$	0.60
0	+1	-1	$\sqrt{3/10}$	0.30
0	0	0	$-\sqrt{2/5}$	0.40
0	-1	+1	$\sqrt{3/10}$	0.30
+1	+2	-1	$\sqrt{3/5}$	0.10
+1	+1	0	$-\sqrt{3/10}$	0.30
+1	0	+1	$\sqrt{1/10}$	0.60

$S = 1$ but $J_z = S_z + L_z$, which can be achieved with various combinations of S_z and L_z .

For a state having a particular J_z , the Clebsch-Gordon coefficients indicate how S_z and L_z are combined to obtain that state. Table 2.1 shows the possible combinations of states along with their Clebsch-Gordon coefficients and the associated probabilities. For example, suppose one measures $J_z = +1$. Then the amplitude for finding the states $L_z = 2$ and $S_z = -1$ is

$$\langle L = 2, S = 1; L_z = +2, S_z = -1 | J = 1, J_z = +1 \rangle = \sqrt{3/5} \quad (2.64)$$

In other words, the $(J = 1, J_z = +1)$ state has a 60% probability to be in the state with $(L_z = 0, S_z = -1)$.

2.7.1 Connecting the Deuteron S -State and D -State

Now suppose $A_{L=0}$ is the asymmetry measured with only $L = 0$ states contributing, meaning there is no D -state contribution. If the target polarization is then measured to be +100%, indicating the deuterons are aligned in the state $J_z = +1$, this would mean that all electrons are scattered from deuterons in which $S_z = +1$. However, if only $L = 2$ states contribute to the scattering, the measured asymmetry would be from electrons scattering off deuterons in the $S_z = +1$ state 10% of the time, 60% of the time from deuterons in the $S_z = -1$ state, and 30% from the $S_z = 0$ state.

For the latter case, with deuterons polarized at +100% in the D -state, let N be the number of detected electrons [49]. Then $N = N_{+1} + N_0 + N_{-1}$ where N_{S_z} is the number of electrons coming from deuterons with spin S_z . The subset of electrons N_{+1} will be scattered with the same asymmetry as for the S -state ($L = 0$) because for both sets the spin $S_z = +1$. Hence, the asymmetry for the electrons in the N_{+1} subset is

$$\frac{N_{+1}^+ - N_{+1}^-}{N_{+1}^+ + N_{+1}^-} = A_{(L=0)}. \quad (2.65)$$

Since the deuterons having $S_z = -1$ are polarized in the opposite direction, the subset of electrons N_{-1} will scatter with opposite asymmetry $-A_{(L=0)}$. Furthermore, the subset of electrons N_0 are scattering off of unpolarized deuterons, so there is no asymmetry, and $N_0^+ = N_0^- = N_0/2$.

Then the total asymmetry for the D -state is:

$$\frac{N_{+1}^+ + N_0^+ + N_{-1}^+ - N_{+1}^- - N_0^- - N_{-1}^-}{N_{+1}^+ + N_0^+ + N_{-1}^+ + N_{+1}^- + N_0^- + N_{-1}^-} = A_{(L=2)}, \quad (2.66)$$

which can be reduced to

$$\frac{N_{+1}^+ + N_{-1}^+ - N_{+1}^- - N_{-1}^-}{N} = A_{(L=2)}. \quad (2.67)$$

Noting that $(N_{+1}^+ - N_{+1}^-) = N_{+1}A_{(L=0)}$ and that $(N_{-1}^+ - N_{-1}^-) = -N_{-1}A_{(L=0)}$, we obtain

$$\begin{aligned} A_{(L=2)} &= \frac{N_{+1}A_{(L=0)} - N_{-1}A_{(L=0)}}{N} \\ &= \frac{(0.1N)A_{(L=0)} - (0.6N)A_{(L=0)}}{N} = -0.5A_{(L=0)}. \end{aligned} \quad (2.68)$$

This gives the asymmetry for the D -state ($L = 2$) in terms of the asymmetry for the S -state, which we can use in breaking the deuteron asymmetry down into proton and neutron asymmetries.

2.7.2 The Deuteron in Terms of the Neutron

Given that the asymmetries for the D -state and S -state of the deuteron are related by $A_{(L=2)} = -0.5A_{(L=0)}$ as shown above, we can determine how to relate the deuteron asymmetry and structure function to those of the neutron. Let ω_D be the probability for the deuteron to be in the D -state. For deuterons polarized to +100% ($J_z = +1$), the asymmetry that we measure is then

$$A = (1 - \omega_D)A_{(L=0)} - (0.5\omega_D)A_{(L=0)} = (1 - 1.5\omega_D)A_{(L=0)}. \quad (2.69)$$

For the S -state ($L = 0$) the asymmetry is basically the average of the proton and neutron asymmetries, weighted by the cross-sections, or equivalently, the unpolarized structure functions. The unpolarized deuteron structure function is essentially the average of those of the proton and neutron:

$$F_1^d = \frac{1}{2}(F_1^p + F_1^n). \quad (2.70)$$

Then we can express the deuteron asymmetry as

$$A^d = \frac{1}{2}(1 - 1.5\omega_D) \left[\frac{F_1^p}{F_1^d} A^p + \frac{F_1^n}{F_1^d} A^n \right] \quad (2.71)$$

Table 2.2: Results of calculations of the deuteron D -state probability ω_D using various models.

Model	Year	ω_D
Bonn [50]	1976	0.0440
Bonn [51]	1987	0.0425
Paris [52]	1980	0.0577
Reid [53]	1968	0.0647

where the asymmetry A can be $A_{\parallel}$, $A_{\perp}$, A_1 , or A_2 . Furthermore, recalling the relation between g_1 and A in Equation 2.32, we find that

$$g_1^d = \frac{(1 - 1.5\omega_D)}{2}(g_1^p + g_1^n). \quad (2.72)$$

The probability for the deuteron to be in the D -state, ω_D , cannot be measured directly by experiments. For this reason, it is inferred by theory and phenomenology, using potentials such as the Bonn-Potential [50] [51], the Paris-Potential [52] and the Reid-Potential [53]. For E143 we used the value $\omega_D = 0.05 \pm 0.01$ in extracting the neutron. The values obtained for ω_D using the above phenomenological calculations are shown in Table 2.2.

2.8 Scaling Violations and Q^2 -Dependence

2.8.1 The Origin of Q^2 -Dependence

In Section 2.1.1, the discovery of *scaling* was presented. Consistency with scaling means that at high Q^2 , the structure functions F_1 , F_2 , g_1 , and g_2 not dependent on Q^2 , but rather they “scale” with the variable Bjorken x , where

$x = Q^2/2M\nu$. For this to occur, the quarks in the nucleon must behave essentially like free quarks, which is most likely to happen at high momentum. However, in experimental regimes Q^2 is finite, and scaling is to a small extent violated. As stated earlier, quarks interact via gluons, and gluons can interact with each other. Although an electromagnetic probe such as the photon does not interact with gluons (which interact via the strong force), the quarks are affected by the gluon field and so are not free. The structure functions describing the nucleons must then be modified based on the Q^2 of the photon-nucleon interaction to provide for some amount of Q^2 -dependence.

It turns out that scaling is violated logarithmically with Q^2 in pQCD, so that as Q^2 increases, the structure functions increase at low x and decrease at high x . This can be thought of physically in the following way: Recall first that photons with a particular Bjorken x value will interact only with quarks that have the same x . Now quarks inside the nucleon emit and then immediately absorb gluons. Due to the time-energy uncertainty principle, $\Delta E \Delta t \approx \hbar$, more high energy gluons than low energy ones stay close to the quarks. For low Q^2 processes, the virtual photon cannot resolve the quark and sees the whole quark-gluon systems. At high Q^2 , however, its resolution is increased and it can distinguish the quarks. If the struck quark has just emitted a gluon, then its momentum x will be lower, and the photon will see low-momentum quarks inside high-momentum quark-gluon systems. Hence, at higher Q^2 more low- x quarks can be seen within the quark-gluon system, so the structure function at low x increases with Q^2 — a well-known result for the unpolarized structure functions. At high- x , as Q^2 increases the photons resolve the quarks rather than seeing quark-gluon systems as well. Yet as x gets large, the photon probe is less likely to see quarks carrying that large a fraction of the nucleon momentum, so the structure function decreases with Q^2 .

2.8.2 Importance of Q^2 -dependence

Understanding of the Q^2 -dependence of the structure functions is needed for several purposes. First, experiments such as E143 measure the structure function at low x with low Q^2 , and at high x with high Q^2 . In the case of E143, we used two spectrometers which gave two different values of Q^2 for the same x due to the different scattering angles measured. In order to obtain a better structure function measurement it is necessary to combine both data sets so as to provide better statistics at each value of x . To combine the data, something must be assumed about how the data vary with Q^2 . Secondly, to test the various sum rules using the structure functions, the integral $\Gamma_1 = \int_0^1 dx g_1$ is needed. This integral must be obtained using $g_1(x, Q_0^2)$ at a common $Q^2 = Q_0^2$. Thus, the structure function data must be corrected for being taken at different Q^2 values.

Structure function corrections can be grouped into two types [54] [55]. QCD corrections in various orders of α_s are called “leading twist” or “twist-2” terms. Corrections in orders of $1/\sqrt{Q^2}$ or larger are called “higher twist” corrections. For example, twist-3 $\sim 1/\sqrt{Q^2}$, and twist-4 $\sim 1/Q^2$, and so forth. The higher twist corrections are due to the scattering of the photon off of two quarks at the same time, or due to finite mass effects. These terms also have QCD corrections of order α_s in them. The higher twist terms are calculated non-perturbatively and are not as well understood as the leading twist correction. They become important at low Q^2 , but will not be included in this discussion.

2.9 Evolution Equations

Typically in the experimental papers, the method used to evolve the structure function data to a common Q^2 is to assume that either g_1/F_1 is independent of Q^2 , or to assume that the virtual photon-absorption asymmetries A_1 and

A_2 are independent of Q^2 . Then the results at different kinematical values can be averaged together, and the integral can be taken. For example, for E143 the average Q^2 of the experiment was about $Q_0^2 = 3 \text{ GeV}^2$. To obtain $g_1(x, 3)$ then, we find $g_1(x, Q^2)/F_1(x, Q^2)$ and multiply it by $F_1(x, 3)$, since F_1 is well known as a function of Q^2 .

This is only approximately correct, however, because g_1 and F_1 do not have the exact same Q^2 -dependent behavior. This method has been valid so far since the difference has been negligible compared to the experimental errors. Yet as the experimental data are growing more precise in a larger range of x and Q^2 , it is becoming important to use a more careful method of evolving the data to a common Q^2 .

2.9.1 Unpolarized Quark Distributions

The Q^2 -dependence of the unpolarized quark distributions is well understood and can be found in textbooks, so here the results are presented following the notation of References [56] and [57].

We have established that an incoming virtual photon with a particular value of x can be absorbed when the struck quark carries momentum fraction x . Since scaling violation occurs when partons interact, we will assume that quarks within the nucleon are absorbing and emitting gluons. Suppose that the quark does not have momentum fraction x initially, but that it either obtained it after it emits or absorbs a gluon, or it is part of a quark-antiquark pair created by a higher momentum gluon. This gluon interaction changes the probability of finding a quark with momentum fraction x . The zeroth and first order QCD diagrams for lepton-quark scattering are shown in Figure 2.3.

In the case where a quark has momentum fraction y of the nucleon but ends up with fraction x after emitting a gluon, the probability distribution to

Figure 2.3: The top figure is the zeroth-order QCD diagram for lepton-quark scattering. Two of the first-order diagrams are shown on the bottom. On the left, the quark loses momentum by emitting a gluon and is left with momentum xP , where P is the total nucleon momentum. On the right, a gluon decays to a quark-antiquark pair, one of which has momentum xP and is struck by the virtual photon.

find a quark with fraction $z = x/y$ of the original quark momentum can be expressed as

$$\mathcal{P}(z, Q^2) = \delta(1 - z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\Lambda^2}. \quad (2.73)$$

Here the delta function $\delta(1 - z)$ is for the zeroth term where there are no gluons involved. Λ is sometimes referred to as μ , and is a lower limit on the transverse momentum used as a cutoff to renormalize divergences. $P_{qq}(z)$ is a “splitting function”, which represents the probability of a quark emitting a gluon so as to reduce it’s momentum by a fraction z , and is expressed as

$$P_{qq}(z) = \frac{4}{3} \left(\frac{1 + z^2}{1 - z} \right). \quad (2.74)$$

[Notice that the second term in the probability contains the factor $\log Q^2$ which reflects the fact that the QPM prediction of scaling is violated by a term logarithmic in Q^2 , as mentioned previously. The violation of scaling is a

signature of gluon emission.]

The virtual photon may interact with a quark of flavor up, down, or strange (u, d, s). The up and down quarks are considered the “valence quarks” in the nucleons and interactions would come easily, while interactions with the strange quarks would come from the background “sea” of quark-antiquark pairs. Contributions to the nucleon structure from the other quarks are suppressed due to their large masses, and so are neglected. Let $q_i(x)$ be the distribution of quarks of flavor i with momentum fraction x . Since this momentum fraction may come from the zeroth-order term or from the first order terms with $x = zy$, the quark distribution can be expressed as:

$$q(x, Q^2) + dq(x, Q^2) = \int_0^1 dy \int_0^1 dz q(y, Q^2) \mathcal{P}(z, Q^2) \delta(x - zy), \quad (2.75)$$

indicating that the quark distribution is modified by the term $dq(x, Q^2)$. This expression can be rewritten following the form of the well-known (in some circles, at least) DGLAP equations [58], or the “Dokshitzer-Gribov-Lipatov-Altarelli-Parisi” equations, although many know them simply as the “Altarelli-Parisi” equations:

$$\frac{d}{d \log Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}\left(\frac{x}{y}\right). \quad (2.76)$$

Typically in the references convolutions are used, but the above has been converted to a convolution integral via:

$$(f \otimes g)(z) \equiv \int_x^1 \frac{dz}{z} f(z) g(x/z). \quad (2.77)$$

Since a quark with momentum fraction x can come from any quark with higher momentum y , the integral in Equation 2.76 over dy ranges from x to 1. The initial shape of the quark distribution has to be input to these equations from either experiment or lattice QCD calculations.

Figure 2.4: Two more of the first-order diagrams are shown here, this time affecting the gluon distribution. On the right, a gluon with momentum xP originates from a quark. On the left, a gluon with momentum xP originates from another gluon.

Another type of first-order modification to the structure functions comes when we allow photon interactions with quarks coming from quark-antiquark pairs created by the splitting of a gluon. The gluon distribution $g(x, Q^2)$ is analogous to that of the quark and gives the probability to find a gluon with momentum fraction x . The term added to the quark distribution equation is

$$\frac{d}{\log Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right]. \quad (2.78)$$

The evolution equation for the gluon distribution is given by

$$\frac{d}{\log Q^2} g(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[\sum_i q(y, Q^2) P_{gq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{gg}\left(\frac{x}{y}\right) \right]. \quad (2.79)$$

The index i represents the three quark flavors and their antiquarks. The quark-gluon splitting functions here are:

$$P_{qg}(z) = \frac{1}{2}(z^2 + (1-z)^2), \quad (2.80)$$

$$P_{gq}(z) = \frac{4}{3} \frac{1 + (1-z)^2}{z}, \quad (2.81)$$

$$P_{gg}(z) = 6 \left(\frac{(1-z)}{z} + \frac{z}{1-z} + z(1-z) \right). \quad (2.82)$$

- $P_{qg}(z)$ corresponds to the probability of gluons breaking into quark-antiquark pairs leaving the quark with a momentum z times the gluon momentum. (Bottom right of Figure 2.3).
- $P_{gq}(z)$ is the probability for a quark to radiate a gluon with z times the quark's original momentum. (Right side of Figure 2.4.)
- $P_{gg}(z)$ is the probability for a gluon to lose a fraction z of its momentum by emitting a gluon. (Left side of Figure 2.4.)

With these evolution equations, and perhaps adding terms of higher order, global fits to the data can be performed.

2.9.2 Polarized Quark Distributions

In this section the procedure and widely-used notation is mainly from Reference [59]. The expressions for the quark distribution $q_i(x, Q^2)$ and the gluon distribution $g(x, Q^2)$ from the last section become the *polarized* quark (for uds quarks and antiquarks) and gluon distributions:

$$\Delta q(x, Q^2) = q^\uparrow(x, Q^2) - q^\downarrow(x, Q^2) \quad (2.83)$$

$$\Delta g(x, Q^2) = g^\uparrow(x, Q^2) - g^\downarrow(x, Q^2) \quad (2.84)$$

The splitting functions for polarized interactions will include the Δ symbol as well. The evolution equation for polarized quark distributions becomes

$$Q^2 \frac{d}{dQ^2} \Delta q(x, Q^2) = \frac{\alpha_s}{2\pi} \left[\int_x^1 \frac{dy}{y} \Delta q(y, Q^2) \Delta P_{qq} \left(\frac{x}{y} \right) + 2 \int_x^1 \frac{dy}{y} \Delta g(y, Q^2) \Delta P_{qg} \left(\frac{x}{y} \right) \right]. \quad (2.85)$$

Here the splitting function $\Delta P_{qq}(z)$ is the same as that for the unpolarized case shown in Equation 2.74. However, in the polarized case for $\Delta P_{qg}(z)$, the probability for a quark to emit a gluon which carries a fraction z of the original

momentum is now:

$$\Delta P_{qg}(z) = \frac{1}{2}[z^2 - (1-z)^2]. \quad (2.86)$$

As mentioned in Equation 2.58, g_1 can be expressed in terms of the polarized quark distribution:

$$g_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x, Q^2). \quad (2.87)$$

Then Equation 2.85 becomes

$$Q^2 \frac{d}{dQ^2} g_1^{p,n}(x, Q^2) = \frac{\alpha_s}{2\pi} \left[\int_x^1 \frac{dy}{y} g_1^{p,n}(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + \frac{2}{3} \int_x^1 \frac{dy}{y} \Delta g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right], \quad (2.88)$$

or equivalently,

$$d g_1^{p,n}(x, Q^2) = \frac{\alpha_s}{2\pi} d \log \frac{Q^2}{\Lambda^2} \left[\int_x^1 \frac{dy}{y} g_1^{p,n}(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + \frac{2}{3} \int_x^1 \frac{dy}{y} \Delta g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right]. \quad (2.89)$$

The α_s term can be approximated [59] by observing that

$$\frac{\alpha_s(Q^2)}{2\pi} d \log \frac{Q^2}{\Lambda^2} \approx -\frac{2}{9} d \log \alpha_s(Q^2) + \mathcal{O}(\alpha_s^2) \quad (2.90)$$

for 3 quark flavors. Also, for a small displacement in Q^2 , the Q^2 -dependence of $g_1^{p,n}$ and Δg on the right hand side of Equation 2.89 can be neglected. Then discarding terms of $\mathcal{O}(\alpha_s^2)$ we now have

$$g_1^{p,n}(x, Q^2) - g_1^{p,n}(x, Q_0^2) = \frac{2}{9} \log \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \left[\int_x^1 \frac{dy}{y} g_1^{p,n}(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + \frac{2}{3} \int_x^1 \frac{dy}{y} \Delta g(y, Q^2) \Delta P_{qg}\left(\frac{x}{y}\right) \right]. \quad (2.91)$$

Substituting in the appropriate splitting functions P_{qq} and ΔP_{qg} , and using the renormalization procedure prescribed in Reference [57] we have for the first convolution integral, sans singularities,

$$\begin{aligned} \int_x^1 \frac{dy}{y} g_1^{p,n}(y, Q^2) P_{qq}\left(\frac{x}{y}\right) &= \frac{4}{3} \int_x^1 \frac{dy}{y} \frac{(1+y^2)}{(1-y)} \left[\frac{1}{y} g_1^{p,n}\left(\frac{x}{y}\right) - g_1^{p,n}(x) \right] \\ &+ \frac{4}{3} \left[x + \frac{1}{2}x^2 + 2\log(1-x) \right]. \end{aligned} \quad (2.92)$$

The second convolution integral containing the term ΔP_{qg} may similarly be evolved. However, since very little is presently known about $\Delta g(x)$, the distribution magnitude and shape must be assumed using theoretical models. Estimates for the contribution $\Delta g = \int_0^1 dx \Delta g(x, Q^2)$ range from the extreme to much lower values. For instance, in References [59] [8] the extreme parametrization:

$$\Delta g(x) = Cx^{-0.3}(1-x)^7 \quad (2.93)$$

was used, normalized such that $\Delta g = 5$ at $Q_{EMC}^2 = 10.7 \text{ GeV}^2$. Newer estimates have given $\Delta g = 1.5 \pm 0.8$ at $Q^2 = 10 \text{ GeV}^2$, $\Delta g \approx 3$ at $Q^2 = 1 \text{ GeV}^2$ [60], and also $\Delta g \approx 1.7$ at $Q^2 = 10 \text{ GeV}^2$ [61].

2.10 Corrections for Nucleon Compositeness

In studying the structure of the nucleon, it would be ideal to scatter off of single nucleons which behave like free particles. However, since the nucleon is bound inside a nucleus within an atom, modifications to the cross-section have to be made to account for the compositeness of the nucleus. In a naive model, the nucleus is a collection of Z protons and $A-Z$ neutrons. Then the DIS scattering would allow the electron (via the virtual photon) to interact with the nucleons as if the other nucleons were non-existent. Yet if we examine

the ratio

$$\rho = \frac{F_2^A}{F_2^N} \quad (2.94)$$

where F_2^A is the structure function per nucleon of the nucleus, and F_2^N is the structure function of a free nucleon, we see that the ratio is typically not equal to 1 [31] [62]. This is due to three different correction factors that need to be taken into account. Although the correction factors have been categorized, the exact boundary where each effect takes over is unclear. For $x \lesssim 0.05$, $\rho < 1$ due to **nuclear shadowing** effects. The ratio ρ is typically lower than one also between $x \approx 0.3$ and $x \approx 0.8$ due to the **EMC effect**. Between $x \approx 0.05$ and $x = 0.3$ is a transition region with $\rho > 1$. Finally, **Fermi motion** takes over at right around $x \lesssim 1$ so that ρ increases to $\rho > 1$. Figure 2.5 shows the fit for three different nuclei: ${}^4\text{He}$, ${}^{15}\text{N}$ and Cu . The regions in x where the various effects are prominent are indicated in the figure. E143 DIS data ranged from $x = 0.02$ (E=16 GeV data) to $x = 0.8$ (E=29 GeV data).

2.10.1 Nuclear Shadowing

In a composite nucleus, since the cross-section for the virtual photon scattering off of a quark is fairly large, the photon is more likely to interact with the quarks in the front of the nucleus than to penetrate the nuclear matter. This means that the quarks in the back don't contribute as much to the scattering process, so that the total cross-section per nucleon is proportional to R^2 , the square of the nuclear radius, rather than proportional to the number of nucleons A ($\sim R^3$). In other words, the photon in general sees a two-dimensional surface area rather than a volume of free nucleons each of which is equally likely to see the photon.

Clearly this effect depends on two things: the total number of nucleons A inside the nucleus and the region in x of the photon probe. For a low A material such as the deuteron, the effect is smaller than for a large A material

Figure 2.5: Fits to the data on σ_A/σ_{deut} versus x for 3 types of nuclei. Shown are the regions where nuclear shadowing, Fermi motion, and EMC effects are present.

such as Iron. In addition, the shadowing effect is stronger at low x where more sea quarks (virtual quark-antiquark pairs) are visible to the photon [31], increasing the probability of interaction at the front of nucleus.

2.10.2 EMC Effect

The “EMC effect”, so dubbed because it was discovered [63] by the European Muon Collaboration (EMC), refers to the fact that the per-nucleon cross-section between $x = 0.3$ and $x = 0.8$ is smaller for nuclei having $A > 2$ than for that of the deuteron. The effect has been confirmed by many other experiments. Indeed, it has been shown at SLAC [64] that the ratio σ_A/σ_d of the cross sections of various nuclei to the deuterium cross-section decrease logarithmically with atomic weight A or linearly with nuclear density. No

commonly accepted explanation for the effect exists yet [65].

2.10.3 Fermi Motion

Within the nucleus the nucleons have a certain momentum distribution relative to the nucleus center of mass called the Fermi momentum. The effect is on the order of a few hundred MeV, and it smears the structure data at all kinematical ranges. The effect only becomes visible, however, when the cross-section varies greatly within the smearing energy range. For our purposes Fermi motion is most important in the region of x near 1, where the cross-section varies rapidly due to resonances.

For this E143 data analysis, the corrections that we used for the compositeness effects, collectively referred to as the “EMC effect” since the different effects are not clearly distinguishable, came from fits to experimental data of σ_A/σ_d for the range $x \geq 0.0085$ [64] [66]. These fits were used to scale the proton and deuteron structure functions [67] [68], and these functions were then used as the structure functions of the nuclei in our experiment.

2.11 The First Moment of g_1

2.11.1 The Bjorken Sum Rule

In 1967, James Bjorken derived a sum rule [4] for the spin structure functions g_1^p and g_1^n using current algebra. In the $Q^2 \rightarrow \infty$ limit he related it to weak vector and axial-vector coupling constants g_A and g_V in the following manner:

$$\int_0^1 (g_1^p - g_1^n) dx = \frac{1}{6} \frac{g_A}{g_V} \quad (2.95)$$

with the summation is done over the variable Bjorken x . The coupling constants were measured from neutron β -decay. With $g_V = 1$, the value for g_A (defined for $Q^2 = 0$) is currently 1.2670 ± 0.0035 [69].

Since then, QCD corrections have been added so that the sum rule now reads [70]

$$\int_0^1 (g_1^p - g_1^n) dx = \frac{1}{6} \frac{g_A}{g_V} \left[1 - \left(\frac{\alpha_s}{\pi} \right) - 3.5833 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.2153 \left(\frac{\alpha_s}{\pi} \right)^3 \right]. \quad (2.96)$$

The Bjorken sum rule is regarded as an important test of the quark model and QCD theory. According to Feynman [71], its “verification or failure would have a most decisive effect on the direction of future high-energy theoretical physics”. More recently, according to Bjorken [72], ‘if the sum rule is violated then there’s something wrong with perturbative QCD’.

2.11.2 The Ellis-Jaffe Sum Rule

While not considered to be quite as fundamental as the Bjorken sum rule, the sum rules derived by John Ellis and Robert Jaffe [5] in 1974 have been interesting to examine so as to gain insight into the QPM and the assumptions that have been made about it in the past. These sum rules were derived for the first moments of g_1^p and g_1^n separately.

Interpreting the spin-structure function $g_1^{p,n}$ within the framework of the QPM as in Equation 2.58, the quark contributions to the total helicity of the nucleon can be expressed as

$$\Delta u = \int_0^1 [u^\uparrow(x) - u^\downarrow(x)] dx \quad (2.97)$$

$$\Delta d = \int_0^1 [d^\uparrow(x) - d^\downarrow(x)] dx \quad (2.98)$$

$$\Delta s = \int_0^1 [s^\uparrow(x) - s^\downarrow(x)] dx \quad (2.99)$$

As mentioned previously, contributions from heavier quarks are neglected. Because isospin symmetry is assumed, the distribution of up-quarks in the proton is the same as that of the down-quarks in the neutron, so that

$$\Delta u^p(x) = \Delta d^n(x) \quad \Delta d^p(x) = \Delta u^n(x) \quad (2.100)$$

Although the masses of these two lightest quarks are slightly different, for this purpose the difference is negligible, so for this purpose isospin symmetry is conserved.

The first moments can then be rewritten as [73]

$$\begin{aligned} \Gamma_1^p &\equiv \int_0^1 g_1^p dx = \frac{1}{2} \left[\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right], \\ \Gamma_1^n &\equiv \int_0^1 g_1^n dx = \frac{1}{2} \left[\frac{1}{9} \Delta u + \frac{4}{9} \Delta d + \frac{1}{9} \Delta s \right], \end{aligned} \quad (2.101)$$

Inserting these expressions into the Bjorken sum rule gives

$$g_A/g_V \equiv (g_A/g_V)_{n \rightarrow p} = \Delta u - \Delta d. \quad (2.102)$$

Here the value for g_A/g_V is again gathered from measurements of neutron β -decay.

In assuming $SU(3)$ flavor symmetry, the masses of the up-, down- and strange-quarks are set equal. Then the quark distributions of protons are related to those of hyperons, whose axial-vector coupling constants g_A/g_V are determined experimentally via semi-leptonic hyperon decay. The couplings can be expressed also in terms of Δu , Δd , and Δs as well as two constants F and D :

$$(g_A)_{n \rightarrow p} = F + D = \Delta u - \Delta d \quad (2.103)$$

$$(g_A)_{\Sigma \rightarrow n} = F - D = \Delta d - \Delta s \quad (2.104)$$

$$(g_A)_{\Lambda \rightarrow p} = F + \frac{1}{3}D = \frac{1}{3}[2\Delta u - \Delta d - \Delta s] \quad (2.105)$$

$$(g_A)_{\Xi \rightarrow \Lambda} = F - \frac{1}{3}D = \frac{1}{3}[\Delta u + \Delta d - 2\Delta s] \quad (2.106)$$

Using the current values [69] for g_A/g_V for the four decays above, one can calculate [74] that $F = 0.4627 \pm 0.0079$ and $D = 0.8039 \pm 0.0080$. Solving for $\Delta u = 2F + \Delta s$ and $\Delta d = F - D + \Delta s$, we can replace them in Equations 2.101 to get

$$\Gamma_1^p = \frac{1}{18}[9F - D + 6\Delta s] \quad (2.107)$$

$$\Gamma_1^n = \frac{1}{18}[6F - 4D + 6\Delta s] \quad (2.108)$$

Adding in leading twist QCD calculations to third order [75] in α_s gives

$$\begin{aligned} \Gamma_1^{p,n} = & \left[1 - \left(\frac{\alpha_s}{\pi}\right) - 3.5833\left(\frac{\alpha_s}{\pi}\right)^2 - 20.2153\left(\frac{\alpha_s}{\pi}\right)^3 \right] \\ & \times \left(\pm \frac{1}{12}(F + D) + \frac{1}{36}(3F - D) \right) \\ & + \left[1 - 0.3333\left(\frac{\alpha_s}{\pi}\right) - 0.5495\left(\frac{\alpha_s}{\pi}\right)^2 \right] \frac{1}{9}(3F - D + 3\Delta s). \end{aligned} \quad (2.109)$$

Equation 2.109 is referred to as the Ellis-Jaffe sum rule, except that when they derived it they assumed that the strange sea quarks do not contribute to the spin of the nucleon ($\Delta s = 0$). The corrections have a large influence on the results [76]. The values of the corrections given are for the $\overline{MS}$ -scheme. Leading twist corrections can allow the comparison of experimental data collected at different Q^2 values, provided that higher twist corrections are negligible. Estimates of the next-to-next-to-leading-order [$\mathcal{O}(\alpha_s)^4$] corrections have been made [77], which could then add a term $-130(\frac{\alpha_s}{\pi})^4$ onto the first correction factor in Equation 2.109 and a term $-2(\frac{\alpha_s}{\pi})^3$ onto the second correction factor.

2.11.3 The Contributions to the Helicity of the Nucleon

The helicity of an energetic nucleon has three sources: the angular momentum (intrinsic spin) of the quarks, the gluon angular momentum (or gluon polarization), and the orbital angular momentum of any of these nucleon constituents. Conservation of angular momentum J_z then implies [78] the sum rule for nucleon spin:

$$\frac{1}{2}\Delta\Sigma + \Delta g + \langle L_z \rangle = \frac{1}{2}. \quad (2.110)$$

Here $\Delta\Sigma = \Delta u + \Delta d + \Delta s$, also referred to as Δq in this thesis, is the helicity contribution of the quarks to the nucleon spin. By definition, the Δq 's contain the anti-quark contributions as well. As mentioned previously in Equation 2.84, $\Delta g \equiv \int_0^1 dx \Delta g(x)$ is the helicity contribution of the gluons, and $\langle L_z \rangle$ is the angular momentum contribution. Although there is relatively little known about Δg at present, it is expected to depend on Q^2 and possibly get fairly large. When this happens, L_z is expected to also grow, but in the opposite direction, so that Equation 2.110 continues to hold.

If in Equation 2.109 the Δs contribution is not set equal to zero, then the measurement of the first moment Γ_1 as well as the experimentally-determined values for F and D can be used to determine Δs . In addition, the values for Δu and Δd can be found using Equations 2.103 to 2.106. Then we can find the total quark contribution to the spin of the nucleon

$$\Delta\Sigma \equiv \Delta u + \Delta d + \Delta s \equiv \Delta q \quad (2.111)$$

Writing the total quark helicity in terms of F and D gives:

$$\Delta\Sigma = 3F - D + 3\Delta s \quad (2.112)$$

In some of the literature [79] the quark contributions are written in terms

of the $SU(3)$ flavor combinations:

$$\begin{aligned} a_3 &\equiv F + D = \Delta u - \Delta d = (g_A/g_V)_{n \rightarrow p}, \\ a_8 &\equiv 3F - D = \Delta u + \Delta d - 2\Delta s = 3(g_A/g_V)_{\Xi \rightarrow \Lambda}, \\ a_0 &\equiv \Delta\Sigma = \Delta u + \Delta d + \Delta s. \end{aligned} \quad (2.113)$$

Here a_0 is the singlet distribution— the spin fraction carried by the quarks— while a_3 and a_8 are non-singlet quantities. With these we can define the parameters

$$\begin{aligned} I_3 &= \frac{1}{12} a_3 \cdot (QCD \text{ corrections}) \\ I_8 &= \frac{1}{36} a_8 \cdot (QCD \text{ corrections}) \\ I_0 &= \frac{1}{9} a_0 \cdot (QCD \text{ corrections}) \end{aligned} \quad (2.114)$$

so that the first moments of the proton, neutron, and deuteron can be expressed as

$$\Gamma_1^{p,n} = \pm I_3 + I_8 + I_0 \quad (2.115)$$

$$\Gamma_1^d = I_8 + I_0 \approx \frac{1}{2}(\Gamma_1^p + \Gamma_1^n). \quad (2.116)$$

Notice that for Γ_1^d the component $I_3 = 0$ so that the term $I_0 \equiv \Delta\Sigma$ dominates. Therefore, a measurement of the deuteron first moment can determine the quark contribution more precisely than Γ_1^p or Γ_1^n .

Here we should note that in deriving the Ellis-Jaffe sum rule, and in extracting the values of F and D from the various baryon semi-leptonic decay parameters g_A/g_V , $SU(3)$ symmetry is assumed. However, in certain instances, $SU(3)$ symmetry is known to be broken. This has led to discussions on its influence on both the sum rules and the measured value of the helicity content of the nucleon [80].

Furthermore, there are many theoretical predictions for quark and gluon helicity contributions to the nucleon, some of which are listed by S. Brodsky in Reference [54]. The naive non-relativistic three-quark proton model is the simplest and predicts $\Delta\Sigma = 1$, which we know to be wrong based on experimental measurements. More intricate theories include the relativistic version of the three-quark model, and the gluon-enhanced three-quark model which includes the axial anomaly term from polarized gluons. In the latter, predictions are that $\Delta g \approx 3.5$ and could potentially reduce the apparent measurement of Δq from 75% to 15%. More recently, many models predict various sizes of the contributions of gluons to nucleon spin [81]. Given a large enough gluon contribution, the measured value of a small $\Delta\Sigma$ could still be consistent with a larger actual $\Delta\Sigma$ if the gluon contribution hasn't been taken into account. However, since the magnitude of the gluon contribution is *not* known, these experiments denote $\Delta\sigma$ (Δq) as being simply the helicity contribution from all quarks, which has typically been measured to be about 0.3 in magnitude.

2.11.4 Other Sum Rules

Various other sum rules exist that are related to the spin structure of the nucleon. Although they will not be a subject of this thesis, they are interesting in the context of the Quark Parton model and QCD. The Gerasimov-Drell-Hearn (GDH) sum rule [82], although not within the realm of deep inelastic scattering (DIS), is still an inquiry into the structure of the nucleon. Whereas the Bjorken sum rule is taken to be at the limit of $Q^2 \rightarrow \infty$ so that asymptotic freedom exists, the GDH sum rule is in the realm of $Q^2 = 0$, so that real photons are the probe rather than virtual photons. The sum rule involves the anomalous magnetic moment κ of the nucleon, and uses cross-sections obtained from DIS data:

$$-\frac{2\pi^2\alpha}{M^2}\kappa_{p,n}^2 = \int_{\nu_{thr}}^{\infty} [\sigma_{3/2}^T(\nu) - \sigma_{1/2}^T(\nu)]_{p,n} \frac{d\nu}{\nu} \quad (2.117)$$

where $\nu_{thr} = Q^2/2M$. The DIS cross-sections are defined in Section 2.4. Experiments at Brookhaven National Laboratory (BNL) and at Thomas Jefferson National Accelerator Facility (TJNAF or JLAB) are planned to study the validity of the GDH sum rule [83] as well as at ELSA in Bonn, Germany.

One can obtain *some* results that are pertinent to DIS from the GDH sum rule. The first moment of the nucleons can be written as

$$\begin{aligned}\Gamma_1 &= \int_0^1 g_1(x, Q^2) dx = \int_0^1 A(x, Q^2) F_1(x, Q^2) dx \\ &= \int_0^1 \frac{\nu - \frac{Q^2}{2M}}{8\pi^2\alpha} (\sigma_{1/2}^T - \sigma_{3/2}^T) dx \\ &= \frac{Q^2}{16\pi^2\alpha} \int_{\nu_{thr}}^{\infty} (1-x)(\sigma_{1/2}^T - \sigma_{3/2}^T) \frac{d\nu}{\nu}\end{aligned}\tag{2.118}$$

after a change of variables from x to ν . Holding ν constant, as $Q^2 \rightarrow 0$, $x \rightarrow 0$, so that with the GDH sum rule we obtain

$$\lim_{Q^2 \rightarrow 0} \frac{\Gamma_1}{Q^2} = \frac{1}{16\pi^2\alpha} \int_{\nu_{thr}}^{\infty} (\sigma_{1/2}^T - \sigma_{3/2}^T) \frac{d\nu}{\nu} = -\frac{\kappa}{8M^2}.\tag{2.119}$$

We have shown the the slope of Γ_1 with Q^2 is proportional to the anomalous magnetic moment. This can provide us with a prediction for the behavior of the first moment as Q^2 approaches zero. Inserting the values [84] $\kappa_p = 1.79$, $\kappa_n = -1.91$, and $M = 0.94$ GeV, we obtain:

$$\begin{aligned}\lim(Q^2 \rightarrow 0) \quad \frac{\Gamma_1^p}{Q^2} &= -0.45 \text{ GeV}^{-2} \\ \lim(Q^2 \rightarrow 0) \quad \frac{\Gamma_1^n}{Q^2} &= -0.52 \text{ GeV}^{-2}\end{aligned}\tag{2.120}$$

There are also predictions for the first moment of the *transverse* structure function, g_2 . The Burkhardt-Cottingham sum rule [85] suggests that as $Q^2 \rightarrow$

∞ ,

$$\Gamma_2^{p,n} = \int_0^1 g_2^{p,n} dx = 0. \quad (2.121)$$

This experiment, E143, was able to make a measurement of the integral of g_2 [19], as well as a prediction for higher moments ($\int_0^1 x^n g_2 dx$ with $n = 2, 4, 6, \dots$) which can be derived from the Operator Product Expansion (OPE) [86]. Although the current data are such that the measurement of g_2 is basically consistent with zero, results will be published soon on more transverse data from SLAC E155. Even more importantly, a dedicated experiment to measure g_2 , E155X, has been approved at SLAC and is expected to run in early 1999. There is quite a bit of theoretical interest [87] in the structure of g_2 and the continuation of g_2 to $x = 0$, but no definite model exists as of now.

Chapter 3

Experiment Apparatus and Performance

3.1 History of SLAC and ESA

The Stanford Linear Accelerator Center, SLAC, was commissioned in 1962 by the U.S. Government's Department of Energy, and was completed in 1966. It covers 426 acres of Stanford property, and is situated west of the Stanford University campus. Research began at SLAC in 1966, when the *linac* was completed, having the capability of producing electron beams up to 20 GeV. The first experiments at SLAC directed the electron beam onto stationary targets (fixed target experiments) to study the structure of matter. The first evidence for quarks, substructure of the proton and neutron, was seen in a fixed target experiment in Endstation A (ESA) in 1968. In 1990, this work was recognized with a Nobel prize shared amongst the leaders of this SLAC experiment: Prof. Richard Taylor of SLAC, and Profs. Henry Kendall and Jerome Friedmann of M.I.T. Since these early experiments began, three other research facilities have been built based on collisions between beams of positrons and electrons,

Figure 3.1: Aerial view of the SLAC research yard. Endstation A is the large rectangular building in the center, and has been the site of many DIS experiments including E143.

rather than fixed target collisions. These are the 8 GeV SPEAR storage ring build in 1972, the 30 GeV PEP storage ring in 1980, and the 100 GeV Stanford Linear Collider (SLC) in 1989. However, throughout the programs which performed physics experiments using colliding beams, SLAC's Endstation A has continued to contribute excellent experimental results to the fields of particle and nucleon physics.

From the commissioning of SLAC in 1966 to 1972, SLAC's research program was centered solely around fixed-target experiments using the two-mile linac. During the 1970s, sources of *polarized* beams, in which the electron's magnetic fields are aligned along the beam axis, were added to the SLAC linac. In 1978, under the leadership of my advisor, Prof. Charles Prescott, the polarized electron beam was used to perform experiments measuring the interference between the electromagnetic and weak forces by looking at Z boson and photon exchange [88]. These were done in the ESA, and had such precision that they could established a clear relationship between the electromagnetic force and the weak force, which is responsible for nuclear radioactive

decay.

Currently, work has continued on a series of experiments in ESA, starting with E142 in 1992 and ending with E155X in early 1999, using the polarized beam to further understand the structure of the proton and neutron, and the nature and dynamics of their quark constituents. The experiment I am reporting on, E143, was the second in this group of five experiments (E142, E143, E154, E155, E155X) aimed at determining the spin structure of the proton, deuteron, and the neutron. E143 was the first in which a new, highly polarized ($\sim 85\%$) beam was used, the description of which follows. Now the A-line that leads to Endstation A has been upgraded to handle 50 GeV beam, the energy used for the final three experiments in the series. The experimental details of the E143 experiment, in which proton and deuteron targets are used, are presented in this chapter.

3.2 Polarized Electron Beam

From early 1992 until mid-1998, SLAC exclusively used a polarized electron beam for its high energy physics experimental programs. Linear accelerators are prime machines for polarized electron beams since during the acceleration there is no depolarization, as occurs in circular machines. However, to incorporate the strained Gallium-Arsenide (GaAs) source predominantly used during this period, several years of work were required to create the necessary stable and reliable accelerator, including changes in electron gun design, transport optics, the laser beam design, and improvements in the high voltage and high vacuum environment [89]. Once this was achieved, however, the polarized electron beam became a very reliable instrument for polarized structure function studies.

3.2.1 Polarized Source

During E142 [15] [90], the first in the series of structure function experiments performed in ESA during the 1990s, an AlGaAs source with a flashlamp-pumped dye laser was used. This combination produced beam polarizations of $\sim 40\%$ at a current of up to $2 \times 10^{11} e^-/\text{pulse}$. Theoretically, this cathode could reach a polarization of 50%, but the dye laser required running to optimize dye lifetime rather than photoelectron polarization.

Beginning with this experiment (E143 [21]) in 1993, a strained GaAs cathode and a unique YAG-pumped titanium-sapphire (Ti:sapphire) laser designed specifically for this purpose [89] [91] [92] has been utilized for the electron source. Polarizations with this configuration have been above 80%, with the average beam polarization for E143 at about 85% at a current of $(1-4) \times 10^9 e^-/\text{pulse}$.

Electron Excitation

The polarized electrons are obtained from the cathode by the process of photoemission from the GaAs, a IV-V type semiconductor [89]. For *normal* (unstrained) GaAs, the ground state of the conduction band, the $S_{1/2}$ state, is approximately 1.5 eV above the filled valence band, and has the quantum numbers $m_j = \pm \frac{1}{2}$. However, the valence band is perturbed by spin-orbit coupling, and so has split energy levels, $P_{1/2}$ and $P_{3/2}$ at a slightly higher energy. The quantum numbers for these valence states are $m_j = \pm \frac{1}{2}$ and $m_j = \pm \frac{1}{2}, \pm \frac{3}{2}$ respectively, as in the left side of Figure 3.2. Labelling the energy gap between the $P_{3/2}$ and the $S_{1/2}$ state as E_g , and the energy difference between the $P_{3/2}$ and the $P_{1/2}$ state as Δ , we have that $E_g = 1.424$ eV and that $\Delta = 0.340$ eV at 300 K [93].

When the cathode is hit by the laser, the circularly polarized light (having

Figure 3.2: Diagram of the energy levels in both normal and strained GaAs lattices. Given an excitation photon incident with energy above E_g and below $E_g + \Delta$ (or below $E_g + \delta$ in the strained case), the solid lines represent allowed transitions and the dashed, forbidden transitions.

angular momentum 1) excites electrons from the valence band into the conduction band. If the incident photon has energy above E_g and below $E_g + \Delta$, then transitions are allowed between the $P_{3/2}(m_j = +\frac{1}{2})$ and the $S_{1/2}(m_j = -\frac{1}{2})$ states as well as between the $P_{3/2}(m_j = +\frac{3}{2})$ and the $S_{1/2}(m_j = +\frac{1}{2})$ states, as indicated by the solid lines in the left of Figure 3.2. In exciting the $P_{3/2}$ state of the valence band, the probability of electron transitions from the $m_j = +\frac{3}{2}$ is three times greater than the probability of transitions from $m_j = +\frac{1}{2}$ state. So for instance, for every four excited electrons, three will be of the same spin and one will be of opposite spin. This one will pair with one of the three, leaving two of the electrons polarized. So, the highest attainable polarization with the GaAs cathode is $(3 - 1)/(3 + 1) = 0.5$, or 50%.

In a *strained* GaAs lattice, however, the cubic symmetry is lost, causing the $P_{3/2}$ state to split into different energy levels for the $m_j = \pm\frac{1}{2}$ and $m_j = \pm\frac{3}{2}$ states, as shown in the right side of Figure 3.2. Let the energy difference between the split $P_{3/2}$ states be called δ . Now, if the incoming excitation photon has energy above E_g but below $E_g + \delta$, the only transition allowed is between the $P_{3/2}(m_j = +\frac{3}{2})$ and the $S_{1/2}(m_j = +\frac{1}{2})$ states. When this occurs,

Figure 3.3: Diagram of the NEA surface band structure. The vertical axis is energy, and the horizontal is the distance from the material surface. The vacuum is on the right of the diagram, the center section is the surface region with the band-bending due to p-doping, and to the left is the crystal interior. V_{BB} is the amount of bending at the surface, S' is the width of the bending region, and χ is the electron affinity.

all of the electrons excited from the valence band into the conduction band have the same polarization, allowing in principle full electron beam polarization.

Electron Emission

With the polarized electrons inside the conduction band, there is still the problem of emitting electrons from the solid and into the vacuum, thus achieving a significant current. This requires reducing the **electron affinity**, (χ), the energy separation between the vacuum level and the minimum of the conduction band [93]. In particular it is ideal to create negative electron affinity (NEA), so that emitted electrons can originate from up to 50,000 Å deep inside the structure, rather than the 100 Å depth when the electron affinity is positive. For GaAs the electron affinity is approximately 4 eV, but it can be lowered by p-doping the material and by coating the surface with cesium and

either O_2 or NF_3 as an oxidizer. The effect of the p-type doping is to bend the conduction band at the surface so that the conduction band minimum is lower, as shown in the center region of Figure 3.3. The co-deposition of the Cs with the oxidizer lowers the vacuum level to 1.1 eV above the Fermi level independent of band gap. Since the band gap of GaAs is 1.42 eV, the vacuum level, shown in the right-hand section of Figure 3.3, remains higher than the conduction potential immediately at the outer surface but becomes lower a short distance into the material. So NEA can be attained with the combined effect of the band bending caused by the dopant and the lowering of the vacuum level caused by the surface coating.

The **quantum efficiency**, (QE), is the number of emitted electrons per incident photon. As the QE decreases, the electrons with lower energy aren't able to cross the surface layer and escape to the vacuum very easily [89]. Since these lower energy electrons are deeper within the lattice (the incoming photons give energy to the surface more easily), they interact more with the lattice on the way to the surface, causing them to lose polarization. The higher-energy surface electrons have shorter paths within lattice and so maintain their polarization more easily. For this reason, the polarization *increases* with *decreasing* quantum efficiency.

As the laser pumped the cathode, the Cs coating dissipated, which was found to decrease the quantum efficiency. This increased the polarization as stated above. However the lower the QE of the cathode, the more laser power was needed to maintain the necessary current— the number of electrons in each pulse. For this reason, the output power of the lasers was increased until the maximum laser power was reached. It was then necessary to “cesiate” the cathode— to deposit more Cs on the surface. This was done periodically throughout the experiment. For more discussion on this, refer to Section 3.3.7.

Figure 3.4: Cross-section of the electron gun. The laser pulses enter through the right opening and hit the cathode. The desired photoelectrons leave through the right side as well.

Source Configuration

The strained GaAs crystal used for the photocathode was grown as a layer of $\text{GaAs}_{0.72}\text{P}_{0.28}$ 100 nm thick on a lattice of normal GaAs. In other words, in the surface layer, 28% of the arsenide atoms were replaced by phosphorus atoms. This strained the lattice which then broke the degeneracy of the $P_{3/2}$ level as described in the above section on electron excitation. In addition, the GaAs crystal was p-doped with Zn atoms at a density of 4×10^{18} atoms/cm³. The cathode was 22.5 mm in diameter, and a layer of cesium and oxidizer was deposited on the surface. It was placed in an ultra-high vacuum and biased with 60 kV for operation, as shown in the electron gun diagram in Figure 3.4 [89]. To achieve photoemission, the cathode was pumped with $2.2 \mu\text{s}$ circularly polarized light pulses from a specially-designed Ti:sapphire laser [91]. The light had a wavelength of 845 nm, optimized for this particular cathode, and had an energy of $80 \mu\text{J}$.

Figure 3.5: Configuration of the polarized electron source, setup for SLC use, which is almost the same as the setup for the ESA experiments starting with E143.

A Pockels cell was used to change the laser light pseudo-randomly from linear to left or right circular polarization [93]. The cell was a 1/4-wave plate that changes optical characteristics depending on voltage bias. Another Pockels cell was used to control the shape and intensity of the light pulse, and to minimize any left or right charge asymmetry. The entire configuration can be seen in Figure 3.5.

During operation, as the ratio of Cs to oxidant decreased, the QE decreased. A lower-power diode laser was used to measure the QE with a beam of continuous 833 nm light. With the diode laser, the number of photons per unit time was very stable, and so measuring the number of electrons emitted gave the QE. This, of course, only measured relative QE changes, which was what was necessary to determine when to cesiate the cathode.

3.2.2 Beam Line

Once the electrons were produced at the source, they were injected into the main accelerator beam pipe at an energy of 120 keV. They were gathered into a “bunch” which was then sent as a beam pulse. The pulses were accelerated down the two-mile long linear accelerator to acquire the necessary energy for the particular measurement we were doing, in our case 29 GeV, 16 GeV, or 9.8 GeV. The SLAC Main Control Center (MCC) tuned, steered, and monitored the beam continuously.

Beam Acceleration

There were 120 pulses per second (pps) during normal data taking. In general, 119 of them were actually used for scattering events while the other pulse was used as a pilot, or “witness”, pulse, to signal the start of the spill. The accelerator was made from machined copper cavities into which 240 klystrons delivered high power 2.8 GHz microwaves. Inside the cavities a standing wave is formed on which the electrons “ride”. The exact frequency of the microwaves and the phase of the standing wave is chosen to match the speed of the electron bunches so that they are always accelerated forward by the field. As a side note, during the running of the SLC, the positron bunches ride on the waves 180° out of phase from the electrons, so that they are accelerated forward by the field as well.

Endstation A (ESA)

At the end of the accelerator, the beam travelled through the Beam Switch Yard, BSY, which is where the decision is made to send the beam to the A-line, to the B-line, to the arcs of the SLC, or to the Final Focus Test Beam (FFTB). For the nucleon fixed-target experiments, including E143, the beam

Figure 3.6: Diagram of the beam line upon entering Endstation A. At the top is the alcove where the Møller target sits. The dotted line from the alcove is the path of the Møller electrons. At the bottom the dotted lines are the scattered electrons accepted into the spectrometers, not shown, at 4.5° and 7.0° with respect to the beam line.

Figure 3.7: Configuration of the chicane magnet system and the effect on the beam direction, as viewed from the side. The system was utilized to correct for the target magnetic field during transverse running.

was diverted into the A line and then sent into ESA [94]. Figure 3.6 shows the beam exiting the alcove at the entrance to ESA, after having hit the Møller target (refer to Section 3.3), and entering into the ESA chicane system from where it proceeds on to the E143 target.

To take the data necessary to measure g_2^p and g_2^d , we had to rotate the target 90° so that the target nucleons were polarized perpendicular to the incident longitudinally-polarized electron beam. In this configuration, the strong target magnetic field is also perpendicular to the beam, resulting in a deflection of the beam away from the target, and away from the beam dump as well. In addition, the target field places a force on the beam electrons, causing their spin to precess so that they are no longer longitudinally aligned with the beam direction. To compensate for the target field effects, a *chicane* magnet system was set up, three of which are shown in Figure 3.6. In front of the target but after the Møller target, three chicane magnets were used to bend the beam downwards and then back upwards. Since the target field deflected the beam downwards again, the beam ended up precisely on target.

Upon exiting the target, however, the beam was still travelling in the upward direction by about 10° . For this reason, a fourth chicane magnet was placed after the target to re-align the beam with its original axis, sending it properly into the beam dump. Yet the final beam was positioned lower than the original beam, so the position reading on the foil arrays had to be corrected. The chicane-beam system is diagrammed in Figure 3.7.

3.3 Møller Polarimetry

Møller scattering is the elastic scattering of incident (beam) electrons off of free electrons in a target. It is one of the most common tools for measuring the polarization of high energy beams [95]. For a precision experiment such as E143, it was important to get a good measurement of the polarization of the beam. In E143, we used two different Møller scattering detectors to provide us with the best possible knowledge of the beam polarization. The single-arm detector was used for E142 prior to E143, but the double arm was built specifically for E143. Both are described in detail here.

3.3.1 Møller Asymmetry

From quantum electrodynamics, the scattering of beam electrons off of free target electrons depends on mutual spin orientation of the two electrons. To measure the polarization of the beam, we exploit this fact, utilizing target electrons with known polarization. For the “single photon interaction” approximation for longitudinally polarized ultrarelativistic electrons [96] we have:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left(1 + \sin^2\tilde{\theta} \frac{7 + \cos^2\tilde{\theta}}{(3 + \cos^2\tilde{\theta})^2} \epsilon_b \epsilon_t \right), \quad (3.1)$$

Figure 3.8: Views of the two Møller polarimeters. The top figure is a top view and the bottom is view from the side. The foil targets are on the left, while the E143 ammonia targets were downstream to the right, off the scale of the picture.

where $\frac{d\sigma}{d\Omega}$ is the differential cross section in the center-of-mass (c.m.) frame, $\frac{d\sigma_0}{d\Omega}$ is for unpolarized particles in the c. m. frame, $\tilde{\theta}$ is the c.m. scattering angle, and $\epsilon_b(\epsilon_t)$ is the beam (target) helicity. Here we see that if ϵ_t is known, then the beam polarization ϵ_b can be determined by simply measuring the cross-section (counting rate) asymmetry for parallel and antiparallel spin orientations

$$A = \frac{N^{\uparrow\uparrow} - N^{\uparrow\downarrow}}{N^{\uparrow\uparrow} + N^{\uparrow\downarrow}}. \quad (3.2)$$

We see that the analyzing power is greatest at $\tilde{\theta} = 90^\circ$, and that the measurement is independent of the azimuthal angle. Furthermore, note that the c.m. scattering angle $\tilde{\theta}$ is related to the laboratory scattering momentum p' as

$$p' = \frac{p_b}{2}(1 + \cos\tilde{\theta}). \quad (3.3)$$

Here p_b is the momentum of the incoming (beam) electron.

3.3.2 Scattering Setup

The configuration of the Møller system in Endstation A is shown schematically in Figure 3.8. The foil targets (see Section 3.3.5) were situated in the alcove upstream from the detectors so that, with the help of a bending magnet, they could be placed away from the beam. A mask was placed several meters downstream of the foils which allowed only Møller electrons of a particular azimuthal angle to go through. A side view of the mask can be seen in Figure 3.8 and a face view is shown at the top of Figure 3.9. A dipole magnet known as 'B0' having a vertical field bent the stream of electrons horizontally, depending on their momentum p' (or c.m. scattering angle $\tilde{\theta}$), forming a parabola at the detector faces. The magnet surrounded the incoming electron beam, so in order to prevent it from deflecting the non-scattered particle beam (which proceeded to the E143 target downstream), metal shielding called a "septum" was placed inside B0, as shown in the middle of Figure 3.9. The septum was

a long, flat piece of 3.2 inch thick metal with a hole in the center to allow the beam to pass. The magnetically-diverted Møller electrons travelled through long plastic bags filled with helium so as to minimize scattering off the larger nitrogen and oxygen molecules in the air. Finally, they reached the detectors which were mounted approximately 15 m downstream of B0. Although the laboratory scattering angles are small for the 90° c.m. scattering case, the bending magnet allowed the detectors to be placed off to the side of the beam line.

3.3.3 Single-Arm Detector

The single-arm Møller detector was built, maintained, and analyzed by collaborators from the University of Wisconsin, Madison [97] [98]. The detector consisted of two sets of silicon pads with 12 channels on each of 4 rows, shown on the top of Figure 3.10. The Møller electrons detected were from the scattering cone at a c.m. angle of approximately 91° . While the peak scattering angle is at 90° c.m. as discussed above, because each scattering event created two electrons at $90^\circ \pm \theta$, double counting was avoided by placing the detectors only in the $>90^\circ$ region. A sheet of lead three radiation lengths thick was situated in front of the detectors to amplify the Møller signals when the electrons knocked off other particles by interacting with the lead. The lead also provided a shield against low energy photons. The majority of Møller electrons scattered onto two of the twelve rows. The remainder of the channels were used to assess the background signal. Rather than reading out each event separately, the total charge deposited on the silicon was measured for each channel. To calculate the asymmetry, a smooth fit to the background signal was made and then subtracted from the total signal. Since the beam helicity was recorded for each spill, the asymmetry, and hence the beam polarization, could be determined.

Figure 3.9: Sample distributions of the Møller-scattered electrons at the z -positions of the mask (top), the B0 magnet (middle), and the shielding wall (bottom) which helped to select particular momenta p' .

3.3.4 Double-Arm Detector

The double-arm detector was built, maintained, and analyzed by collaborators at the University of Basel, CH [99] [100]. While such detectors had been used in various experiments, only recently have the electronics been fast enough to allow their use in high rate experiments such as E143. The detector was based on shower technology, and contained 14 lead glass blocks which formed two arms along the Møller acceptance parabola, as shown in the lower part of Figure 3.10. It was positioned so as to detect electrons scattered within the c.m. scattering angle range from 70° to 110° . This detector was placed behind the single arm detector, but since the two detectors only overlapped in a small region, and since the silicon pads were thin, the effect on Møller detection was negligible. The lead glass also benefitted from the thin lead plate mentioned above, which was deliberately made large enough to cover the double-arm detector acceptance.

3.3.5 Target Foils

Inside the ESA alcove, the target foils were placed inside Helmholtz coils with a homogeneous magnetic field strength of 0.01 T (100 gauss), mounted at a 20.7° angle to the beam. The foils were ferromagnetic, made from *Vacoflux*, an alloy of 49% Fe, 49% Co, and 2% Va by weight [101]. There was a range of thicknesses from 20 μm to 154 μm . The field magnetized the foils to near saturation, and the polarity was reversed between data sets to help minimize systematic errors. The field established an overall electron polarization of 8%, affecting about two of the 3 “d” (M-shell) electrons. The polarization of the target foils was measured to a relative error of 1.7%, the dominant error contributing to the overall systematic error on the beam polarization measurements. Prior to E143 a systematic study of foil magnetization measurements was done in which more than thirty foils 20, 30, 40, and 154 μm thick were

Figure 3.10: Scattering profile of Møller electrons on the single-arm (top) and double-arm (bottom) detectors. Note that as indicated on the ordinate, the double-arm detector was situated behind the single-arm. The coordinates are with respect to the beam line.

measured to try to get the error as low as possible. In the end, the average polarization for the 20 μm foils was used separately from the average 30 and 40 μm foils, as measured after E143 from the foils used. The foil polarizations were 0.0816 for the 20 μm foils and 0.0827 for the 30, 40, and 154 μm foils.

3.3.6 Data Collection

The beam polarization was measured almost every day. During the Møller runs, no regular data was taken because the foil targets were moved via remote control into the beam line, and the beam was refocused on the foils. A single Møller run took only about 10 minutes. However, foils of several of the four thicknesses mentioned above were used each time, and the Helmholtz coil was reversed between runs. In addition, it took time to refocus the beam and to change the spot characteristics, so data collection typically lasted at least an hour. Hits on the detectors were sent through ADC discriminators and TDC modules. Output was logged onto separate Møller tapes. A dedicated drive was used for Møller runs so that the tapes did not have to be unmounted and remounted each time we stopped to take Møller data. For the double-arm detector, Møller coincidences were found offline, and were not selected real time by hardware, so all of the data went to tape.

3.3.7 Beam Polarization Analysis

As described in Section 3.2.1, the circularly-polarized laser beam pumped the cathode to knock out polarized electrons. Over time, the cesium coating on the cathode became degraded, perhaps by migrating and combining with newly incident oxides [89], thus reducing the effective ratio of Cs to oxidant. The effect of the lower ratio was that only the electrons with the highest energy, which were nearer to the surface of the GaAs crystal, would escape the lattice,

Figure 3.11: (a) The quantum efficiency QE of the photocathode showing the exponential decay as a function of time. (b) Double Arm Møller beam polarization results as a function of QE. Box errors indicate the spread in measurements for that QE range, and circles give the results with the averaged errors. The solid line is a fit to the data.

resulting in a lower quantum efficiency, QE [89]. To maintain the current, laser power was increased, which further reduced the Cs ratio in the coating, and hence the QE. The QE as a function of time throughout the experiment is shown at the top of Figure 3.11. Each jump to a peak in QE is right after “cesiation” of the cathode, when the Cs coating was replenished. Note that towards the end of the experiment, the cesiations became more frequent but were not as strong. This is due to the fact that they could be done more quickly (a few minutes), and that it was discovered that the lower QE corresponded to **higher** polarization, as explained in Section 3.2.1.

QE measurements were taken throughout the experiment at short intervals—not more than two hours apart—and were written to a computer file. The decay of the quantum efficiency followed an exponential curve, so the data points between cesiations were fit with a function of the form $QE = Ae^{Bt}$, with t the time from cesiation and A and B fitting constants. Using this information, the QE was available for all times, as shown in the plot.

The dependence of the beam polarization on the quantum efficiency is shown for the double-arm detector measurements in the bottom portion of Figure 3.11. For each QE data point, the average beam polarization is shown with two different error bars [99]. The error bars on the left (the larger ones) show the spread of the measurements for that particular QE range. The smaller bars on the right indicate the standard deviation of measurements in that range. The solid line through the points represents a curve fitted to the data, not backed by a particular theory. The curve has the form

$$P_{beam} = a + \frac{b}{e^{c(q-d)} + 1} \quad (3.4)$$

where P_{beam} is the beam polarization, q the QE, and the fit parameters are: $a = 0.828$, $b = 0.035$, $c = 300$, and $d = 0.055$. Combining this expression with the QE determined via the exponential decay expression the beam polarization for each individual run could be determined.

The Levchuk Effect

After E143 took data it was discovered by Levchuk [96] that the effect of the motion of bound electrons in the foil target atoms can result in an overestimate of the Møller asymmetry. In many cases, the overestimate can exceed the uncertainty in the target foil polarization itself, and so must be taken into account when analyzing the data. The electrons in the outer energy shells have negligible momentum, whereas the electrons in the inner shells, specifically the K- and L-shell for the E143 foils, have a momentum of about 100 keV, similar in magnitude to the electron. Using the scattering and recoil calculation for Møller scattering off the high-momentum inner shells, it turns out that the scattering angle can be off by as much as 10% compared to scattering from the outer electrons. In addition, the inner electrons typically have less polarization, meaning that in calculating the Møller asymmetry, care must be taken as to which part of the scattering parabola is used. For instance, if one uses mostly scattered electrons from the center of the parabola, a deceptively high number of outer-shell events would determine the asymmetry and it would be overestimated. If one took electrons mainly from the two ends of the parabola, most of these would be scattered from the inner shell and the asymmetry would be underestimated. Prior to this calculation, it was generally assumed that the asymmetry was independent of the scattering angle.

The single-arm detector had a large enough acceptance to detect all Møller electrons, including those shifted due to the intrinsic atomic motion, but it also had a high enough resolution that some of the Møllers were sent into channels used to calculate subtracted background. A correction was made to these data for the so called Levchuk-effect, which resulted in a lower asymmetry by close to 3%. The acceptance of the double-arm detector was large enough to detect all Møller electrons and to put them properly into the asymmetry, so the Levchuk correction was less than 1% [102]. The double-arm data therefore needed only small dead-time and acceptance corrections.

Results

For the double-arm detector, at a beam energy of 29 GeV, scattered electrons with center of mass (c.m.) angles between 70° and 110° were collected. For the incident beam energy of 16 GeV, the angle became much smaller, as the amount magnet deflection decreased. For the 9 GeV data set, no double-arm measurement could be made, and only the single-arm detector acquired data. However, double arm results were applied to the 9 GeV data by assuming that the beam polarization was independent of beam energy, and then extrapolating the known relationship between the QE of the cathode and the beam polarization to the 9 GeV data set.

Since the double-arm detector had better statistics and smaller systematic errors than the single arm detector, the results from the double arm analysis was used for the first 5 E143 published papers [16] [17] [18] [19] [20]. The beam polarization used for these publications ranged from 83% to 86%, with a quoted mean of 0.85 ± 0.02 . More specifically, we used the double-arm results to find the beam polarization for each measured QE, using the function in Equation 3.4.

However, further analysis of both the single and double-arm Møller data during 1997 presented some discrepancies between the two results and with what was expected. After further analysis, agreement was reached between both data sets, and an average of the single arm and double arm results was taken. Therefore, for the final E143 paper [21], as well as for the results presented in this thesis, the beam polarization used was calculated from the formula

$$P_{beam} = a - b * QE \quad (3.5)$$

where here $a = 0.866$ and $b = 0.34$. The final error remains the same, an absolute error of ± 0.02 , being dominated still by the error on the foil polarization. This change represented only small differences in the overall

results of the experiment.

3.4 Polarized Target

The spin structure measurements require not only a highly polarized beam, but a polarized nucleon target as well. For a precision experiment such as E143, we required a target that could achieve a high polarization and maintain it at high beam intensities. For this reason, we chose a polarized solid target.

3.4.1 Target Material

For the proton spin structure measurement, polarized ammonia, $^{15}\text{NH}_3$, was used, and for the deuteron measurement, $^{15}\text{ND}_3$. Ammonia has been shown to provide high polarization as well as to be radiation resistant [103]. It also has a large number of polarizable nucleons per molecule: 17% for protons and 29% for deuterons. The asymmetries had to be corrected for the nitrogen polarization, but by choosing ^{15}N instead of the most common isotope, ^{14}N , we eliminate an extra polarized neutron, reducing the amount of corrections needed to the model of ND_3 as being simply polarized deuterons. The nitrogen correction is discussed in Section 5.3.3. Furthermore, since ^{15}N is a spin-1/2 nucleus, it is easier to measure its polarization.

Since ammonia is not solid, the material had to be frozen. A container of high purity (^{15}N was better than 98% and deuterium was better than 99%) ammonia gas was immersed in liquid nitrogen where it slowly froze [104]. When frozen, the ammonia was solid and had to be crushed into granules for packing into the target cups. The granules were filtered with 1-2 mm wire mesh, so that the pieces would be similar in size. Before the experiment, the densities of the material was measured at 77° K. The NH_3 was found to be

0.917 g/cm³ and the ND₃ to be 1.056 g/cm³ with a 1% relative error.

To polarize the ammonia, we had to create small paramagnetic centers throughout the lattice that could then help in the transfer of spin to the nucleons. To create these centers one can pre-irradiate the ammonia beads or dope them chemically [105]. It had been found previously that the pre-irradiation works the best for ammonia target materials, so several collaborators took the ammonia to relatively low energy electron sources to accomplish this. Such irradiation was done at MIT-Bates Laboratory using a 350 MeV beam, at HEPL Lab on the Stanford campus with 35 MeV electrons, at the Saskatoon Accelerator in Canada using a 250 MeV beam, and at the Naval Postgraduate School in Monterey with a 65 MeV beam.

As the incident electrons strike the ammonia lattice, protons can be knocked out leaving the ammonia molecules ionized (NH₂⁻, ND₂⁻). As long as the material is cold, the extra electron can remain localized in the lattice and then can serve as a paramagnetic center. To keep the material cold, a bath of liquid argon was used, since liquid nitrogen can become explosive under long radiation exposure due to the build-up of nitrates [106]. The pre-irradiation doses were typically about 10¹⁷ e⁻/cm² with average beam currents of 1.0 μA. This created spin centers of around 10¹⁹ spins/cm³, a fairly dilute spin system.

3.4.2 Dynamic Nuclear Polarization

The ammonia and deuterated ammonia target material is polarized using a technique called Dynamic Nuclear Polarization, or DNP. This is done by applying an external magnetic field as well as microwave power of an appropriate frequency to the nucleon-electron systems within the ammonia, as explained here,

When a particle is perturbed by a magnetic field, the spin of the particle

Figure 3.12: Diagram of the split energy levels of the electron and proton when ammonia is placed in a magnetic field. The larger splits are due to the electron magnetic moment, while the smaller are due to that of the proton (refer to text).

can contribute to the total energy an amount

$$\epsilon = -\boldsymbol{\mu} \cdot \mathbf{B} \quad (3.6)$$

where μ is the magnetic moment vector of the particle, which depends on the spin $\mathbf{S}$ of the particle, and $\mathbf{B}$ is the magnetic field vector. For the electron, $\mu = g\mu_e\mathbf{S}$ and for a nucleon or nucleus, $\mu = g\mu_N\mathbf{S}$, where μ_e is the Bohr magneton, μ_N is the nuclear magneton, and g is the g -factor of the particle.

Proton

As mentioned previously, the pre-irradiation of the ammonia creates paramagnetic spin centers which are basically electrons localized in the lattice. For the spin-1/2 nucleon or nucleus case, as we have with the proton, the electron-proton system has the Hamiltonian [107]

$$H = -\boldsymbol{\mu}_e \cdot \mathbf{B} - \boldsymbol{\mu}_p \cdot \mathbf{B} + H_{int} \quad (3.7)$$

Figure 3.13: Diagram of the transitions achieved by pumping the ammonia with the appropriate microwave frequency. A lower frequency is used to force the suppressed transition on the left while a higher frequency is needed for the larger gap on the right.

where H_{int} is due to the coupling between the two spins, and where I have given the magnetic moments subscripts for clarity. Without the interaction term, there are four energy levels, **a**, **b**, **c** and **d**, shown in Figure 3.12. The **a**-**c** and **b**-**d** splitting is due to the electron spin interacting with the magnetic field. The **a**-**b** and **c**-**d** splitting, much smaller, is due to the proton spin. Due to dipole selection rules, the allowed transitions between states are the ones in which either the electron spin or the nucleon spin flips, namely $c \leftrightarrow a$, $b \leftrightarrow a$, $d \leftrightarrow b$, and $d \leftrightarrow c$. The forbidden transitions involve the flipping of both the electron and proton spins, $d \leftrightarrow a$ and $c \leftrightarrow b$. However, if the interaction term is kept, the two forbidden transitions become allowed, but they are suppressed relative to the other transitions.

To polarize the protons we apply microwaves tuned to the proper frequency to drive the suppressed transitions, where both the spins are flipped. This done, the electrons relax via the usual $c \leftrightarrow a$ and $d \leftrightarrow b$ transitions and the proton spin relax via the $b \leftrightarrow a$ and $d \leftrightarrow c$ transitions to the natural equilibrium. The key to dynamic nuclear polarization [105] is that the electron spin relaxes

much faster (~ 1 ms) than the proton (nucleon) spin (a few tens of minutes to a few hours), so that the protons stay polarized while the electrons are used again and again to drive other transitions with nearby protons (nucleons). As demonstrated in Figure 3.13, the microwave frequency can be chosen to drive either the $d \leftrightarrow a$ or the $c \leftrightarrow b$ transition. This will polarize the protons parallel or antiparallel to the magnetic field, referred to in the data set as a “positive enhancement” or a “negative enhancement”, respectively.

This mechanism of polarization works for polarizing nucleons that are near to the paramagnetic centers. However, since the pre-irradiation only creates a dilute system of centers, the ratio of centers to polarizable nucleons is small. To establish polarization throughout the target material, we then rely on *spin diffusion* to spread the polarization out. Because there is a dipole-dipole coupling between the nucleons, as the pumped nucleon makes the relaxation transition to the stable state, it can transfer its spin to a nearby nucleon. The nearby nucleon can then transfer its spin to a neighbor, and so forth, until the polarization diffuses throughout the target material.

Because the spin-lattice coupling for the nucleons is small, the relaxation time is quite long as stated earlier, which allows us to generate and sustain high nucleon polarizations in such low temperature solids. Yet what enables the nucleons to eventually relax is mainly the nucleon coupling to the spin centers, so as long as the number of paramagnetic centers stays small, the material will stay polarized. However, as the high energy beam hits the target ammonia during data taking, more centers are created at 1° K, the temperature at which the target is maintained. With more centers, the nucleon relaxation increases which then starts decreasing the polarization of the target. When the polarization falls below a level considered useful, it is necessary to turn off the beam and allow the ammonia to heat up to approximately 85° K for five minutes or so, a process called *annealing*. This allows most of the paramagnetic centers created at 1° K to recombine, so that when the ammonia is cooled down again, the polarization is restored to a sometimes even higher polarization than

before the radiation [106].

Deuteron

The above description is valid for a spin-1/2 nucleon or nucleus, as in the case of NH_3 . However, the deuteron is a spin-1 particle, so some minor differences are worth noting. The magnetic moment of the deuteron is about 1/3 of that of proton (0.857 compared to 2.793 for the proton). This means that the coupling to the paramagnetic centers is weaker, making a much slower spin diffusion rate, and allowing more deuterons to relax before the polarization spreads throughout the material. For this reason, polarizing the ND_3 was more challenging and resulted in lower overall polarizations. In addition, to achieve maximum polarization, ND_3 needed to have some in-situ 1° K irradiation. This created a different type of spin center, one in which the polarization rate and maximum increased. After pre-irradiation only, 13% polarization was achieved, while the polarization increased to 40% after exposure to the beam. Another aside is that the deuteron has a small quadrupole moment which can interact with the lattice electric field and cause another weak level splitting. While it is too small an effect to change the polarization, it can be seen in the NMR signal profile, distinguishing it from the proton signal.

3.4.3 Apparatus

The target was cryogenic and used a ^4He evaporation refrigerator together with a large vacuum manifold to pump away the helium so as to achieve the running temperature of 1.06° K with a heat load of about 1.3 W from both the beam and the applied microwaves. For the DNP process we required a large magnet employing superconducting Helmholtz coils to surround the target, providing a 4.82 Tesla magnetic field homogeneous to $\frac{\Delta B}{B} \lesssim 10^{-4}$ over a 3 cm spherical target volume surrounding the target ammonia [108]. The field

was chosen to match the frequency of the microwave tube, a narrow band centered on 136.5 GHz. To flip the target polarization, done periodically to avoid systematic effects, we changed the direction of the current through the coils. To take data with the target polarization transverse to the longitudinal beam polarization, the magnet was rotated 90° . The magnet orientation was controlled remotely from the ESA counting house. Figure 3.14 shows a cross-section of the entire target configuration including the cylindrical holder for the magnet and the cryogenic baths.

The ammonia beads were held in a target cup, 3.0 cm long and 2.5 cm in diameter, made from Torlon with aluminum foil windows. Torlon has a low free proton density, so any contribution to polarized protons is negligible. It also has a very high radiation damage resistance. The cups were held by a *target insert* roughly 1 m long. Each insert contained three such target cups stacked one on top of the other. One cup held the NH_3 , another the ND_3 , and the third was left empty to be used for calibration. Beneath the cups we attached a solid target made of carbon or aluminum for calibration and acceptance measurements. This was made to be approximately the same radiation length as the ammonia targets. A microwave guide came down from the top of the insert and ended in a horn situated just above the target cups. This was used to pump the suppressed transitions necessary for the DNP polarization process previously discussed. The ND_3 was placed in the uppermost target cup to receive the most microwave power since it is more difficult to polarize. NMR signal cables came down from the top of the insert as well, connecting to the NMR pickup coils used for the target polarization measurements.

Figure 3.14: Cross-section of the target configuration for E143. The figure shows two target cups on the insert instead of the three that were used.

NMR

Two 0.5 mm NMR pickup coils of 30/70 copper/nickel wire were placed in each target cup. At a field strength of 4.82 T, the resonance frequencies were:

$$\nu_{res} = 2\mu B/h = 2\mu_N 2.79B/h \approx 210 \text{ MHz for protons} \quad (3.8)$$

$$= 2\mu_N 0.86B/h \approx 32 \text{ MHz for deuterons} \quad (3.9)$$

Since the hydrogen frequency was high, the wire impedance had to be small, so the coil was short, straight, and traversed the cell diameter vertically. This was used to measure the proton polarization in $^{15}\text{NH}_3$ as well as the residual proton polarization in $^{15}\text{ND}_3$. The deuteron frequency was low, so a small helical coil having four turns of diameter 2 cm was used. This was used to measure deuteron polarization in $^{15}\text{ND}_3$ and the ^{15}N polarization for both target materials. The incident beam electrons periodically hit the coils creating unwanted events, which had to be corrected for in the dilution factor. However it was decided that it was better to have the coils inside the target cups to be sure the ammonia polarization values were reliable. The signal cables for the NMR ran up the insert as mentioned earlier. They were 0.085 in diameter and were made of beryllium and copper.

For the polarization measurement, the target coils were analogous to an inductor in a tuned LCR circuit. The ammonia beads within the coil were the ones whose polarizations were read with the NMR system. The frequency of the circuit was swept back and forth by ± 300 kHz so as to find the resonances of the nuclei. When the nuclei are at resonance, the power absorbed by the ammonia lattice is proportional to its polarization, so the trick is to stay on the resonance and to read the power absorption accurately. The output signals for the proton and deuteron are shown in the top and bottom of Figure 3.15, respectively. The area under each curve is related to the polarization. The deuteron signal has two peaks due to the quadrupole splitting mentioned

before.

To find the proportionality constant, and thus to determine the polarization from the NMR signal, the signal has to be calibrated by measuring a substance with a known polarization. For this purpose, the reference signals chosen were signals from the same materials but at *thermal equilibrium*, TE. When its at TE, a material's polarization is known from the Maxwell-Boltzmann distribution. At thermal equilibrium, only the two lowest states are populated with an equal number in each. The small gap between the states causes a small polarization. So for protons at TE and in an external magnetic field, the number of particles with extra energy ϵ due to the interaction with the field is given by

$$N \sim e^{\epsilon/kT}, \quad (3.10)$$

with k the known Boltzmann constant and T the temperature. The proton polarization is then defined as

$$P_{1/2} = \frac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}}. \quad (3.11)$$

Together, this gives the following expression for the proton polarization at TE:

$$P_{1/2} = \tanh \frac{\mu_p B}{kT}. \quad (3.12)$$

Since deuterons are spin-1 particles, however, their polarization is defined by

$$P_1 = \frac{N_{+1} - N_{-1}}{N_{+1} + N_0 + N_{-1}} \quad (3.13)$$

resulting in the expression

$$P_1 = \frac{4 \tanh \frac{\mu_D B}{2kT}}{3 + \tanh^2 \frac{\mu_D B}{2kT}}. \quad (3.14)$$

Figure 3.15: The NMR signals corresponding to the power absorbed by $^{15}\text{NH}_3$ (top) and $^{15}\text{ND}_3$ (bottom) during a frequency sweep. The area under the curve is proportional to the polarization.

In a magnetic field of 4.87 T and at 1.5° K, the TE polarizations turn out to be 0.33% for protons and 0.068% for deuterons. These small polarizations gave a low signal-to-noise ratio, so several measurements were made for each target material. The proton and deuteron calibrations consisted of 25 measurements of 500 and 1000 frequency sweeps respectively. To find the area under the signal peaks, a baseline signal had to be subtracted from the curves. The baseline was found by shifting the magnet field slightly, about 1%, away from the desired field. Then the sweep never hit resonance and the baseline could be measured and subtracted.

3.4.4 Performance

The target polarization was monitored continuously during the DNP process. It was calculated using the relation

$$P_{DNP} = \frac{P_{TE}}{A_{TE}} A_{DNP}. \quad (3.15)$$

Here A_{DNP} is the area under the peak measured during DNP. The TE signal calibrations were done about five times for each target throughout its use, typically after a target was loaded and after each anneal. The calibration constants had a statistical error of 0.2% for the proton and 3.0% for the more difficult deuteron.

During the run the target was controlled remotely. It was set up so that the target inserts could be moved up and down to put the different target cups in the beam position. Most of the target setup could be monitored and run using Macintosh computers running LabView software. LabView was programmed to automatically perform NMR measurements to read out the polarization, which was then logged and sent to the main online stations to be written to tape at each checkpoint (every 5 minutes). The average target polarization, weighted by incident current, was found for each run and written to a file

called “hdwsumry.out”, which contained numerous statistics for each run that was recorded. This was all done real-time.

After the experiment, the target group from the Univ. of Virginia took more TE data and corrected the calibration constants for a more accurate target polarization value. It turned out that there was a certain period of time when the TE constant was 10% larger than what was used online, so that changes had to be made to the polarizations for these runs.

Ideally the polarization of the NH_3 target could reach 90%. During the experiment, however, the polarization was less than 80%. Beam heating effects as well as the fact that the microwave horn was directly over the deuteron target so that the protons absorbed less microwave power were thought to be the reason for the reduced polarization. For ND_3 , in-situ radiation helped the polarization reach more than 40%. Figure 3.16 shows typical plots of target polarization versus time for the proton and deuteron targets. The relative error on the polarization was $\Delta P_p/P_p \sim 2.5\%$ for the proton and $\Delta P_d/P_d \sim 4\%$ for the deuteron, calculated from the spread in TE measurement values.

Beam Heating Effects

The polarization of the ammonia depended upon the material being at very low temperatures. Yet when the beam hits the ammonia, the temperature of the beads goes up causing the polarization in that region to drop. The beam was rastered across the face of the target (see Figure 5.1) so that the area it hit had a chance to cool back down before the next time it was hit, but the time between hits was not enough for the material to completely recover. This resulted in a lower polarization than if the beam had not hit the region. Since the raster pattern did not completely cover the entire region of ammonia, there was some material outside the beam path that was not affected, thus having a higher polarization. However, the NMR polarization measurement took the

Figure 3.16: Histograms of the target polarization versus time for NH_3 (top) and ND_3 (bottom).

average of the entire target.

To correct for this effect, a model of the target depolarization was created using knowledge of the thermodynamic properties of the target materials, the known size of the rastered region, the position of the coils, and the size of the target cups [109] [110]. With this model, a relationship between the polarization measured and the polarization actually seen by the beam was determined to be

$$P_t = P_{meas} \left(1 - C_{heat} \frac{I_b}{4} \right) \quad (3.16)$$

where C_{heat} was found to be 0.0081 for $^{15}\text{NH}_3$ and 0.0197 for $^{15}\text{ND}_3$, and I_b is the beam current in units of $10^9 e^-/\text{spill}$. The correction was applied during the analysis, and the size depended on the beam current, as expected, and ranged from 0.8% to 2% relative to the measured polarization.

3.5 Magnetic Spectrometers

Two large acceptance spectrometers, situated at 4.5° and 7.0° , were used to detect the electrons scattered off of the polarized target. The spectrometers were almost identical to those used in the E142 experiment [15] [90], but the magnets were operated at somewhat higher fields to accommodate the larger momenta scattered electrons due to the higher beam energy. Each spectrometer contained two dipole magnets bending in opposite directions in the vertical plane, two gas threshold Čerenkov barrels, two scintillation hodoscope arrays, two scintillation trigger counters, and an array of lead glass total absorption shower counters, 24 radiation lengths in depth. The spectrometers are shown in Fig. 3.17.

The reverse bend magnet design was chosen to have maximum acceptance over a wide momentum range. The asymmetries to be measured were expected to be small, of the order of 10^{-3} , so to minimize beam running time, the

Figure 3.17: Two Arm Spectrometer for E142/E143.

spectrometers needed to have the largest possible solid angle of the momentum acceptance range from 6 to 20 GeV/c, which the dipole doublet configuration provided [94].

In addition, considerable photon background produced by the electron beam interacting with the thick target due to bremsstrahlung radiation, radiative Moller scattering, and the decay of photoproduced π^0 mesons was expected. To shield the detectors from the unwanted photons, a two-bounce system was needed, which the reverse bend design provided. The probability for photons to scatter into the spectrometer acceptance after bouncing twice on vacuum walls or magnet gaps was small.

A further benefit of the reverse bend design, other than the convenience and simplicity of having the detector package at beam height, was that the bend plane position of the scattered particles was only weakly dependent on the particle's momentum, as seen in Figures 3.5 and 3.5. The resulting loss in

momentum resolution was not as important to the experiment as the beneficial spreading out of the pion background onto a large detector area, allowing accurate particle detection at higher pion background rates.

The quadrupole magnet in the 4.5° spectrometer served to spread the electrons over a larger detector area along the horizontal (non-bend) plane, thus increasing the range in x_{BJ} coverage. The quadrupole provided focusing in the vertical (bend) plane, however, enhancing the position-dependence of the electron momentum along the bend plane [94], thus improving the momentum resolution.

During the experiment the currents of the magnets were set so that the momentum of the central ray for the 29 GeV runs were 11.5 GeV/c and 12.5 GeV/c for the 4.5° and the 7.0° spectrometers respectively. For the both the 16 GeV and the 9 GeV running the central rays were set at -7.5 and -8.0 for the two spectrometers. Since we had no independent check of the momentum for E143, we monitored the stability of the dipole magnets by watching the current and by taking NMR readings from the probes located inside of the magnets.

The detector elements of the spectrometer were chosen to have various necessary functions, as described in this section. The Čerenkov barrels were used to discriminate against a large pion background. The hodoscopes were used to reconstruct the trajectory of each event, which in turn could be used to determine the momentum and other kinematic variables. Finally the shower array provided the energy measurement as well as some particle identification information.

3.5.1 Hodoscope System

For each spectrometer, two scintillator hodoscopes arrays provided track

Figure 3.18: Electron ray trace for the 4.5° spectrometer. (a) Vertical bend plane. (b) Horizontal non-bend plane. Particle momentum ranges from 7.5 to 19 GeV/c. Figure drawn such that the 10 GeV/c ray at the center of the acceptance is a straight line.

Figure 3.19: Electron ray trace for the 7.0° spectrometer. (a) Vertical bend plane. (b) Horizontal non-bend plane. Particle momentum ranges from 7.5 to 19 GeV/c. Figure drawn such that the 10 GeV/c ray at the center of the acceptance is a straight line.

Figure 3.20: Layout of the hodoscope planes with respect to the other detector elements for E143.

information for the scattered particles. Tracks were then used to determine the particle's momentum from the reverse matrix elements of the spectrometer magnet. Each array consisted of horizontal(y), vertical(x), and slanting(u) planes of fingers. Although most of the hodoscope planes were the same ones used in E142, the hodoscope arrays were slightly modified to handle a higher instantaneous rate. The upstream hodoscope array contained four planes: u , x , and two y in order from upstream to down, and the downstream hodoscope contained x , y , and u planes. Each spectrometer had 7 planes total. The arrangement of the hodoscope planes with respect to the other detector elements is shown in Figure 3.20.

The first y plane in each hodoscope array consisted of scintillator elements of 3.0 cm width, while the elements of the second y plane in the upstream hodoscopes were 4.76 cm wide. The elements in the first x plane of the 4.5° spectrometer were 2.0 cm wide, while the remaining x plane elements were 3.0 cm wide. The u -planes contained elements that were 4.5 cm and 7.5 cm in width for the front and back hodoscopes respectively. Table 3.1 shows the specifications for the various hodoscope planes. Within each plane the elements overlapped by $1/3$ of the width on both edges, as shown in Figure 3.21,

Figure 3.21: The schematic for the overlap of the fingers of the H1U hodoscope plane. All planes had such overlap to make sure particles hit at least one finger on each directional plane.

resulting in a bin width of $1/3$ of the element width. Fine hodoscope segmentation (~ 210 scintillator elements per spectrometer) was chosen to tolerate the large expected photon and neutron backgrounds and to reconstruct with sufficient resolution the trajectory of the scattered particles. The signal from each finger was sent through a discriminator and then fed into a multi-hit TDC which recorded all signals within a 100ns window around each trigger.

The separation of the two hodoscopes was 5.0 m in the 4.5° arm and 5.1 m in the 7.0° arm. The angular tracking resolution of the hodoscopes [94] was ± 0.7 mrad for the 4.5° spectrometer and ± 0.9 mrad for the 7.0° spectrometer; the position tracking resolution was ± 0.3 cm for the both spectrometers.

The scattering angle resolution in the non-bend plane was ± 0.3 mrad for both spectrometers, whereas for the bend plane, it was ± 0.9 mrad for the 4.5° arm and $\sim \pm 0.3$ mrad for the 7.0° arm. The momentum resolution was dependent upon the absolute value of momentum and varied from $\pm 0.3\%$ to $\pm 3.2\%$ for the 4.5° arm and from $\pm 0.6\%$ to $\pm 3.8\%$ for the 7.0° arm.

Figure 3.22: Diagram of the H1U hodoscope plane used for E143.

Each finger ended with a lightguide that fed the signal into a phototube. Phototubes used for the H1U plane were Hamamatsu H3167 tubes with a $3/4$ " diameter. For the remainder of the planes, the tubes were $1/2$ " Hamamatsu R4014 phototubes. High voltage for the phototubes on each finger was supplied by LeCroy HV4032A power supplies which were upstairs in the counting house. For this reason, more than 120 SHV coaxial cables were stretched from the counting house down to the spectrometers in ESA. The voltage lines were split and powered four phototubes each. Figure 3.22 shows the front half of the H1U hodoscope plane with the lightguides and phototubes attached to each finger.

The signals from the phototubes were processed by LeCroy 4418 discriminators mounted in two CAMAC crates in each spectrometer hut. Discriminator

Table 3.1: Specifications of the various hodoscope planes. Where the numbers differed for the 4.5° and the 7.0° hodoscope planes, the left number describes the 4.5° plane and the right number describes the 7.0° . The u-hodoscopes had fingers tilted at 45° from the top left side to the bottom right side, when viewed from upstream looking downstream. The timing resolutions were those assumed by the tracking program and not necessarily the actual ones.

Plane	Built by	No. of fingers	Width (mm)	Length (mm)	Timing Resol. (ns)
1 u	Tohoku	25	45	200-740	1.4/1.3
2 x	Syracuse	34/23	20/30	589/690	1.1
3 y	Syracuse	31/36	30	430	1.1
4 y	SLAC	20	47.6	356/483	1.4/1.0
5 x	Syracuse	27	30	1070	1.1/0.9
6 y	Syracuse	55	30	510	1.1/1.0
7 u	Saclay	21	75	200/820	1.4/1.2

thresholds were -30 mV for H1U and -50 mV for all the rest. ECL output signals were then sent back to the counting house with twisted pair cables so as to preserve signal height and minimize cross-talk. Signals were then sent through a gate card, which accepted signals only when the gate was open, and into a LeCroy 2277 TDC that could process up to 16 signals per channel.

Hodoscope Efficiency

The efficiency of the hodoscope detector package was determined by identifying electron events using only the shower counter and the Čerenkov detectors and then finding how many of these electrons were identified by the tracking system (Čerenkovs, hodoscopes, shower counter, electronics, and tracking program). In particular, an electron was tagged by a MAIN-TRIGGER, a Čerenkov ADC height of at least 25, and a neural network response 3.5.4 of at least 0.95. For these electrons, a track was required to be within 62 mm and 10 ns of the shower cluster. The percentage of electrons that did not have such a track, or

the detector inefficiency, was 9.2% for the 4.5° spectrometer and 3.5% for the 7° spectrometer [111].

Finding Tracks

The two sets of hodoscope planes were used, along with the position of the time-correlated hit in the shower counter, to find a track. Since the hodoscope signals were only recorded if a particle was above threshold, it was necessary to match hits with those from other planes and fingers. To calibrate the timing for all fingers, a track was found and the time of each hit on the track relative to the overall trigger was recorded. These times were corrected for propagation delays in the fingers and distances between planes. The timing distribution of each finger was then fit with a Gaussian, and the mean value was used as a timing correction to the hits coming from that finger. In general the corrections were less than 0.5ns.

The tracking code, written originally for E142 and then adapted for E143, reconstructed particle events using hits from each detector. Hits in the shower counter and the Čerenkov counter were treated as if they were really hits on a hodoscope located at the position of each detector. Furthermore, the following information was used by the tracking algorithm in finding tracks:

- The **Čerenkov** hit time was given by the TDC values passing the lowest discriminator thresholds, Č1L and Č2L. The timing resolution used was 1ns. Hits were placed on an imaginary horizontal finger at the Čerenkov z -position. Since the tanks cover the spectrometer acceptance, a 100m spatial resolution was used.

- The timing resolution varied for each **hodoscope** plane, and is listed in Table 3.1. The spatial coordinates for each finger were defined by the position of its center, the length of the finger, and the direction the fingers point. The

Table 3.2: List of the various track classes defined by the tracking algorithm. Class one tracks were found first, followed by two, and so on, until all the hits were used or found to not belong to a track.

Track Class	Required # of hits:			Total
	Čerenkov	Hodoscope	Shower	
1	2	4	2	8
2	2	4	0	6
3	0	4	2	6
4	0	6	0	6

spatial resolution was taken to be the width of the finger divided by $\sqrt{12}$.

- Since the **shower calorimeter** clusters were only recorded if there was a MAIN-OR trigger (see Section 3.6.5), and since the counter did not have TDCs, the time of the hit was the same as the time of the MAIN-OR trigger, with a resolution of 1ns. The position of the hit was represented by two hodoscope plane hits, x and y , located at the center of the cluster, with a resolution of 10mm. The z -position was set to be zero at the counter face.

The tracking program reconstructed four different classes of tracks. The requirements for a track to be put into a certain class were some number of hits in time in each detector. Table 3.2 defines the track classes. We required an electron event to have a class 1 track, requiring both Čerenkov detectors to fire (2 hits), a shower counter cluster (2 equivalent finger hits), and four hodoscope planes to fire (4 hits). A pion event was likely to only fire the hodoscopes, since the Čerenkov tanks were set to reject pions and since pions shower inefficiently in the electromagnetic calorimeter.

To reconstruct a track, first a set of hits was selected in time. Then the χ^2 of each hit compared to the track was defined in the following manner: Let t_{tr} be the track time, or the best average time for the selected hits, each hit time having been adjusted for the time to reach the position of the detector element that fired. Let t_i be the time of the i th hit. Also, let δs_{min} be the

closest distance of the track to the detector element. Then the χ^2 of the i th hit with respect to the track is

$$\chi^2 = \left(\frac{t_i - t_{tr}}{\sigma_t} \right)^2 + \left(\frac{\delta s_{min}}{\sigma_s} \right)^2 \quad (3.17)$$

where σ_t and σ_s are the time and spatial resolutions of the detector element.

The tracking algorithm selected a set of hits for each track class using only hits that had not yet been included in a track, and that had not been marked as unusable for that track.

- **Class 1** track hits were found by finding a hit in Čerenkov 2 that was close in time to the Čerenkov 1 hit, with a $\chi_i^2 < 16$. Then a shower cluster in time was found, as well as all hodoscope hits with a $\chi_i^2 < 16$.
- **Class 2** track hits found matching Čerenkov hits, as above, but then added hodoscope hits. Shower hits were added in the end, if there were any. If such hits were found, the track was reclassified before accepted.
- **Class 3** track hits started with a shower cluster and then added Čerenkov and hodoscope hits in time.
- **Class 4** track hits were selected from any hits with similar time tags. If the minimum was found for a class 4 track, a track was built from these.

Once the hits were selected, the time of each hit was compared to its track time using equation 3.17, setting $\delta_s = 0$. Fitting was done until all saved hits had $\chi_i^2 < 16$. Other hits were thrown out and marked as unusable. Then the tracking program compared both time and space using all of equation 3.17. Hits with $\chi_i^2 > 16$ were dropped and the remaining hits were fit to a new track. Once all the hits fit the track in time and space, the track was saved. Finer details of the tracking algorithm as well as a flow chart can be found in reference [112].

Tracking Efficiency

The efficiency of the tracking program itself was defined to be the percentage of times that the program found a track when the various counters provided enough good hits for a track: ie– the MAIN-TRIGGER fired, the Čerenkov ADCs were at least 25, and the neural network response was at least 0.95 as before. In addition, only one cluster was allowed per trigger so that tracking was not confused by *ghost clusters*, or clusters that came during an overlapping trigger time. Also, no clusters were allowed at the edge of the shower counter, and for each cluster, there were enough hits in the hodoscopes within the time of the cluster to be able to make a track. The tracking program inefficiency was determined in this manner to be 1.8% for the 4.5° spectrometer, and 1.0% for the 7° spectrometer.

3.5.2 Scintillation Trigger Counters

We used two scintillation trigger counter detectors in each spectrometer for E143, one placed in front of each hodoscope array. The coincidence of these counters served as a trigger for the hodoscope gate and for pions, the rate being prescaled by various amounts as described in section 3.6.2. The trigger counters were made from Kuray SCSN38 scintillator, which fed into two phototubes at each end via light guides, four phototubes total.

The front counter was placed between the first Čerenkov tank and the hodoscope H1U plane. It was 700 x 450 x 6 mm, and was read out by Hamamatsu H3690 phototubes. The two pmt outputs at each end were combined with a linear fan-in, and then were discriminated with a 30 mV threshold. The coincidence of the outputs from each end was sent to the trigger electronics. The fan-in outputs were also sent to a TDC separately.

The rear counter was placed between the second Čerenkov tank and the

Hodoscope H5X plane. It was 1110 x 550 x 13 mm, and the signals were sent through twisted acrylic light guides and into Hamamatsu H1161 phototubes. The outputs of these phototubes were also combined using a linear fan-in, and then were discriminated with a 50 mV threshold. The coincidence of these discriminator outputs was sent to the trigger electronics as well, and the signals from the rear counter were also recorded by a TDC.

The timing resolution of both the front and rear trigger counters was determined using cosmic rays along with two other counters placed in front of and in back of each counter. The resolution of the front counter was found to be 0.71 ns and the resolution of the rear counter was 0.56 ns.

3.5.3 Čerenkov Counters

For a complete description and analysis of the threshold Čerenkov counters used for part of the MAIN-TRIGGER, see Chapter 4.

3.5.4 Shower Calorimetry

The shower calorimeters were used to measure the energy of the electrons, to help provide an online electron trigger, and to identify electrons using both a neural network algorithm and an energy E versus momentum P cut, detailed in Chapter 5. The counters were composed of 200 lead glass blocks of dimension 62 x 62 x 750 mm stacked 20 high by 10 wide, as in Figure 3.23. Schott type F2 glass having 41.8% lead content by weight, an index of refraction of 1.58, and a *radiation length* of 31.7mm was used for the blocks. [After traveling through one radiation length of material, the electron has approximately $1/e$ of it's original energy left.] The electrons saw approximately 24 radiation lengths of lead glass and so lost more than 99.9% of their energy in the glass.

Figure 3.23: Diagram of the lead glass shower calorimeter.

To measure the energy accurately, it was necessary to stop the electrons completely in the counter, so the calorimeters were placed at the ends of the spectrometers. High energy electrons passing through the spectrometer produced an electromagnetic shower upon entering the lead glass blocks. The shower is primarily due to bremsstrahlung processes, and produces a large number of electrons, positrons, and photons. The electrons and positrons from the shower then lose their energy through the ionization of atoms in the glass and the emission of Čerenkov light. The amount of Čerenkov light emitted is proportional to the total path length of the leptons in the shower, which is then proportional to the energy of the initial incident electron. Thus for the energy measurement, the light emitted was collected at the rear of each block using photomultiplier tubes. Due to sampling fluctuations, the energy resolution in the shower counter is proportional to $E^{-1/2}$, and is better at high energies as long as noise and calibration errors do not enter.

Figure 3.24: The phototube output was split into 5 signals. Four went to the ADCs and one was used for online triggering and TDC readings.

The less predominant process by which the electrons lose their energy is through ionization. All heavier charged particles such as pions lose their energy via this mechanism as well. The Bethe-Bloch formula [113] which depends on the atomic number, atomic weight, and density of the absorbing material describes the energy loss. The particles collide with and excite atoms in the lattice, sometimes causing ionization and other times mere deflection. The material's effect on a particle depends on the nuclear interaction length λ_I , or the mean free path:

$$\lambda_I \approx 35 \frac{g}{cm^2} A^{1/3}.$$

Each phototube output signal was split into five signals as in Figure 3.24. One of the signals was split again to be sent to a TDC and to be used in the MAIN-TRIGGER and in the Efficiency Triggers. The other four signals went to an ADC via a 200 ns delay cable. Each ADC channel was read and the buffer was initialized after every spill, allowing each channel to collect data once per 2.2 μs .

Energy Calibration

The light collected in the phototubes attached to each lead glass block is proportional to the energy deposited by the particle. To determine the energy of the electrons, we had to calibrate the blocks by finding the constant of proportionality, K_i^j :

$$E_i^j = K_i^j \times ADC_i^j \quad (i = 1, 200 \quad j = 1, 4)$$

where E_i^j is the energy the electron deposits in the j th trigger of the i th block during a spill, and ADC_i^j is the corresponding ADC value obtained. Each calorimeter contained 200 blocks with four ADC channels per block, for a total of 800 constants were needed per detector.

Because the electrons deposited basically all of their energy in the lead glass, the ratio of the energy to the tracking momentum $E/P=1$. To determine the electron energy, a Cellular Automata [114] program was used to determine what constituted a cluster. The sample of clusters used to calibrate the blocks for electron energy had to have no pion contamination, as for pions, $E/P < 1$. To be certain they were electron clusters, we required that a track be associated with a cluster within 10ns in time and 50 mm in x and y; that the energy deposited in the central block was greater than 5 GeV; that at least 65% of the cluster energy was deposited in the central block; and that the cluster was designated an “electron cluster” by the neural network (see Neural Network section to follow).

Since all of the energy of an electron is deposited in the central block and the surrounding eight blocks, we required that the total energy deposited in the nine blocks have $E/P = 1$. The block constants were corrected with each iteration as:

$$(K_i^j)' = \frac{K_i^j}{\langle E/p \rangle_i^j}$$

where $\langle E/P \rangle_i^j$ is the averaged E/P for that iteration and $(K_i^j)'$ is the new constant.

For clusters centered on edge blocks, $E/P \neq 1$ in general because some of the shower energy is not collected. The edge block and the five adjacent blocks contain a certain fraction of the energy however, and this was estimated from the calibrated non-edge blocks to be about 92%. Therefore, the constant was determined by:

$$(K_i^j)' = \frac{0.92K_i^j}{\langle E_{edge}/p \rangle_i^j}$$

where $\langle E_{edge}/p \rangle_i^j$ is the average energy versus momentum deposited by the six blocks instead.

The edge blocks were *also* calibrated using the fact that the edge block contains about 6.5% of the total electron energy when the electron passes through the adjacent block:

$$(K_i^j)' = \frac{0.065K_i^j}{\langle E_{edge}/p \rangle_i^j}.$$

For these two methods, whichever method gave the smallest correction than the other was selected so as to avoid the divergence of the constant due to statistical fluctuations.

Furthermore, corner blocks were calibrated using events in which the electron passed through the inside diagonal block. In this case, the E/P for the corner block was found to be about 2% of the total energy, giving:

$$(K_i^j)' = \frac{0.02K_i^j}{\langle E_{corner}/p \rangle_i^j}.$$

Finding Clusters

To reconstruct a particle shower it was necessary to determine the energy and shape of each particle cluster. This consisted of scanning the calorimeter for *hit blocks* and then finding the clusters using a particular algorithm. A *hit block* was defined to be a lead glass block in which at least 50 MeV was deposited, so as to reduce backgrounds due to photomultiplier noise and ADC pedestal fluctuation. The reconstruction algorithm used for the clusters was based on a **cellular automaton** [114] which provided for the separation of overlapping clusters.

In this case, the cellular automaton (CA) used had 200 cells, one for each of the lead glass blocks. The position of the cells corresponded to the spatial position of the blocks in the detector. For each trigger, the energy deposited in each block was used as the cell's initial state. Then the cell values evolve according to three rules:

- **Rule 1:** A given cell is only sensitive to its eight neighbors. If a cell has a higher value than each neighbor, it's considered a source cell, as well as the center of the cluster.
- **Rule 2:** At a given step a cell will take the value of its highest energy neighbor.
- **Rule 3:** A cell whose state has already been changed by a *source cell* in an earlier stage can no longer be changed.

Utilizing the above rules, the algorithm is able to separate most overlapping clusters provided that the source cells are not adjacent. Figure 3.25 shows an example using a test sample of two overlapping electron showers from Geant 3.16. Here each box represents a CA cell. Using rule 1 on the initial state, the cells containing 7.0 and 8.0 are source cells, which means they are the center of the cluster. Using rule 2, we reach the second state in which all of the cells neighboring the source cells take on the source value. In state two, however, a steady state has not been reached, so it evolves further into state three, the

			0.2		
			3.0	1.0	0.2
0.3	0.2	0.4	7.0	2.0	0.2
2.0	8.0	1.0	0.4		
0.2	0.6	0.3	0.2		

Initial State

			3.0		
			7.0	7.0	2.0
8.0	8.0	8.0	7.0	7.0	2.0
8.0	8.0	8.0	7.0		
8.0	8.0	8.0	1.0		

State Two

			7.0		
			7.0	7.0	7.0
8.0	8.0	8.0	7.0	7.0	7.0
8.0	8.0	8.0	7.0		
8.0	8.0	8.0	8.0		

State Three

Figure 3.25: Example of two overlapping clusters being resolved using the rules of the cellular automaton algorithm.

Figure 3.26: Diagram of the multi-layered neural network. The lines joining the inputs to the hidden layer neurons, and the hidden layer to the output, are synapses through which information is passed.

final state. Here rule 3 prevents the CA from evolving further so that the 7.0 cells do not become absorbed by their 8.0 neighbors.

Once the clusters are identified, the energy of the particle is found by summing the actual energy deposit in each block of the cluster. However, the blocks located between the two source cells of overlapping clusters shared their energy. The performance of the CA was compared to a more standard “vector approach” [114] and it was found that the CA method is much more efficient when the cluster pile-up rate increases.

Neural Network

To differentiate an electron from a pion without using tracking and only using information from the shower calorimeter, we developed a neural network [114]. The network originally contained 13 input cells, and had two

outputs corresponding to pion-like or electron-like events. The input cells had *synapses* connecting them to each neuron in the second, hidden layer through which information was passed, as shown in Figure 3.26. The signal passed was either to “excite” the neuron in the next layer or to calm it, using +1 or -1 respectively, depending on whether the value of the input cell was typical of an electron or not. Then, the condition of a neuron O_j in the second layer was determined by a Sigmoid function summing up the information from the input cells:

$$O_j = \frac{e^{ka} - 1}{e^{ka} + 1} \quad a = \sum_i W_{ij} x_i,$$

where x_i is the condition of the input cell, and W_{ij} is an amplitude of the signal from the cell to the neuron, serving to weight the importance of the synapse signal. To obtain these weights, the neural network had to be trained using 3000 sample electron clusters and 3000 pion clusters from a Geant simulation of the calorimeter. The next layer consisted of two outputs, and was determined similarly using as input cells the neurons in the second, hidden layer.

For our particular situation, we wanted to distinguish between electron and pion clusters, so we used as inputs properties of the shower signals that gave us information as to whether the particle’s shower was like that of a pion or an electron. The inputs were:

- $V_1 = E_{tot}$: total energy in the 9-block cluster.
- $V_2 = E_5/E_{tot}$: ratio of the central block E_5 to total energy.
- $V_3 = E_{16}$: energy in the ring beyond the 9 blocks.
- V_4 to $V_{12} = E_1$ to E_9 : energy in each of the 9 blocks.
- $V_8 = E_5$: energy in the central block.
- $V_{13} = N_{blocks}$: number of hit blocks in the cluster.

The training was done so that the weight values W_{ij} were modified during

Figure 3.27: Sample output of the neural network for a typical run. Note the log scale. The neural network placed most clusters into the pion or the electron category, with values close to -1 or +1 respectively.

each iteration according to an algorithm using the error function, E :

$$E = \sum_{m=1}^N (O_m - d_m)^2$$

where O_m is the output for the m th cluster and d_m is the desired output. Then the weight factors are changed at each iteration t using the gradient back propagation [114] of E :

$$W_{ij}(t) = W_{ij}(t-1) + \Delta W_{ij}(t),$$

$$\Delta W_{ij}(t) = -\eta \frac{\delta E}{\delta W_{ij}} + \alpha \Delta W_{ij}(t-1).$$

Here η and α are learning parameters.

To keep the neural net as fast and simple as possible, the number of input

Table 3.3: Efficiency for electron identification and pion contamination using the neural network response.

	4.5°	7.0°
Efficiency (%)	98.1 ± 0.0	98.8 ± 0.0
Pion Contamination (%)	0.3 ± 0.0	0.3 ± 0.0

cells as well as the number of layers can be optimized according to how much information is needed. After the neural net was trained and the weights W_{ij} were determined, we were able to eliminate four input cells, V_4 , V_6 , V_7 , and V_{12} because the discriminating power of these were found to be weak. Thus, we ended up with a first layer of 9 input cells, a second layer of 4 neurons, and a third layer of two outputs. The output of the neural network gave values between -1 and +1, depending on the confidence level as to if the particle was a pion (-1) or an electron (+1). Figure 3.27 shows the distribution of output from the neural network on a log scale. The majority the values are close to one end or the other, indicating a very good ability to discriminate between the two particles.

The efficiency for electron identification and pion contamination was determined by defining an electron as having both Čerenkov ADC's above 40 and an E/P ratio between 0.8 and 1.2, and by defining a pion as having both Čerenkov ADC's less than 40 and an E/P ratio less than 0.8. The efficiencies obtained are shown in Table 3.3. These are higher than the $\sim 90\%$ efficiency for electron ID and $\sim 10\%$ pion contamination obtained using a conventional method [114].

3.6 Electronics and Triggers

For E143 most of the electronics modules were located in the ESA counting house. Each spectrometer had separate but almost identical electronics packages. Signals from each of the detectors were sent into logic gates that formed various triggers. These triggers were sent into an overall trigger called the MAIN-OR, and whenever the MAIN-OR fired, data were written to tape. Because of expected high rates in the experiment, four MAIN-OR triggers per beam spill were accepted. The setup is described below.

3.6.1 Hodoscope Output

The signals from the hodoscope fingers were sent into LeCroy 4418 discriminators which were the only modules located downstairs inside the spectrometer huts. Discriminators were necessary to reduce noise from the fingers by accepting only strong enough signals that were likely due to a real particle. The signals passing threshold values were sent upstairs to the counting house via cables that I specifically selected for good signal transmission, after testing many designs. Each signal cable contained 17 twisted-pair cables which reduced cross-talk (noise) between the wires. Since the hodoscope signals were then sent to multihit TDCs and no ADCs (see HODOGATE section below), the information was sent as ECL signals to further prevent degradation of the pulse shapes and hence timing information.

In addition, each discriminator module had an overall OR output in which a signal is formed if any of the channels receives a signal above the set threshold. The signals from each plane's OR were then sent via HELIAX cable to the counting house and into the scintillator coincidence logic described in Section 3.6.5.

3.6.2 Scintillator Output

The signals from the four phototubes in each scintillator plane were added and sent to a discriminator. The ADC signals from the fingers of the H2X hodoscope plane were added and sent to a discriminator as well. The output of these three discriminators were then sent through a logical AND, the output of which was called **S**, the scintillator trigger. This was the most general trigger possible, firing when almost any particle went through the spectrometer. To prevent the **S** triggers from overwhelming the electronics and DAQ, the signal was fanned out into four other signals, each of which was prescaled (divided by) a particular number, **N1**, **N2**, **N3** and **N4**, thus reducing the number of triggers read. The value of the numbers changed depending on the experimental conditions at the time, but the **N1**, **N2** and **N3** factors were always identical, and the **N4** factor was always set to a lower or equal value. For the run-dependent pre-scaler values, see Reference [115]. The outputs of the scaled scintillator trigger were labeled **S/N1**, **S/N2**, **S/N3** and **S/N4** and were used as inputs to the trigger logic. The rates were also tracked to see if the prescaler values needed adjustment.

3.6.3 Čerenkov Output

The phototube output signals from the four Čerenkov counters were sent to the Counting House through HELIAX. The signals were then split with a fan-out module and sent into one of four ADCs (one for each of four possible triggers per spill) and into four discriminators having low, medium, high, and very high threshold settings. These were denoted for the first Čerenkov as **C1L**, **C1M**, **C1H**, and **C1VH**, and similarly for the second Čerenkov, **C2L** and so forth. Each discriminator output was then split and used as input to trigger logic as well as to a dedicated TDC for each level. See Figure 4.13 in Section 4.2.4.

3.6.4 Calorimeter Output

The output of each phototube in the shower counter went via 50 Ω foam cables to the counting house, where the analog signal was split into 5 parts, each about 17% of the original. Four of these signals were delayed 200ns so that they would fall within the HODOGATE, and then sent into four separate ADC crates gated for each of up to four triggers per spill. The digitizing was performed by LeCroy 2282 12 bit ADCs, each crate being controlled by a LeCroy 2280 processor. The fifth outputs of each block were summed by a series of LeCroy 628 Linear Fan-Ins so that the summed signal was proportional to the total energy deposited in the calorimeter. The summed output went to a discriminator with five different threshold settings: **SH[VL]**, **SH[L]**, **SH[M]**, **SH[H]**, and **SH[VH]**. The signal output from each threshold went to a dedicated TDC channel and then was used for various triggers as well as online scaler counters.

In addition, signals from four non-adjacent lead glass blocks were combine and sent to discriminators and TDC modules so that more accurate cluster timing would be possible. However, the timing measurements were not exploited during setup and running and so were never fully operational. Hence, this timing information was excluded from the data analysis.

3.6.5 Triggers

Electron event reconstruction required a large amount of information from the ADCs and TDCs. Because of constraints on electronics and data acquisition speed, it was desirable to limit the amount of information allowed into the system. First, we allowed only the first four triggers per spill to be recorded by the ADCs, and the first 16 triggers were recorded by the TDCs. Also, we only allowed information to flow to the DAQ during the BEAMGATE (**BG**).

Figure 3.28: Basic E143 trigger logic. Pion-Or triggers when any charged particle passes, but note that it is suppressed by prescalers. Main-Or triggers after a logical “Or” of the Pion and Main triggers. Hodogate triggers and opens TDC gates when any charged particle enters, as well.

At the start of each spill the **BG** signal was formed by a logical AND of the **A2N** signal sent by the Main Control Center (MCC) to ESA and a computer ready/run status signal. The **A2N** signal arrived $2.5 \mu\text{s}$ before the electron spill, which was typically $2.3 \mu\text{s}$, and lasted for $6 \mu\text{s}$, thus covering the entire spill. For this reason the **BG** signal was used as input to gate logic for the electronics and for computer interrupt for DAQ. The TDCs were reset on the leading edge of the **BG** signal, at the start of each spill. For calibration and pedestal runs, a **BG** signal was fabricated to allow the flow of data.

The basic idea of the trigger logic was to accept all electrons, typically 1 per spill, depending on running conditions. Since pions were far more abundant than electrons, we wanted to open the ADC and TDC gate for *all* electrons but for only a fraction of pion events, so as to reduce deadtime (see section 3.6.5). The basic structure of the E143 trigger electronics is shown in Figure 3.28.

HODOGATE

The HODOGATE (**HG**) was used to trigger the hodoscope TDCs. Each TDC was started by the **BG** and then gated by the **HG**. The (**HG**) was formed by a logical OR of $[\text{S}/\text{N4} \cdot \text{C1M}]$, $[\text{S}/\text{N4} \cdot \text{C2M}]$, $[\text{C1L} \cdot \text{C2L}]$, $\text{S}/\text{N1}$,

S/N2, and **S/N3** followed by a logical AND with the **BG**. Each finger of each hodoscope plane was attached sent through a discriminator and then into a TDC channel. Every channel could record up to 16 values with a 1ns resolution and a total dynamic range of 65 μ s. This range was plenty large because the TDCs were read and cleared at the end of every 2-3 μ s spill.

MAIN TRIGGER

The main electron trigger, or MAIN TRIGGER **MT**, was the primary signal of an electron passing through the spectrometer. The **MT** was made from the logical and [**C1L** · **C2L** · **SH[L]**]. This was used because the Čerenkov counters were set with a pion threshold such that only a small number of high momentum pions radiated light in the counter. On the contrary, all electrons emitted light in the counters. Requiring that both C1 and C2 fired eliminated most pions. Requiring that there be a cluster in the shower calorimeter above the low threshold further got rid of noise and pion background. The **MT** was used as one of the inputs to the MAIN-OR (see MAIN-OR section below).

PION-OR

The PION-OR was included in the MAIN-OR trigger to record possible pion events. As mentioned previously, only a percentage of pion events were recorded so as not to inundate the electronics and the DAQ. To facilitate this, the pion trigger was in general composed of the prescaled scintillator signals. It was formed by a logical OR of **S/N1**, **S/N2** · **SH[VL]**, and **S/N3**. However, since during E143 the values of **N1**, **N2**, and **N3** prescalers were always the same, the signal was the logical equivalent of **S/N1** and thus served as a trigger for any particle. The label PION-OR was still appropriate since pions dominated our rates in the E143 kinematical region. The PION-OR was used as another input to the MAIN-OR.

Efficiency Triggers

The efficiency triggers were used to measure how well one of the detectors signaled an electron when the other triggers indicated that there was an electron. The 2m Čerenkov (C1) efficiency trigger consisted of the logical AND: $[S/N4 \cdot C2M \cdot SH[M]]$. The 4 m tank (C2) efficiency trigger consisted of the logical AND: $[S/N4 \cdot C1M \cdot SH[M]]$. The shower counter efficiency trigger was made from the logical AND: $[S/N4 \cdot C1M \cdot C2M]$. Note that the medium setting on the discriminator was used for these to help ensure that the signal was actually from an electron. These triggers were sent into the MAIN-OR as well, and the ADC values were recorded on tape.

MAIN-OR

The MAIN-OR trigger used with a logical AND of the **BG** was used to decide if an event would be written to tape. This opened up the ADC gates for the shower counter energy measurements and the Čerenkov ADC spectra. The MAIN-OR consisted of a logical OR between five triggers: the **MT**, the PION-OR, and the three Efficiency Triggers (see Figure 3.29). Therefore, in the case of a probable electron or a pion (prescaled), data were recorded from the spectrometers. Since the Čerenkov counters and the shower counters had no TDC measurements, we used the time of the MAIN-OR trigger for the time of the shower clusters and of the events that fired the Čerenkov detectors. The MAIN-OR could accept up to four triggers per spill as limited by the shower counter electronics.

Figure 3.29: Logic design of the Main-Or trigger which fired for a prescaled number of random charged particles (Pion-Or) and for electrons passing through the spectrometers. The Main-Or trigger opened up the ADC gate for the Čerenkov counters and the shower calorimeters.

3.7 Data Acquisition

Data readout and control of the experiment was done from the upstairs “counting house” of EndStation A (ESA). The central data acquisition (DAQ) computer was a VAX 4000/200 running VAXeln, which is a realtime operating system. It read ADC and TDC information from the CAMAC crates via a QBUS connected to three Jorway interfaces that were attached to the beam and Møller CAMAC racks, the 4.5° spectrometer CAMAC rack, and the 7.0° rack. The computer ran a program called DatServ which read first the beam data, then the spectrometer data. Approximately every five minutes, at “check-points”, it would also read in special data like pedestals, target polarization, beam polarization, and so forth. The DAQ computer had no monitor, so a VAX 3100 was used as a terminal for some amount of control, monitoring, and debugging of DatServ. After it read the data it sent them over a dedicated network to a DEC 4000/60 Vax Station which ran the TapeServ program on VMS. Due to DECNET limitations Datserv could only send 4096 bytes of data between spills to TapeServ, so if rates became high the spills would get truncated. This was monitored throughout the experiment on the run sheets.

The dedicated network between the DAQ VAX and the TapeServ workstation was connected to the Ethernet ESA subnet across a bridge that shielded the two stations from extraneous traffic. The experimenters interacted with DAQ through a program called DAQCNTL which was run on a VAX 4000/300 that was connected to the subnet side of the bridge. The experimenters could record pedestals, choose the run type, stop, start and pause runs, mount and eject tapes, control the Møller foil positions, take LED (spectrometer) and toroid calibration runs, and so forth using DAQCNTL. In addition, the DAQCNTL workstation interfaced with CAMAC racks holding magnet controls, power supplies, the scalers, and certain target monitoring modules. Programs for magnet and voltage control, however, were run on dedicated terminals in the counting house.

The Vax Station running TapeServ was connected to three EXABYTE 8mm tape drives via its SCSI bus. One drive was used to record data, another to allow a full tape to be ejected and a new tape to be mounted, and the third was dedicated to recording the Møller runs. In addition, the TapeServ machine was connected to four other workstations that sampled data from TapeServ for the purpose of online analysis. The electronics modules and other hardware of the two spectrometers were monitored by VAX 4000/90 machines using online analysis code that was so detailed and time intensive that only a couple of spills per second could be processed. Only about 3% of the data was sampled, but this was enough for monitoring the detectors. The beam and Møller analyses were run on separate VAX 4000/60 workstations. The beam analysis was able to look at about 50% of the spills which was important for monitoring the beam raster pattern and the position and stability of the beam spot.

Control and monitoring of the target was done on a Macintosh Quadra system running LabView software. The experimenters could use LabView to change the target insert position, to control the refrigerator and target field magnets, and to monitor target polarization and temperature. Cables from the ESA target apparatus were connected to the Macintosh system through a GPIB interface. LabView was also used to send information to the DAQCNTRL program through a FIFO connection to a control module in a CAMAC crate.

Chapter 4

The Čerenkov Counters

The four Čerenkov counters in E142 and E143 were crucial in determining the electron events we would trigger on. Furthermore, I was responsible for their setup and operation in E143, having been handed the baton by David Kawall, then a Ph.D. candidate from Stanford University. These same detectors were modified and lengthened, and then used for SLAC experiments E154 and E155, so their configuration and performance was important for four separate Endstation A experiments. Likewise, they will be used for E155X, scheduled to run in 1998. Here I devote extra space for a careful description of the design and performance of these detectors.

4.1 Design Choices

4.1.1 Designer

The E142/E143 Čerenkov detectors were designed and tested by David Kawall, mentioned above, of Stanford University, working under advisor Dr. Zein-Eddine Meziani, who was an Assistant Professor at Stanford University at the time. Professor Meziani is currently at Temple University in Pennsylvania.

4.1.2 Purpose of Čerenkov Detectors

For the E142 and E143 asymmetry measurements, it was necessary to select a clean sample of scattered electron, and to reject a large background of pions and other debris from the collisions. Therefore, in designing the spectrometers, we chose to use two threshold Čerenkov detectors in each of two spectrometers, which were placed at 4.5° and 7.0° with respect to the incident beam direction [94] as described earlier. See Figure 4.1. These detectors were configured to signal the passage of electrons through them, while ignoring all but the highest momentum background pion particles. They were then used in coincidence with the lead glass shower counter to provide a high efficiency electron trigger, referred to as the **Main Trigger**.

4.1.3 Pion Thresholds

Reasons for Threshold Values

As stated previously, the spectrometer package used for E143 consisted mostly of detectors originally used in SLAC E142 [94]. Since it was expected that the number of pions scattered into the spectrometer, per electron, would range

Figure 4.1: Two-arm spectrometer for E142/E143.

from 14 at pion momenta of 7 GeV/c dropping to 0.4 per electron at pion momenta of 13 GeV/c [90], a pion threshold of 13 GeV/c was chosen for the larger Čerenkov tanks in each spectrometer, referred to as “C2”. This means that in general only pions with momentum of 13 GeV/c or higher would produce a signal in the detector, and this high threshold would greatly limit the number of pions entering the data stream. The smaller tank, “C1” was designed with a 9 GeV/c threshold for pions. C1 and C2 were used in coincidence for triggering on electrons to provide a clean electron sample and to reject random pulses.

During the initial design it was thought that E143 would use the same incident beam energy as E142, 22.66 GeV, and hence would have similar backgrounds. Yet as the experiment approached, it became clear that E143 would be able to achieve a highly polarized beam with an energy of 29.11 GeV. At this beam energy, the pion rate per electron was expected to range from about

30 at 6.5 GeV/c to almost zero at 25 GeV/c. Because of the higher pion-to-electron ratio, we investigated raising the 13 GeV/c pion threshold on the larger tank. For a gas radiator at constant temperature, $p_{th} = 1/\sqrt{\bar{P}}$, with p_{th} the momentum threshold and $\bar{P}$ the pressure. This relation can be obtained using the ideal gas law, equation 4.1, and equation 4.2, the Lorenz-Lorentz relation. To raise the threshold momentum, we would have had to lower the pressure in the detector from an already low 2.99 psia.

Lowering the pressure much below 3 psia, however, posed a problem. Both the phototube and the high voltage base were mounted inside the tanks, and the base was known to arc and become noisy at pressures below 3 psia. Our tests showed that the base could even burn out at these low pressures. We then investigated the possibility of building a container to enclose the bases and maintain them at atmospheric pressure. Unfortunately, we had limited time between E142 and E143, and we found that designing a mechanically sound container that could withstand such a pressure differential and could maintain a seal against the delicate phototube glass was a time-consuming venture, and there would not have been time to design, build, and test it.

Furthermore, we calculated that at 29 GeV, raising the threshold of C2 to 14.5 GeV/c, which would correspond to a pressure of about 2.4 psia— the lowest pressure in which the high voltage base could function, would only decrease the number of pions triggering the Čerenkov detectors by about 2.5% relative to the 13 GeV/c threshold. This 2.5% would further be reduced by our main electron trigger anyway, in which we require that the shower counter fire in coincidence with the two Čerenkov tanks. For these reasons, we decided that we would leave the threshold of C2 at 13 GeV/c. Likewise, we left the threshold of the smaller tank, C1, at 9 GeV/c. We then investigated the use of positive high voltage bases to see if the arcing problems would be decreased in this configuration. In my studies, I found that the positive high voltage bases performed slightly better than the negative high voltage bases while in low pressures, so we chose to use these during E143. However, once the bases

were put in the low pressure tanks, they seemed to arc and burn out more readily than the original negative bases, so we switched the phototube bases back to the negative ones for data taking.

Radiator

The desired threshold for pions in the Čerenkov detector determines the index of refraction needed, and hence the appropriate radiator can be chosen. For Čerenkov light to be emitted by the pions we have the following requirement:

$$\cos \theta \leq \frac{1}{n\beta_\pi} \quad (4.1)$$

At the threshold for Čerenkov emission, $\cos \theta = 1$. Here θ is the angle of emission of Čerenkov photons with respect to the particle's trajectory and β_π is the ratio of the velocity of the pion to the speed of light in a vacuum v_π/c . We can also relate β_π to the desired pion threshold momentum p_π using:

$$\beta_\pi = \sqrt{\frac{p_\pi^2}{p_\pi^2 + m_\pi^2}}.$$

Then for a 13.0 GeV/c threshold, $\beta_\pi = 0.9999424$ ($\beta_e \approx 1$). Using equation 4.1 we see that this requires a dielectric with an index of refraction $(n - 1) = 5.763 \times 10^{-5}$, which suggests a gas radiator.

In choosing a gas radiator, one has to look at how the gas behaves under the conditions needed. Ideally, the gas should transmit the Čerenkov light efficiently to the phototube, absorbing very little within the wavelength range of the radiated light. Also, since the radiator gas can potentially scintillate, sending non-Čerenkov light into the phototube which results in false signals, we want the radiator to have minimal scintillation properties. Furthermore, knock-on electrons, or δ -rays, can be emitted when the incoming electrons strike the radiator molecules and then can produce false signals as well. For

Figure 4.2: Transmission curves for several potential gas radiators.

this reason, we want a fairly light gas.

Originally CO₂ was looked at for the choice of radiator gas because it has a high Čerenkov radiation transmission from 190 nm to 700 nm [116], which is in the desired wavelength range. CO₂ also has low scintillation properties [117] and knock-on rates. However, once it was determined that the phototube output improved with the use of a wavelength shifter, p-terphenyl (discussed in section 4.1.5), it became advantageous to use a radiator that has high transmission of Čerenkov radiation into the UV. Nitrogen gas, N₂, has transmission that extends down to 150 nm. It also has low production rates for knock-ons and fairly good scintillation properties. Figure 4.2 shows the transmission versus wavelength for several gases. Note how the CO₂ transmission drops suddenly at wavelengths below 195 nm [118], whereas transmission for N₂ remains high extending down below 180 nm, as seen in Figure 4.19.

Thus, for E142 and E143, we used research grade nitrogen gas, which was

> 99.999% pure. We pumped the air from the tanks using a single 50 cubic feet per minute pump, which had a liquid nitrogen trap and a molecular sieve to condense and filter contaminants such as oil, oxygen, or water, preventing them from entering the tanks. Then the nitrogen gas was delivered to the vessels via electropolished stainless steel tubing, passing first through oil, water, and mesh filters to ensure the purity of the nitrogen entering the tanks.

We installed pressure and temperature gauges with remote readouts in order monitor the set pion threshold during the experiment. To obtain the threshold value, the index of refraction of the radiator can be found via the Lorenz-Lorentz relation:

$$\frac{n(\lambda)^2 - 1}{n(\lambda)^2 + 2} = K(\lambda)\rho \quad (4.2)$$

where ρ is obtained from pressure and temperature via the ideal gas law and $K(\lambda)$ is the gas dispersion constant. For N_2 , $K(\lambda = 546nm) \approx 0.163cm^3/g$ [119]. Then knowing the index of refraction, and using equation 4.1 above, we find the threshold momentum value β_π .

The temperature measurements were made with Yellow Springs Instruments Model 4320 Thermilinear thermometers which were valid to $\pm 0.1^\circ C$. Pressure in the small tanks was measured using Setra Model 270 pressure transducers which were accurate to $\pm 0.1\%$. In the large 4 meter tanks, TransMetric Model P21A transducers were used with the same accuracy. During the holiday break in E143, the tanks were evacuated and refilled once to minimize potential effects of small amounts of air leaking in. Leak-up rates were estimated to be from 30-60 mtorr/day.

4.1.4 Dimensions

Length, Diameter, and Windows

Figure 4.3: Čerenkov spectrum and efficiency curves.

The number of photoelectrons detected due to Čerenkov radiation can be predicted by

$$\langle N \rangle = \epsilon 2\pi \alpha L \int_{\beta n > 1} \sin^2 \theta(\lambda) \frac{QE(\lambda)R(\lambda)}{\lambda^2} \exp(-\sigma(\lambda) \frac{\rho L N_A}{A}) d\lambda \quad (4.3)$$

where $\sin^2 \theta(\lambda) = (1 - \frac{1}{\beta^2 n^2(\lambda)})$, $\sigma(\lambda)$ is the absorption cross section of N_2 , N_A is Avogadro's number, A is the atomic mass number of the gas, $n(\lambda)$ is the index of refraction, L is the length of the radiator and ρ is its density, β is the velocity of the electrons, α is the fine structure constant and ϵ contains the inefficiencies due to things like losses of photoelectrons during the stage from the photocathode to the first dynode, and nonuniform response across the phototube face. This expression is a convolution of the Čerenkov spectrum, $\propto \frac{1}{\lambda^2}$, the quantum efficiency of the phototube $QE(\lambda)$, the mirror reflectivity $R(\lambda)$, and the absorption of the radiator. Figure 4.3 shows schematically how the factors affect the Čerenkov spectrum.

From equation 4.3 we see that the number of photoelectrons detected is

Figure 4.4: Schematic of 4.0 meter detector, C2. Electrons enter from the left face and emit light which is then reflected off of the mirrors and onto the PMT face.

proportional to the length L of the detector. In our case, the lengths of the Čerenkov tanks were limited by the space available in Endstation A for all of the downstream detector elements. There was room in each spectrometer for a large tank 4.30 meters long with an effective radiator length of 4.0 meters and a smaller tank 2.24 meters long with an effective radiator length of 2.0 meters. However, it was only because the pion thresholds were so high that we needed the detectors this long. For example, 4.0 meters at a 13 GeV/c pion threshold is equivalent in performance to a 1.14m tank at a threshold of 7 GeV/c.

The large Čerenkov detectors C2 had an 80 cm inner radius to cover the large acceptance of the spectrometer, and was made from half inch thick 6061-T3 aluminum. The small detectors C1 were also made from 6061-T3 aluminum and had an inner radius of 23.5 inches. To minimize δ -ray production and multiple scattering effects, the windows on the tanks were designed to be very thin. For the big tanks, the windows were made from a hydroformed

sheet of 1mm thick 2024-T3 aluminum. They withstood more than 20 tons of force without any problems. The small tanks were made from 6061-T3 aluminum and were also about 1mm thick. They did not, however, sustain as much pressure on them, about 6.24 psia, so they did not need to be carefully preformed as did the windows for the big tank. Furthermore, the endwindows of the large tanks were held on by 10 cm wide flanges with 118 3/8 inch bolts on each. A similar but smaller setup was used for the small tanks. Figure 4.4 shows a schematic drawing of the 4.0m tanks with the phototube inside facing the three spherical mirrors.

Mirrors

The size of the mirrors (and hence, the inner diameter of the tanks) was dictated by the trajectories of the scattered electrons within the acceptance of the spectrometers. In the 4 meter tanks the scattered electrons covered an area greater than 65 cm by 115 cm. After tracing the Čerenkov photon trajectories from these electrons, it was found that they could be focused with a spherical mirror of 1.63 m radius of curvature onto a spot size of 10 cm diameter without the use of a Winston cone. Figure 4.5 shows the simulated distribution of the Čerenkov light on the face of the phototube, having been reflected and focused by the proposed mirrors. Mirror molds existed at CERN that allowed us to order three 44 cm(x) by 71 cm(y) spherical mirrors for the large tank. The mirrors were made by slumping a 3 mm thick 836 mm diameter disk of float glass into a stainless steel mold. [120] The glass was cut, cleaned, and coated with 80 nm of aluminum followed by a 30 nm coating of MgF_2 (transparent down to 115 nm [121]) for protection. The mirrors are 3 mm thick and cause fewer than 1.8×10^{-3} knock-ons/pion. They have a multiple scattering angle of 0.29 mrad.

Similarly, for the small tanks two mirrors having radius of curvature of 1.2 m were found to be sufficient. These were 40 cm(x) by 51 cm(y), and 38 cm(x)

Figure 4.5: Monte carlo simulation of the Čerenkov photons on the face of the photomultiplier tube.

by 51 cm(y), and were also 3 mm thick. They were made in a similar fashion, and carefully coated with aluminum as well to optimize mirror reflectivity. The importance of the mirror reflectivity can be seen from equation 4.3, where it enters into the numerator. Figure 4.6 shows a typical reflectivity curve as measured by the fabricators of the mirrors at CERN. The reflectivity started at 70% at wavelengths of 140 nm and quickly went up to 90% at 200 nm and greater.

The mirrors were held at the sides by aluminum clamps cut to the same curvature as the mirrors. The clamps squeezed pieces of rubber tubing around the mirror edges for better grasp and to cushion the squeeze on the mirrors. The clamps were in turn held in an aluminum frame that was mounted 19 cm upstream from the downstream window. See again Figure 4.4.

Figure 4.6: Sample mirror reflectivity curve from CERN.

4.1.5 Phototubes

Choice of PMT

In order to focus all the Čerenkov light onto a single phototube in the detectors, a large diameter tube was needed. The choice came down to a five inch photomultiplier tube from Burle, or one of the same radius from Hamamatsu. Because the Burle tube was slightly smaller than five inches in diameter, and because the Hamamatsu tube had a higher quantum efficiency (QE), the Hamamatsu R1584-01 photomultiplier tube with a UV glass window and a bi-alkali photocathode was selected for the detectors.

Figure 4.7: Specifications for the Hamamatsu R1584 photomultiplier tube.

PMT Base

The high voltage bases used for the Čerenkov phototubes in E143 were the same as those used in E142, the standard negative high voltage base that comes with the R1584-01 five inch phototube. The circuit diagram is shown in Figure 4.7. The phototube case provided by Hamamatsu provided some magnetic shielding against the fringe fields of the spectrometer dipole magnets, primarily a problem for the 7° C1 tank where fields reached nearly 10 gauss. For this reason, the shielding was supplemented with a layer of Permendur (μ -metal). In addition, the C1 tanks in each spectrometer were wrapped with a few layers of Permendur to further reduce the fields.

During the E143 checkout period in November 1993, we tested some positive high voltage bases, R1584-03, that we had specially ordered. In the laboratory they seemed to perform better (i.e. arc less) at the lower pressures than the negative bases. However, when we put them in the tanks and looked

at the signals we found that the positive bases were burning out capacitors, presumably due to arcing at the low pressures. It was particularly noticeable in the 4.0 meter tanks, which we pumped down to about 2.99 psia. We finally decided to put back the negative high voltage bases in all four tanks. Then, in order to get down to 2.99 psia in the large detectors, we had to pump the tanks down slowly and ramp up the high voltage carefully, allowing the bases time to settle down and warm up. The high voltage tripped a few times due to the arcing at the low pressures, but for the experiment we were able to run the bases at a voltage of -2600V, allowing us the same strong signals that were achieved in E142.

Wavelength Shifter

Because the Čerenkov photon spectrum falls off as $1/\lambda^2$ and phototubes typically have low QE below a wavelength of about 200 nm, it was desirable to coat the face of the PMT with a wavelength shifter. The shifter was chosen so that the detector could capture the many photons that are in the UV region. For the E143 Čerenkov detectors, the phototubes were coated with 2430 nm (0.3 mg/cm^3) of para-terphenyl. P-terphenyl [122], as it is sometimes called, has the property of absorbing light down below 300 nm and re-emitting it via fluorescence at longer wavelengths. Figure 4.8 shows the absorption and emission spectra of p-terphenyl. A layer of 25 nm thick MgF_2 was added over the shifter [123] to protect it from evaporation and aging. The thickness of the film was controlled during the vacuum deposition by a crystal thickness monitor. The equipment used was from the laboratory of E.L. Garwin, and Dave Kawall performed the deposition prior to the E142 experiment. There was some worry that for E143 the wavelength shifter would have lost its effectiveness with age. However, the detectors performed comparably to E142. Their performance will be discussed in section 4.3.

For para-terphenyl, the fluorescence maximum is at around 385 nm [122].

Figure 4.8: Absorption and emission spectra of para-terphenyl.

This is well matched with the region of high QE for the Hamamatsu phototubes (Figure 4.21). In addition, the 1-2 ns decay time [122] for the emission allowed us to maintain accurate timing information, which was important for triggering.

Prior to the first use of the Čerenkovs in E142, a test was done using one of the 2.0 meter tanks at Brookhaven National Laboratories. During the test, the phototube was used both with and without the para-terphenyl coating. As can be seen from Figure 4.9, the performance of the detector with the wavelength shifter on the PMT was far superior to its performance with the PMT uncoated. Here raw ADC channel is plotted against the number of counts, and since the number of photoelectrons is roughly equal to the $(mean/\sigma)^2$ of the distribution, we can see that in this test the wavelength shifter provided for a gain of roughly 90% in the number of photoelectrons obtained. Figure 4.10 shows a plot of the number of photoelectrons obtainable with and without the wavelength shifter as a function of wavelength of the Čerenkov light. At small wavelengths the response improves quite a bit with the shifter added. Also, as mentioned earlier, with the use of the wavelength

Figure 4.9: Effect of the wavelength shifter on the output of the Čerenkov phototubes. Collection of the Čerenkov light is much more efficient with the p-terphenyl coating.

shifter it was better to choose a radiator with high UV transmission and a mirror with high reflectivity for wavelengths above and near 150 nm, making nitrogen the gas of choice. Since this allowed a large percentage of small wavelength light to reach the phototube, we were able to receive a high flux of photons for conversion by the wavelength shifter.

4.2 Setup and Alignment

The two Čerenkov detectors in each spectrometer were positioned end-to-end, with the 2.0 meter tank upstream of the 4.0 meter tank, as shown in the diagram in Figure 4.11. Note that the phototube ports are shown with the high voltage and signal cables connected. The nitrogen supply pump line is also shown, along with the pump line to evacuate the tanks.

Figure 4.10: Effect of the wavelength shifter on response of PMT to Čerenkov light.

4.2.1 Ray Traces

In order to align the mirrors in the four Čerenkov detectors so that all of the Čerenkov light rays generated by electrons within the spectrometer acceptance were focused onto the phototube, we use a computer simulation that traced the rays coming from the magnet into the detectors. The program, written by Dave Kawall, takes the particle trajectories, generates Čerenkov rays at the appropriate cone angle, and bounces them off the mirror and onto the phototube. Only one photon is looked at per ray because for alignment purposes we only need to be sure that the outermost rays are focused to a spot within the face of the PMT. The output of the ray trace program gives us the position of the incident ray on the face of the detector, using a coordinate system centered on the face of the detector. It also gives the matching point on the face of the phototube where the ray should end up in order for all rays to be focused. We can then use this information to adjust the mirrors to the proper positions, as

Figure 4.11: The Čerenkov counters were placed end-to-end in both spectrometers. C1 was upstream of C2. Shown here are the various instruments and valves needed to run the detectors and monitor them during the experiment.

detailed below.

4.2.2 Laser Alignment

In order to adjust the mirrors in the tanks properly, we used a small solid state laser with minimal divergence, enabling us to see the the light rays and their final position on the phototube. The apparatus was fairly simple. We borrowed an X-Z stage onto which we mounted the laser at the proper 4° angle in Y (surveyor coordinates, not transport). We placed a thin Plexiglas window onto the front of the detector so that the laser could shine through it to the mirrors and so that we could draw our coordinate system grid on it. Likewise, we covered the PMT face with a paper grid so that we knew the position of the reflected ray on it. We directed the laser beam through a large range of grid

Figure 4.12: Apparatus used to align mirrors.

points on the Plexiglas window and then adjusted the mirrors until each ray was focused to the proper point on the tube. In this way we ensured that all of the possible rays of Čerenkov light would make it onto the phototube face and be detected. A drawing of the apparatus used is shown in Figure 4.12.

4.2.3 Cleanliness

When the Čerenkov detectors were built, and before they were used in E142, the interiors were treated with a mild acid etch and steam-cleaned to remove any aluminum oxide or other surface contaminants as part of a general effort to keep the tanks clean. However, prior to E143 we opened up the Čerenkov tanks to realign the mirrors. which means dirt and contamination could get inside the tanks. If these contaminants were to remain in the radiator, they would absorb Čerenkov light and reduce the detector efficiency. In addition, dust and debris on the mirrors would reduce their reflectivity. For this reason,

we had to clean out the tanks thoroughly. We used a can of pressurized air to blow off the mirrors and we vacuumed the tanks. Then we wiped down the inside of the tanks as well as the phototube port and holders with ethyl alcohol to clean it, using rubber gloves so that oils from our skin wouldn't rub off onto any surfaces. In addition, we cleaned the port and window areas that were near the O-rings with acetone followed by ethyl alcohol to get any stray vacuum grease off the surfaces.

4.2.4 Electronics

The outputs from the phototube in each tank were sent via fast Heliac 50 Ω ($\beta=0.96$) cables to the ESA Counting House. It was then split eight times with LeCroy (LRS) fan-outs. Four of the signals were delayed and sent to our ADC's, each going separately to four crates. The other four signals were sent through LRS 821 discriminators. The signals passing the low, medium, high, and very high thresholds (C1L, C1M, C1H, C1VH respectively for the small tank) continued on to four separate TDC's, one for each threshold level, where the event times were recorded. The low threshold was set at 50 mV, just above one photoelectron, and signals from there went on to form part of the Main Trigger and the hodgegate. Čerenkov signals passing the medium threshold (C1M or C2M) of 70 mV formed a portion of the efficiency triggers. The high and very high thresholds (C1H, C2H, C1VH, C2VH) were set at 90 and 120 mV. Signals from these had the times recorded, but did not form any triggers. Figure 4.13 shows a schematic of the electronics for the Čerenkov detectors.

The pressure and temperature measurements in the tanks were read out by Slow Analog Monitors (SAMs) every five minutes at checkpoints, and were written to tape. Furthermore, LED's were installed in each tank to be used in conjunction with those in the hodoscope detectors to facilitate timing of the electronics. They were also useful in determining the effect of the magnetic fields from the nearby dipole magnet on our phototubes.

Figure 4.13: The signals from the Čerenkov tanks were split with a fan-out, and then sent through discriminators or on to the ADC's. Signals passing the lowest discriminator were sent on to form part of the main trigger and the hodogate. All signals were timed with TDC's.

4.3 Performance

4.3.1 Efficiency

ADC Distributions

One way to determine the efficiency of the Čerenkov detectors is to calculate the number of photoelectrons N_{pe} obtained in each detector. To find the average number of photoelectrons, I looked at the raw ADC distributions from the detectors due to electrons. I then fit the peak of the distribution with a Gaussian. Figure 4.14 shows one such distribution for the 4.0 meter tank in the 4.5° spectrometer.

The number of photoelectrons can be found by the square of the mean of

Figure 4.14: Raw ADC distribution for 4.5° 4.0m tank; peak fit with gaussian.

Table 4.1: Typical performances of the four Čerenkov detectors in E143. “p.e.” stands for photoelectrons.

	4.5°		7.0°	
	C1	C2	C1	C2
$\langle ADC \rangle$	165.6	127.1	158.8	137.2
σ	56.6	42.3	54.2	55.5
# p.e.	8.6 ± 0.5	9.0 ± 0.5	8.6 ± 0.5	6.1 ± 0.5
Efficiency	$99.98 \pm < 0.01$	$99.90 \pm < 0.01$	$99.98 \pm < 0.01$	$99.78 \pm < 0.11$

the distribution divided by the sigma: $(\mu/\sigma)^2$. For this distribution the average number of photoelectrons is about 9. The probability of N_{pe} fluctuating to zero is $e^{-N_{pe}}$. Then, the efficiency is defined to be $(1 - e^{-N_{pe}})$. Table 4.1 below gives typical results for each detector. The cuts used to define these electrons were basics cuts on tracks and track/cluster matching, as well as the requirements that the shower neural network id > 0.95 and $|e/p - 1| < 0.2$. Obviously there were no cuts made on Čerenkov ADC.

Figure 4.15: Efficiency as a function of Čerenkov ADC cut.

Efficiency v. Minimum ADC

Another way to study the performance of the Čerenkov detectors is to look at their efficiencies while varying the cuts on the minimum ADC value for accepting the event. Pion contamination levels should decrease as the minimum ADC cuts get higher. As an example, to study the efficiency of the 4.5° C1 (2.0 meter) detector, one can create a fairly strong electron definition (same cuts as above). Then one can plot the ADC spectrum from the 4.5° C2 (4.0 meter) detector while requiring different minimum ADC values for the C1 detector. The efficiency of C1 with a minimum ADC cut at "x" is then the ratio of the number of entries in the C2 ADC plot cut at $C1 > x$ to the number of entries with no cuts on C1. The curve showing the efficiency of the detectors as a function of ADC cut is in Figure 4.15. This is useful in deciding which ADC cut to use in the analysis so that most pions are rejected, where a higher cut is

Figure 4.16: Čerenkov ADC response to electrons (white) and pions (grey). cutting out low ADC values rejects a large number of pions while keeping most electrons.

better (see Figure 4.16), and most electrons are kept, which would correspond to a high efficiency value for electrons.

4.3.2 Pion Thresholds

From Data

In order to verify that the Čerenkov detectors were doing their job, that is, rejecting pions below the threshold momentum, it is useful to look at the Čerenkov ADC distributions for pions versus pion momentum. In a two dimensional scatter plot, what one expects to see is a clustering of pions at around ADC= 0, which are pions below the threshold momentum that don't emit Čerenkov light. However, pions with momenta about the 9 and 13 GeV thresholds will start to produce some radiation in C1 and C2 respectively, resulting in a slow "turn-on" of ADC values for pions above the threshold.

Figure 4.17: Pion momentum versus ADC with turn-on function superimposed.

Pion ADC values should roughly follow the functional form

$$ADC = \mu(1 - (\frac{p_{th}}{p})^2)$$

where μ is the mean value of the regular ADC distribution and is proportional to the gain, p_{th} is pion threshold momentum, and p is the actual momentum of the pion.

In order to have a good number of pions, I looked mainly at “positron runs”. Then, to define a pion, I looked at each trigger and found those triggers that do not have any event in them with a value for e/p greater than 0.7. This is to avoid plotting the Čerenkov ADC for a trigger which contains an electron, because the ADC value would be artificially high and wouldn’t be the correct ADC value for the pion. For these triggers, I then accepted only shower calorimeter clusters with shower neural network id less than zero. These are mostly all pions. The plots of Čerenkov ADC v. momentum for pions are shown in Figure 4.17. The functional form of the theoretical curve

Figure 4.18: Nitrogen index of refraction as a function of wavelength. At low wavelengths, the index increases, allowing some pions to fire the Čerenkov detectors below the set momentum threshold.

is superimposed, and is in pretty good agreement. However, since the index of refraction of nitrogen (Figure 4.18) is a function of wavelength, the actual pion threshold will depend on wavelength. At smaller wavelengths, the index of refraction is slightly larger, allowing some pions below the set threshold momentum, defined at a wavelength of 250 nm in our tanks, to radiate light. This is partially counterbalanced by placing discriminator thresholds at 1 photoelectron or higher, but we still see a number of pions producing light below threshold.

4.4 Simulation

In order to study the detectors carefully and to understand all the factors that go into capturing the Čerenkov light and detecting it properly, I have done a detailed Monte Carlo simulation of one of the detectors. The detector I studied was the 4.5° 4.0m tank.

4.4.1 Rays to PMT

For the input electrons, "rays" were obtained from Makis Petratos, who simulated the spectrometer acceptance of electrons from the target. These rays were weighted by cross section, and were given with momentum, slope, and point of exit of B81. The rays were read in and then advanced to the front of the tank, from where the simulation begins. The same ray-trace program that Kawall had created and that was used for alignment of the mirrors formed the skeleton of the simulation. It was then modified to accept a large number Čerenkov photons from each electron ray, and all the other factors were put in to make it realistic from there.

For each electron ray, a random number of Čerenkov photons are generated to be radiated from the electron. The number of Čerenkov photons generated follows a Poisson distribution, centered at the mean number of Čerenkov photons we expect, about 72, from the following formula:

$$\langle N \rangle = 2\pi\alpha L \sin^2 \theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} \quad (4.4)$$

Here the Čerenkov angle, θ , is assumed to be wavelength independent since the dependence is small. The limits on the wavelength λ in the present case come from two places. The lower limit, λ_1 , is due to the absorption curve of Nitrogen gas for light with wavelengths below about 160 nm. The upper limit, λ_2 , is due to the higher bound on quantum efficiency for the phototube, about 630 nm. Light at other wavelengths is not of interest because the apparatus cannot detect it.

Figure 4.19: Transmittance of nitrogen for 1atm-m for high and low wavelengths.

Absorption of Light

Nitrogen gas absorbs light with a certain probability, given by

$$1 - \exp(-\sigma(\lambda) \frac{\rho L N_A}{A})$$

Here $\sigma(\lambda)$ is a characteristic absorption cross section for nitrogen. ρ is the density of the gas and L is the length. The Čerenkov photons emitted by the electrons are emitted at random positions in Z over the flight of the electron through the tank. Therefore, the length used in the calculation, L , is not the approximately 4.0m of the electron's flight through the tank in front of the mirrors, but rather the average length the photon travels, after it is emitted, to reach the phototube. The distance to the mirrors as well as the distance back to the PMT are added to obtain the total length in which the photon is in contact with the gas and has the potential to be absorbed.

In order to get a good approximation of how much light reaches the PMT in the detector, such things as the absorption of the light in the gas have to be taken into account. Impurities in the gas would cause even more absorption

of the light. However, such problems were minimized by using research grade nitrogen, which is greater than 99% pure, and by cleaning everything in the tank thoroughly to prevent dust and oils from contaminating the radiator. For these reasons, it was assumed that the radiator in the gas was only nitrogen. The transmittance spectrum for the nitrogen came from two documents, one that measured the transmittance in the low wavelength region [123] as in Figure 4.19, and the other in the higher wavelength region [118] shown here as well. It is clear from the figure that the transmittance drops severely for wavelengths below 160 nm, and that it is, on average, close to 40%. However, in the higher wavelength region, the nitrogen becomes quite transparent, at around 97-99% transmittance. Note that in both cases the study was done for 1atm-m of gas, so the 3psia was converted to the appropriate length.

Mirror Reflectivity

Another inefficiency in the transmission of the Čerenkov light to the phototube is the reflection off the mirrors. These mirrors were made at CERN and are in general very good reflectors above wavelengths of about 200 nm. Figure 4.6 indicates that for a typical mirror the reflectivity is close to 80% at 160 nm, but then quickly rises to around 90% and becomes fairly stable. The reflectivity was factored in, as well, in determining how many quanta reach the phototube.

Multiple Scattering

Because the radiator used is nitrogen gas at very low pressures, the scattering of the light off of the gas molecules is negligible. Prior to E142, Kawall looked at this question and found that it made no difference at all whether or not multiple scattering is considered.

4.4.2 Collection at PMT

Once it was decided whether or not a photon makes it to the PMT, it was necessary to determine if the photon is detected properly. Each photon that has survived to reach the phototube has a certain wavelength associated with it. Many of them are in the UV, at small wavelengths, which is the region of poor QE for the photomultiplier tube (see Figure 4.3 and Figure 4.21). For this reason, an organic wavelength shifter was used to coat the tube so as to shift the light wavelength higher, as mentioned previously in Section 4.1.5.

Wavelength Shifter Effects

For this detector, the range of light that needed to be shifted upward in order to be better detected by the PMT was about 160-300 nm. In this region, the quantum yield of p-terphenyl (same as para-terphenyl) is wavelength independent [124]. The emission spectrum [122] is shown in Figure 4.20. In addition, above 1000 nm thickness (2430 nm was used for our PMT coating), the output of the shifter is thickness-independent [122]. Because of the stability of the emission with thickness and with wavelength, the emission curve could be parameterized, Figure 4.20, and then used to find the output wavelength of a shifted photon (randomly selected within the curve). Quanta with wavelengths much above the peak of the emission spectra basically passed right through the shifter with the original wavelength. The wavelength shifter had a particular efficiency, as well. The quantum efficiency of p-terphenyl for absorption and re-emission has been found to be about 82% [125]. This was taken into account when shifting a photon.

Figure 4.20: Emission spectrum of p-terphenyl excited at 310 nm.

Quantum Efficiency

Once a good portion of the UV photons had been shifted to higher wavelengths, it was easier to detect them, as seen in the QE curve given by Hamamatsu for the R1584 tube, Figure 4.21. Along with the curve Hamamatsu provided a table of measured points and the corresponding QE. At the peak, the QE is only about 22%, and goes to zero at about 175 nm at the low end and 630 nm at the high end. Hence the PMT QE was responsible for the loss of a large amount of Čerenkov light.

Uniformity of Response and Collection Efficiency

The response of a phototube always varies somewhat across the face. The measurements obtained from Hamamatsu showed that on average, the response across the face of our phototubes is about 90% relative to the incident light. This can be seen by measuring the signal on the cathode for photons incident upon different (x,y) points on the face. See Figure 4.22. Here one can also see

Figure 4.21: Quantum efficiency curve for Hamamatsu R1584 PMT, plotted on a log scale.

the collection efficiency: the percentage that makes it to the first dynode and subsequently the anode. The collection efficiency is, on average, fairly low—about 75% averaged across the PMT face. Both of these (in)efficiencies are factored in to the determination of whether a quanta survives.

Photoelectrons

Once all of the photons had been generated, transported to the phototube and focused, and all the inefficiencies had been taken into account, the number of photoelectrons for each event could be obtained. This then gave the average number of photoelectrons per event. From this Monte Carlo simulation, an average of about 7.7 photoelectrons were obtained from the phototube for each electron signal. A typical spectrum is shown again in Figure 4.23, and from a Gaussian fit to the peak, 8.4 photoelectrons is found. This number can

Figure 4.22: X-Y uniformity and collection efficiency across the face of the PMT.

change a little depending on the range of the fit, giving an error of about ± 0.5 photoelectrons. An error is also assigned to the simulation based on how well the efficiencies are known, and how well the extrapolations and approximations are made. This adds an error of about ± 1.2 photoelectrons. One can conclude that the simulation agrees with the data to within error.

4.4.3 ADC response

Once the average number of photoelectrons per event has been found, it is useful to obtain an ADC distribution for the signals. To find the ADC distribution, one then needs to obtain the gain of the system. One can also use the single photoelectron peak to extrapolate the shape of subsequent peaks. Since Christina Berisso had digitized a single photoelectron peak using the same PMT and base as was used in E143 (see Figure 4.24), it was possible to

Figure 4.23: Čerenkov ADC spectrum, fitted with a gaussian. The number of photoelectrons can be found by dividing the mean² by the sigma² of the fitted curve. Here we have about 8.4 photoelectrons.

Figure 4.24: Single photoelectron peak out of E143 PMT and base.

Figure 4.25: Single photoelectron peak, extrapolated to 2,3,4, and 8 photoelectrons. Gain is arbitrary. Used to compare with ADC distribution generated by the simulation at 8 photoelectrons, shown in Figure 4.26.

then find the probability distributions for each successive photoelectron peak after the single peak. Such distributions have been plotted in Figure 4.25. The system she used had a slightly different gain than that of the data, but the shape and basic behavior of the peaks are the same.

Knowing the probability distributions for each peak, one can then form a new ADC distribution. Taking the mean number of photoelectrons for each event, one can then find a random number out of the corresponding photoelectron distribution and plot it as an ADC value. This was done for each the input rays to the detector. The result is shown in Figure 4.26. Note that after the gain is adjusted, which had initially been arbitrary for the single photoelectron distribution, the simulated distribution corresponds to about 8.25 photoelectrons on average, which is approximately what we see in the data.

Figure 4.26: Generated ADC distribution, fit with a gaussian. Dividing the mean^2 by the sigma^2 gives about 8 photoelectrons, as we expected.

Therefore, the simulation reproduces the behavior of the Čerenkov detector quite well.

Chapter 5

Data Analysis

The experiment E143 ran in SLAC Endstation A from mid-November 1993 to February 6, 1994. However, at the end of November there was a "repair opportunity day" (ROD) during which several experimental problems were fixed. These included a tilt in the bottom mirror of the 4.5° 4 meter Čerenkov tank (a misalignment thought to have been caused by the strength of the vacuum pumps when the tank was pumped down), some electronics problems in the 7.0° spectrometer, and so forth. The data from before the ROD day was salvaged using software fixes during our analysis. However, due mainly to insufficient knowledge of the target polarizations, it was later determined by the collaboration that the results would be more reliable if we excluded the pre-ROD data.

Two weeks of data in the "transverse mode" at the 29 GeV energy were logged during January 1994, while two weeks of longitudinal data at 16 GeV were taken in December, and one week at 9 GeV in February. Table 5.1 describes the various data sets, which overall contained approximately 2×10^8 electrons. In addition, 10-15% of the runs were taken with one of the detectors set at the opposite momentum as the electron runs so as to measure

Table 5.1: Information on the various data sets taken during E143. Run numbers below 1000 were considered test runs and weren't analyzed. Also, runs below 1244 are the pre-ROD runs, and were not used in the final analysis.

Energy & Asymmetry	Central Mom. (GeV)		No. of e^- x 10^6 (Prot./Deut.)	Run Range	Dates Taken
	4.5°	7.0°			
29 GeV $A_{ }$	-11.5	-12.5	4/9	1000-1244	11/24/93 - 12/01/93
29 GeV $A_{ }$	-11.5	-12.5	12/18	1245-1745	12/01/93 - 12/14/93
16 GeV $A_{ }$	-7.5	-8.0	34/12	1766-2063	12/15/93 - 12/21/93
16 GeV $A_{ }$	-11.5	-12.5	5/5	2072-2160	12/21/93 - 12/23/93
29 GeV $A_{\perp}$	-11.5	-12.5	20/13	2164-2642	01/04/94 - 01/18/94
29 GeV $A_{ }$	-11.5	-12.5	19/27	2651-3137	01/18/94 - 01/31/94
9 GeV $A_{ }$	-7.5	-8.0	30/28	3142-3378	02/01/94 - 02/06/94

backgrounds due to the positively charged particles, π^+ and e^- .

To select good electron events, many requirements and cuts had to be placed on the data. This involved examining the data stream by run, by spill, and by event to make sure that the data were obtained under acceptable conditions, and that the events analyzed were indeed electrons. Both beam conditions and spectrometer conditions were examined to be sure our electron sample was as clean as possible.

5.1 Run and Spill Selection

First only what were considered to be “good” electron runs were accepted into the analysis pool. Then, within the accepted runs, each spill was examined and tolerance limits placed on the spill as described below.

5.1.1 Bad Run Determination

Typical runs in E143 were about an hour long and contained approximately 500,000 interrupts. However, during the analysis it was found that several runs should be excluded due to what was considered bad running conditions. The runs that were chosen not to be analyzed had the following bad conditions:

- Short runs (less than 15 minutes) and checkout runs. The checkout runs usually contained only tiny amounts of data and were used for tests, as noted in the log books. The other short runs were usually ended because something went wrong during the run or something was set wrong when the run was started.
- Target polarization was too low, in which case there were large uncertainties in the polarization measurement.
- Detectors or electronics were bad, such as failed NIM crates, bad ADC modules, dead shower counter blocks, or misaligned Čerenkov mirror.
- Trigger rates were too high for the given current, implying that the beam might be misaligned and hitting the target holder, causing a spray of bad events to be sent into the detector.
- Beam conditions were undesirable, such as when the beam position drifted a great deal, the spot size was too wide, beam intensities fluctuated too much, or good and bad spill counter signals were abnormal.

The total number of runs used for the 29 GeV analysis were 85 runs of longitudinal proton data, 76 runs of perpendicular proton data, 119 runs of longitudinal deuteron data, and 50 runs of perpendicular deuteron data. The results for the analysis excluding the pre-”repair opportunity day” (ROD) data will be presented in Chapter 6.

5.1.2 Beam Spill Cuts

The incoming electron beam was “rastered” across the target face, as shown in Figure 5.1, so that each spill was sent to a different position on the target. This was done to prevent non-uniform target heating and depolarization effects. Because the analysis program was called spill by spill, requirements could be placed on each spill at the start of the analysis to determine if the spill was acceptable. The beam cuts used on several quantities were dynamic, and were calculated using properties of the individual distributions of each run. The first 235 spills were used to accumulate statistics for the distributions. Then a cut was calculated on each individual quantity. The reason dynamic cuts were used is because for each run, the distribution widths and means change due to varying electronics and beam conditions. Therefore, it would have been inefficient to place the same fixed cut on a quantity for all of the runs.

Good and Bad Spill

Both an upper cut and a lower cut was used on the good spill. The high cut was the mean value of the good spill distribution multiplied by 1.75. The low cut was the mean value divided by 2. These factors were chosen after studying the good spill distributions of a great many runs to see what reasonable cut values would be. See Figure 5.2(a).

The bad spill cut value was only an upper limit cut: the mean bad spill value multiplied by 3. However, if the calculated value of the cut went above 150, the maximum cut value was 150. This maximum was set because for a few runs there were large tails to the distributions which shifted the calculated mean value, and it was desirable to cut off most of these tails. Figure 5.2(b) shows a typical distribution and cut.

Figure 5.1: Beam raster pattern: the spill-by-spill beam distribution on the target as measured by the foil array. Axes are the x- and y- positions of the spill. All of the beam's 253 points are visible. The spots represent the center of the beam position, but the beam width of a few mm allowed coverage of the entire target area.

Figure 5.2: Cuts and typical values for the good and bad spill monitors. White is accepted spills, grey is spills that were cut, dark line is the cut value, which is sometimes dynamic.

Toroid Cut

The toroid #2 reading was used for our current information, and so requirements were placed on this value to be sure the spill contained a reasonable current. Both high and low cuts were used on the toroid #2 value. current information. The low toroid cut was the mean of the distribution minus $0.75(\times 10^9 \text{ electrons})$, and the high cut was the mean plus 0.75. However, if the low cut became less than 0.5, the cut was placed at 0.5 as a minimum cut value. See Figure 5.3.

Beam Spot

Several cuts were placed on the beam spot width. The first two were dynamic low and high cuts. First we found the mean beam width $\bar{w}$. Then we made the

Figure 5.3: Toroid Values and Cuts

low cut $\bar{w} - 3\sigma$ and the high cut $\bar{w} + 3\sigma$. The second two were fixed maximum and minimum cuts, so that if the calculated low cut is less than 0.5mm, we set the cut to be 0.5, and if the calculated high cut is greater than 5.0, we set the cut to be 5.0. Figure 5.4(a) shows the beam spot measurements and the dynamic cuts.

Beam Position

The position of the beam spot was measured downstream of the target by a foil array. Because of the nearby magnets, we determined the actual beam spot position by multiplying the measured position on the foils by a magnet correction factor so that the center of the foils coincided with the center of the target. Then two cuts were made on the actual beam spot position. The first cut was a requirement that the spill be within a radius of 12 cm from the center of the foils. See Figure 5.5. The second cut was to provide for cases in which the rastering pattern was offset with respect to the center of

Figure 5.4: a) The measured width of the beam spot along with the calculated dynamical cuts. b) Beam position and cuts.

the foils. In these cases, the mean foil position was calculated and declared to be the center of the raster pattern. Then, only spills within the pattern, i.e. within a radius of 12 cm for the 29 GeV data and 14 cm for the lower energy data, was accepted. The accepted spills then lay in area A in Figure 5.5. See Figure 5.4(b) for the values and cuts on the beam position.

Finally, if the spill passes all of these beam cuts, we accept the spill and go on to inspect the events in the spill.

5.2 Electron Event Selection

After eliminating bad runs and spills, the next step in the data analysis consisted of identifying electron events amidst a background of mostly pion events. With these good electrons we could then form raw asymmetries in the rates of scattered electrons coming from each incident electron polarization. These

Figure 5.5: Beam position cuts. The dark circle represents the raster pattern centered on the foil center. The light circle represents a raster pattern offset from the foil center by some amount. Accepted spills are within the grey region (A) if pattern is offset, the dark circle if it is centered.

asymmetries are functions of the kinematics of the scattered electron, such as it's scattering angle and the momentum transferred in the scattering process, so for each scattered electron it's scattering angle and momentum were noted.

5.2.1 Main Trigger

The electron trigger was the first requirement for identifying electrons. For each spectrometer this trigger consisted of a time coincidence between signals from both Čerenkov counters and the shower counter. Requiring pulses in time from both Čerenkov detectors helped to reduce random signals. Since the pion thresholds for the two Čerenkov counters in each spectrometer were 9 GeV and 13 GeV, requiring signals from both counters only allowed pions with momenta greater than 13 GeV to fire the trigger. Because the cross section for pion production at high momenta is small, the majority of the these

background pions were rejected by the electron trigger. In addition to these high momentum pions, there were also a small number of lower momentum pions which entered the spectrometer in time with an electron. The shower counter signal in the electron trigger had a minimum ADC pulse requirement so that the incident particle had to deposit a minimum energy in the shower counter. Since the electromagnetic shower counter is inefficient for pions, this rejected a portion of these low momentum pions.

5.2.2 Association of Tracks with Clusters

Only events in which the track pointed to a cluster were accepted, so that the information from both the energy and the momentum could be used to identify it as an electron. To match the tracks with clusters, first each track was extrapolated to the z position of the cluster on the shower counter. Then the chisquare difference between the track position in x and y and the cluster position was calculated. The same is found for the time between the track and the shower cluster. Combining the chisquare in x, y, and t via the formula:

$$\chi^2 = \left(\frac{dx}{\sigma_x}\right)^2 + \left(\frac{dy}{\sigma_y}\right)^2 + \left(\frac{dt}{\sigma_t}\right)^2 \quad (5.1)$$

the track that has the minimum chisquare total with the cluster is found. Here dx , dy , dt is the separation of the track and the cluster in x, y, and time, respectively, and $\sigma_i = \sqrt{\sigma_{i_1}^2 + \sigma_{i_2}^2}$ where σ_{i_1} is the error on the i position of the cluster and σ_{i_2} is the error on the i position of the track and so on. Once the track with the least χ^2 is found, this track is labelled as being associated with the cluster. Also, the quantities dx, dy, and dt are saved for the track-cluster pair and later are examined to determine if it was a good match.

Figure 5.6: (a-c) Delta x, delta y, and delta t of the track and cluster association. (d) Difference between the x position of the target and the track.

5.2.3 Tracking Cuts

Particle tracking was performed using the spatial and timing information provided by the hodoscopes and shower counter. By measuring the particle trajectory and knowing the optics of the magnetic spectrometer, it is possible to reconstruct the scattered particle's momentum, p . The momentum can then be used as one of the kinematical values in the asymmetry analysis, discussed in (ref. Ch. 1 eqns).

To determine the momentum of the electrons, we had to be sure we were using good tracks. We made a cut on how well each track pointed in space and time to its associated cluster. We required that the electron track point to the cluster to within 40 mm. in x and y, and to within 8 seconds in time. Figure 5.6(a-c) shows the distributions in dx , dy , and dt discussed above in Section 5.2.2. We also required that each track point back to the target to

Figure 5.7: Typical Čerenkov ADC spectrum for an electron. Pions can be seen in the dark region with predominantly low ADC values.

within 15 mm in x so as to eliminate tracks not coming from the target area. See Figure 5.6(d).

5.2.4 Cuts on Čerenkov Response

As mentioned in Chapter 4, the Čerenkov counters were configured to reject all but high momentum pions. Most of the pions that did give a signal in the counters had low ADC values, since the Čerenkov response slowly increases for pions. For electrons, a software cut requiring a minimum Čerenkov ADC pulse height was made so as to reject the contaminating pions. Therefore we excluded all events with pulse heights of less than 40 in either counter, or an equivalent of less than about 2 photoelectrons. This cut out the majority of the pions, but cut very few electrons, as can be seen in Figure 5.7

5.2.5 Cuts on Shower Calorimeter Response

The shower counter provided two methods of electron identification: comparison of the particle energy with its momentum, and association of the shape of the electromagnetic shower with either a hadron or an electron.

Shower Cluster Energy

Once the counter was calibrated, the energy, E' , of the scattered particle was found by adding up the energies from all the blocks in the cluster. The small number of events with clusters on the outer edges of the shower counter were rejected because of possible incomplete clusters and hence incorrect energy measurements. Furthermore, only events with shower clusters were used as electron candidates.

The particle energy obtained from the shower cluster was then compared to the momentum of the particle as measured by the hodoscope tracking system. Because our tracking was sensitive to all charged particles, and our calorimetry was not efficient for pions, the electron events had an E'/P peak centered around unity, whereas the E'/P values for pions were in general much less than one. By making a cut around the electron peak of $0.8 < E'/P < 1.25$, we were able to reject the majority of pion contaminants left in our data sample. The E'/P requirement was approximately 96% efficient (*i.e.* - it included about 96% of the electron sample), and left a pion contamination of less than 1%. Figure 5.8(a) shows a typical E'/P spectrum for electrons and pions. Note that the electron values are centered at 1.0 as expected.

Figure 5.8: (a) Energy divided by momentum for electrons(grey) and pions(dark). (b) Neural network output for electrons and pions.

Particle Identification by Neural Network

In addition to the E'/P cut, we used a shower counter neural network algorithm [114]. See section 3.5.4. This modeled a typical electron cluster spatial profile to determine to very good accuracy which events were electron events and which were background. The output of the algorithm was to give a value of 1 if the particle was an electron and a value of -1 if the particle was a pion. A very small percentage of the events fell between these values. In our electron data sample, we allowed only particles with neural network values of greater than 0.9, cutting the rest. Figure 5.8(b) shows the neural network response for electrons and pions, plotted on log scale. The neural network was an extremely effective method of particle identification. It was approximately 98% efficient for electrons, and using it alone left a pion contamination of about 0.5%.

5.2.6 Kinematical Cuts and Binning

Figure 5.9: (a) x_{BJ} for the 4.5° spectrometer. (b) x_{BJ} for the 7.0° spectrometer. (c) Q^2 values for the 4.5° spectrometer. (d) 7.0° spectrometer Q^2 values.

The bins in x_{BJ} and Q^2 are calculated separately with three methods of determining E' : the energy of the cluster, the momentum of the track, and the unweighted average of energy and momentum. However, since we considered our momentum measurement to be reliable, for our results we used $E' = p$ in general. We used 38 bins in x equally spaced logarithmically from 0.01 to 0.9 and 13 bins equally spaced in Q^2 from 0 to 13. At this point we discard events whose x or Q^2 values do not fall within these ranges. We keep events in the range of x and Q^2 larger than the data actually covers to avoid missing events, but we discard events whose values are unreasonable. Figure 5.9 shows values of x and Q^2 for both spectrometers. We chose 38 bins as per Charlie

Table 5.2: The bins that were used for the analysis varied by energy, as did the kinematically attainable ranges in x . The “high” and “low” labels refer to the two settings of accepted momentum for the 16 GeV data.

	Energy(GeV)	Lowest Bin	Highest Bin	x -Range
4.5°	29	10	35	0.029 - 0.631
	16 high	13	29	0.041 - 0.310
	16 low	7	29	0.020 - 0.310
	9	18	27	0.033 - 0.135
7°	29	18	37	0.075 - 0.800
	16 high	21	33	0.107 - 0.498
	16 low	15	33	0.053 - 0.498
	9	18	27	0.075 - 0.245

Prescott’s memo in April, 1994 in order to make approximately 28 bins in the data range, $0.029 < x_{BJ} < 0.8$. This was done so that the resolution in x was not much finer than the binning, as was the case in a more preliminary report of results.

The kinematical range of that we kept for use in the asymmetry measurements was dependent upon both the acceptance of our detectors, and the location of the closest resonance region. The detector acceptance was determined by the positions and sizes of the detectors, magnets and collimators. Because of tracking inefficiencies and resolution smearing, some data were collected in kinematic bins that would seem unphysical. However, the actual physical limit was indicated by a sharp drop in event rate in the particular bin. We determined the cut-off by studying histograms to see which bins in E' still contained good electron samples. Taking the average scattering angle with this information, we calculated the highest and lowest x -bins to keep.

The most important resonance within the range of E143 data was the $\Delta(1232)$ peak, which sat at invariant mass-squared of $W^2 \approx 1.5 \text{ GeV}^2$. Other resonances exist at higher energies, but they are not as large and there are

enough that they fade into the DIS region rates. Since resonances have particular orbital and total angular momentum, their production cross-section depends on the helicity of the incoming electron, so the spin asymmetries we measure are influenced by such resonances. For this reason we placed a cut on the resonance region at $W^2 = 4$, which is generally regarded as a clean cut and leaves most resonances out of the data set. This set the upper limit for the average of the x -bin. Table 5.2 shows the bins used for this analysis for each energy.

The Δ resonance region contained little of our data at 29 GeV and 16 GeV. These data were primarily in the deep inelastic regime. Figure 5.10 shows the kinematical region obtained by E143 for each of the three energies at which data were collected. The dot-dash line that comes down from the top right indicates the $W^2 = 4$ GeV² cut-off. Notice that the 9 GeV data contains a large portion of data within the resonance region. In that region, a separate study was made of the data [126] [20], in which the dilution factor, the radiative corrections, and the resolution smearing all had to be modelled by a Monte Carlo simulation. The results in this thesis exclude this resonance region, since it is handled more carefully elsewhere.

5.2.7 Final Electron Sample

If the event satisfies all of these requirements, it is then accumulated into an array using the appropriate x_{BJ} and Q^2 bins, along with information on spectrometer number, left or right beam polarization, and a number referring to which cuts were used. At the end of a run, this array, along with the incident left and right charge and other needed information, is output to a disk file. We are then able to use the run-by-run information to reconstruct the appropriate raw physics asymmetries.

Figure 5.10: The top two regions show the kinematical range of the data for the 29 GeV data subset for each spectrometer. The middle two regions indicate the 16 GeV data, and the bottom two, the 9 GeV data. The dot-dash line on the right side indicates the resonance region cut-off value at $W^2 = 4 \text{ GeV}^2$.

5.3 Determination of Cross Section Asymmetry

Electrons passing the event selection cuts were binned in x and Q^2 such that the resolution in x was not much finer than the binning. The electrons were also tagged according to their relative target and beam helicity states, $N^{\uparrow\downarrow(\uparrow\uparrow)}$, and which spectrometer they entered. After combining the Q^2 bins within

each x bin, the asymmetries $A_{\parallel}$ and $A_{\perp}$ were formed:

$$A_{\parallel} \text{ (or } A_{\perp}) = C_1 \left(\frac{1}{f P_b P_t} \frac{N_L - N_R}{N_L + N_R} - C_2 \right) + A_{rc}. \quad (5.2)$$

Here f is the dilution factor, P_b and P_t are the beam and target polarizations, A_{rc} is the radiative correction to the asymmetry, and $C_{1(2)}$ are the corrections needed due to the presence of nitrogen in the targets, with C_2 disappearing for the proton target. Also, here $N_{L(R)}$ is the number of left or right-handed helicity events corrected as

$$N_{L(R)} = N_{L(R)}^{(raw)} d_{L(R)} / Q_{L(R)} \quad (5.3)$$

where $d_{L(R)}$ is the appropriate deadtime correction and $Q_{L(R)}$ is the appropriate incident charge. All of the corrections that have to be made to the raw rates to form physics asymmetries, as well as the unpolarized structure functions, are discussed below.

5.3.1 Dead Time

For this experiment, there were two ways in which we could miss incoming data. First, the electronics were set up such that for each spill, only the first four MAIN-OR triggers could be read by the ADC modules. Any triggers that came after the first four were not recorded. We also missed data due to the actual electronics dead time: once the MAIN-OR was triggered, any signal arriving within the next 32ns could not trigger the MAIN-OR again. When a trigger occurred the discriminators opened a 32 ns gate which then allowed signals from the ADC's and TDC's to enter. If another trigger signal then entered the data stream within the next 8 ns, the discriminator would not see it. This 8 ns is known as the “double pulse” resolution of the discriminator. If the next trigger signal came within the following 24 ns but still within the 32 ns gate, the discriminator gate was extended for another 32 ns from that

time, and the signal was not recorded as a separate trigger. The discriminators were run in this “updating mode” so as to prevent loss of data if the detectors became particularly noisy.

In measuring the asymmetry $A_{||}$ at large x , the cross section for events when the proton and electron are antiparallel is larger than for events in which the proton and electron are parallel. This means that the rate of events into the detectors is higher with one electron helicity versus the other, so the probability of overlapping signals increases, and hence the loss of data due to both electronics deadtime and the four-per-pulse maximum is larger with this helicity. Because this then reduces the measured asymmetry, it was necessary to correct for the data we failed to acquire.

The deadtime correction was made by generating a multiplicative coefficient d for the raw number of events counted.

$$d = \frac{N_{real}}{N_{meas}} > 1. \quad (5.4)$$

To obtain N_{meas} , for each run the number of spills N_i with $i = 0, 1, \dots, 16$ triggers were recorded from the TDC outputs. The number of MAIN-OR triggers per spill was rarely more than 12, so $i = 16$ is taken here to be the maximum. Then

$$N_{meas} = \sum_{i=0}^{i=16} \bar{i} N_i, \quad (5.5)$$

where $\bar{i} = \min(i, 4)$ to provide for our four-per-pulse ADC limit.

The actual number of MAIN-OR triggers was found using a probability matrix, P whose elements $P(i, j)$ denote the probability that i hits will be measured when there are j real triggers. This matrix was obtained from a Monte Carlo simulation in which it was assumed that the spills were distributed randomly throughout a 2200 ns spill length, and that the electronics dead time

Figure 5.11: Dead time correction factors as a function of rate for both spectrometers.

was 32 ns. The number of real events was then

$$N_{real} = \sum_{j=1}^{j=16} j \sum_{i=1}^{i=16} P^{-1}(i, j) N_i. \quad (5.6)$$

The statistics for the quantity N_{meas} were accumulated for each run, for each spectrometer, and for each beam helicity separately. The deadtime correction, d , was then applied to the rates N_L^{raw} and N_R^{raw} for each x and Q^2 bin. The rates then used for the asymmetry in Equation 5.2 looked as follows, normalizing them by the incident charge:

$$N_L = d_L \frac{N_L^{raw}}{Q_L} \quad \text{and} \quad N_R = d_R \frac{N_R^{raw}}{Q_R}. \quad (5.7)$$

Figure 5.11 shows the deadtime corrections for the proton 29 GeV runs. Typical 4.5° dead-time correction coefficients were $d_{L,R} \approx 1.05$, while for the 7° they were closer to $d_{L,R} \approx 1.01$ due to the lower event rate.

5.3.2 Dilution Factor

In counting the number of electrons that scatter off of the target and into the spectrometers, we are counting both electrons that have scattered off of the desired polarized protons and deuterons in the target as well as those that have scattered off other materials in the target such as nitrogen and helium. Since there is no asymmetry in the scattering off of such materials, the measured asymmetry is effectively diluted. For this reason, we have to apply a **dilution factor** correction to the measured rates so that our electron rates that form the asymmetry can be presumed to come only from our desired polarized target nucleons. The raw asymmetry formed from electron rates can be written as follow:

$$A_{raw} = \frac{N_p^{\downarrow\uparrow} - N_p^{\uparrow\uparrow}}{N_p^{\downarrow\uparrow} + N_p^{\uparrow\uparrow}} = \frac{N_p^{\downarrow\uparrow} - N_p^{\uparrow\uparrow}}{N_p^{\downarrow\uparrow} + N_p^{\uparrow\uparrow} + N_{other}^{\downarrow\uparrow} + N_{other}^{\uparrow\uparrow}} \quad (5.8)$$

where N_p is the number of electrons scattered off of the polarized protons and N_{other} is the number of electrons scattered off all other materials in the target. The contribution from the other materials cancel in the numerator since there is no asymmetry from them. To cancel the rates from the “other” materials in the denominator, we must divide the A_{raw} by the dilution factor, f , defined as:

$$f = \frac{N_p^{\downarrow\uparrow} + N_p^{\uparrow\uparrow}}{N_p^{\downarrow\uparrow} + N_p^{\uparrow\uparrow} + N_{other}^{\downarrow\uparrow} + N_{other}^{\uparrow\uparrow}}. \quad (5.9)$$

For the deuteron, the rates N_p are replaced by the rates N_d . Here the rates have not yet been radiatively corrected, and this correction will come at the end.

The number of electrons N scattered by a particular material is proportional to:

$$N = \frac{\rho l}{A} (N_p \sigma_p + N_n \sigma_n) g, \quad (5.10)$$

where ρ is the material density, l is the length of the material, A is the atomic mass number of the nucleus, N_p and N_n are the respective numbers of protons and neutrons in the nucleus of the material, σ_p and σ_n are the proton and neutron cross sections, and $g(x, A)$ is the correction for the EMC effect needed for heavier nuclei taken at the kinematics (x_{BJ}) of the current bin and the mass number A of the heavy nucleus. This formula neglects constants such as Avogadro's number and the beam flux.

Packing Fraction

The target cell was filled with both ammonia and helium. To obtain the length, l , of each material, it was necessary to accurately measure the **packing fraction** p_f , or the percentage of volume occupied by NH_3 . The packing fraction was measured in three different ways. Because the NH_3 density is known, we could first find the packing fraction by accurately measuring the weight of ammonia as it was put into the target cell. The goal was to measure the weight to $\pm 4\%$ so that the typical error on the dilution factor would be about 1.5%. However, it is difficult to estimate the accuracy of the weighing. The second method was to make an x-ray attenuation measurement of the target cells after they were used, which would potentially be very accurate except that some ammonia beads could be lost in transport, and the NMR coils are opaque to the xrays so that large corrections must be made.

The third method was to compare the relative rates in each spectrometer of the ammonia targets with the carbon and empty targets. Carbon was a good candidate because it is similar to the ammonia targets in weight and radiation

length, and its thickness was known to 1%. The ammonia and carbon targets typically had about 1.6gm of material and about 0.5gm of other material such as helium, aluminum, and NMR coils. The drawbacks were that we didn't take very many carbon runs and that the beam wasn't rastered as it was for ammonia. The empty target (actually filled with He⁴) was useful to compare with because there were many empty target runs and the beam was rastered. Yet because the ammonia and helium densities were so different we had to make a large correction for the ratio of radiative corrections of the two targets as well as careful corrections for rate-dependence in spectrometer efficiencies and for dead time.

To make the comparisons, a prediction was generated for the relative counting rates between the ammonia and the reference target, using a cross section model for each target component, which included the radiative and the EMC corrections. Then the measured ratio was compared to the predicted ratio and the ammonia weights were adjusted until the ratio approached unity. The packing fractions obtained using carbon as a reference agreed with those using the empty target, and each spectrometer agreed as well. Because this third method turned out to be the most reliable, packing fractions generated by averaging the results from both carbon and empty target comparisons were used in the analysis. The numbers used are shown in Table 5.3.

With the packing fraction known, we can express the number of electrons scattered off of the ammonia, the hydrogen, and the helium as follows:

$$N_{NH_3} = \frac{\rho_{NH_3} l_{cell} p_f}{18} [3\sigma_p + (7\sigma_p + 8\sigma_n)g_N] \quad (5.11)$$

$$N_H = \frac{\rho_{NH_3} l_{cell} p_f}{18} [3\sigma_p] \quad (5.12)$$

$$N_{He} = \frac{\rho_{He} l_{cell} (1 - p_f)}{4} [2\sigma_p + 2\sigma_n]. \quad (5.13)$$

where 18 is the atomic mass number of NH₃, l_{cell} is the length of the target cell, and g_N is the EMC effect correction for nitrogen nuclei.

Table 5.3: Packing fractions p_f used for the dilution factor calculation for each target, labelled by insert number. For run numbers 2482 through 2817, data were collected only on $^{15}\text{NH}_3$ targets.

	Insert No.	Used for Run Nos.	p_f
$^{15}\text{NH}_3$	1	≤ 1205	0.665 ± 0.000
	2	1206-2289	0.643 ± 0.017
	3	2290-2481 & 2818-3378	0.588 ± 0.010
	4	2482-2817	0.570 ± 0.043
	5	2482-2817	0.574 ± 0.040
$^{15}\text{ND}_3$	1	≤ 1205	0.502 ± 0.000
	2	1206-2289	0.632 ± 0.012
	3	2290-2481 & 2818-3378	0.600 ± 0.012

Here we assume that the portion of the cell not filled with ammonia is filled with He^4 . The other materials in the target that electrons might encounter include the aluminum vacuum windows, the aluminum liquid nitrogen shield windows, the titanium tailpiece windows, the aluminum target windows, the copper/nickel NMR coils, the helium outside of the target cell, and the helium bag were all measured or modelled and taken into account. Finally, the dilution factor is:

$$f = \frac{N_H}{N_{\text{NH}_3} + N_{\text{He}} + \Sigma N_{\text{others}}} \quad (5.14)$$

The ratio σ_n/σ_p is approximately equal to F_2^n/F_2^p , which is calculable from the well-measured F_2^d/F_2^p . The dilution factor was calculated for each run and each x bin separately. Since the EMC effect corrections depend on x and the unpolarized cross-sections F_1 and F_2 depend on x and Q^2 , the mean values of each bin were used. Also, the packing fractions varied for different target cells and materials. Typical dilution factors for the proton targets are plotted as a function of x in Figure 5.12 and for the deuteron in Figure 5.13. Typical values were 0.16 for NH_3 and 0.23 for ND_3 , indicating that about one in six counts originated from hydrogen for NH_3 and one in four from deuterons in ND_3 .

Figure 5.12: Proton target dilution factor for both spectrometers.

Figure 5.13: Deuteron target dilution factor for both spectrometers.

Errors are predominantly from packing fractions and radiative corrections.

5.3.3 Nitrogen Correction

In measuring the proton and the deuteron asymmetries, it was necessary to correct for events which scattered from other polarizable nuclei in the target aside from the desired protons or deuterons. The targets were made of $^{15}\text{NH}_3$ and $^{15}\text{ND}_3$, and both the ^{15}N and the $\approx 2\%$ contamination of ^{14}N were polarizable. In addition, the $^{15}\text{ND}_3$ target was contaminated to $\approx 1.5\%$ with residual polarizable protons from $^{15}\text{NH}_3$.

The polarization of ^{15}N and the residual protons was measured after the experiment by selecting a slightly different resonance frequency with the target NMR, and a fit was made to obtain the polarization. We know that an ^{16}O nucleus has an equal number of protons and deuterons, which pairwise add up to spin zero and therefore to zero magnetic moment. Removing one proton from such a nucleus, leads to ^{15}N with a magnetic moment approximately equal but opposite in sign to the proton magnetic moment. Thus the correction for scattering from such a nucleus involves knowing the proton asymmetry.

For the target material $^{15}\text{NH}_3$, the following fit was used to express the ^{15}N polarization P_N in terms of the polarization of the protons P_p :

$$P_N = 0.136 P_p - 0.183 P_p^2 + 0.335 P_p^3. \quad (5.15)$$

The correction C_1 to the proton asymmetry ($C_2 = 0$) which is referred to in Eq. (5.2) is given by:

$$C_1 = 1 - \frac{1}{3} \frac{1}{3} \frac{P_N}{P_p} g_{\text{EMC}}(x). \quad (5.16)$$

Here $g_{\text{EMC}}(x)$ is the correction for the EMC effect taken at atomic mass number 15. The first factor $-\frac{1}{3}$ comes from Clebsch-Gordan coefficients involving the nitrogen wave function. The second factor $\frac{1}{3}$ reflects the fact that ammonia has three hydrogen atoms for each nitrogen atom. The error on the second term in C_1 was estimated to be about 20% relative. Here the contribution of ^{14}N to the asymmetry was neglected. Figure 5.14 shows the average nitrogen correction factor C_1 for the proton at 29 GeV.

For the target material $^{15}\text{ND}_3$, the correction was more complicated, since the proton asymmetry had to be used as one of the inputs. The ^{15}N polarization P_N is given by

$$P_N = -0.40 P_d, \quad (5.17)$$

Figure 5.14: Typical correction factor for nitrogen in NH_3 at 29 GeV.

where P_d is the polarization of the deuterons. The residual proton polarization is expressed as:

$$\begin{aligned} P_p^{res} &= 0.191 + 0.683 P_d \quad \text{for } P_d > 0.16 \\ &= 1.875 P_d \quad \text{for } P_d \leq 0.16 \end{aligned} \quad (5.18)$$

The polarization of ^{14}N was obtained from the measured ^{15}N polarization by assuming that the polarization was equal and opposite in sign to that of ^{15}N .

The corrections used in Eq. (5.2) for the $^{15}\text{ND}_3$ target are given by:

$$\begin{aligned} C_1 &= \frac{1}{1 - \eta_p + D_n/\gamma} \approx 1.02, \\ C_2 &= \frac{U_p F_2^p}{U_d F_2^d} (D_n - D_p) (A^p - A_{rc}) \approx -0.03 (A^p - A_{rc}). \end{aligned} \quad (5.19)$$

A^p is the final proton (Born) asymmetry $A_{\parallel}$ or $A_{\perp}$, and by subtracting the appropriate proton radiative correction A_{rc} we are left with the radiated asymmetry. U_p and U_d are the radiative corrections to the unpolarized cross-sections.

Model	Year	ω_D
Bonn [50]	1976	0.0440
Bonn [51]	1987	0.0425
Paris [52]	1980	0.0577
Reid [53]	1968	0.0647

Table 5.4: Calculations for the value of ω_D for various models. For E143 we use the value $\omega_D = 0.05 \pm 0.01$.

The remaining factors are defined as

$$\begin{aligned}
 \eta_p &= \frac{\text{number of protons}}{\text{number of deuterons} + \text{number of protons}} \approx 0.015, \\
 D_n &= \eta_N \frac{P_N}{P_d} \frac{g_{\text{EMC}}(x)}{9}, \\
 D_p &= \eta_p \frac{P_p^{\text{res}}}{P_d} + (2\eta_N - 1) \frac{P_N}{P_d} \frac{g_{\text{EMC}}(x)}{9}, \tag{5.20}
 \end{aligned}$$

$$\begin{aligned}
 \eta_N &= \frac{\text{number of } ^{14}\text{N}}{\text{number of } ^{14}\text{N} + \text{number of } ^{15}\text{N}} \approx 0.02, \\
 \gamma &\simeq 1 - 1.5\omega_D \approx 0.925. \tag{5.21}
 \end{aligned}$$

Here ω_D is the probability of the deuteron to be in the D-state. Since it cannot be measured directly, it is inferred from phenomenological potentials such as the Bonn Potential [50] [51], the Paris-Potential [52], and others [53]. Table 5.4 shows these estimates. For E143 we use the value $\omega_D = 0.05 \pm 0.01$.

5.3.4 Charge-Symmetric Background Correction

The data collected in each of the spectrometers included some number of background events coming from a small amount of misidentified pions and from electron and positron pairs that were created by photons from the decay of π^0 particles. This background was responsible for up to 10% of the events

in the lowest x -bin, but close to zero events for $x > 0.3$. The pions and electrons from π^0 decay had no measurable asymmetry, and to avoid diluting the actual DIS electron asymmetry, a correction had to be applied. To do so, we took data at spectrometer momentum settings of the opposite momentum as the electron settings, *e.g.*, +11.5 GeV instead of -11.5 GeV. These runs were “positron” or “pion” runs, in which only π^+ and e^+ were accepted into the spectrometer data stream. Assuming that the production of π^+ particles was charge symmetric, and the π^+ particles were produced at the same rate as the π^- particles, and assuming the same for the e^+ ’s from pair production, we could subtract these background events using the counts obtained from the positron runs.

To apply the correction, a pure subtraction of rates would not account for the fact that these “positron” runs might have different target or beam polarizations, or other experimental conditions, than the electron runs we were correcting. Instead, we formed a positron run asymmetry, A_+ , applying the same cuts to get rid of pions as in the electron runs, and then subtracted it from the electron asymmetry, A_- , weighted by the rates in the following way:

$$A = A_- \frac{N_-}{N_- - N_+} - A_+ \frac{N_+}{N_- - N_+}. \quad (5.22)$$

Here, N_- , N_+ are the number of events per incoming charge for electron and positron runs. Mathematically, this is almost equivalent to subtracting events directly, except that differing experimental conditions are taken into account.

5.3.5 Radiative Corrections

The cross section for deep-inelastic scattering of electrons off nucleons is dominated by the one photon exchange process, which makes up the **Born cross section**, shown in Chapter 2, Figure 2.1. The actual scattering process that we measure, however, consists of other processes as well. For instance, a

Figure 5.15: Diagrams of processes that contribute predominantly to the radiative corrections.

photon can be exchanged between the incoming and outgoing electron lines (vertex correction), soft photons can be exchanged between the electron and nucleon lines, a photon can be emitted from the incoming or outgoing electron lines (bremsstrahlung), or a particle-antiparticle loop can form within the virtual photon propagator (vacuum polarization). Such processes cause the electrons to lose energy, which then shifts them into different x bins and changes the measured asymmetry. For this reason, we have to correct for such processes to obtain an accurate single-photon exchange measurement. The diagrams that contribute most significantly (to lowest order in α_{QED}) to the deep-inelastic cross section are shown in Figure 5.15. These processes are provided for in the **internal corrections** and they consist of two categories: the internal bremsstrahlung radiation diagrams are real photon processes, and the remainder are virtual photon processes. Note that nucleon vertex and nucleon bremsstrahlung processes were negligible and were not considered.

The incoming electron can also interact with external fields from materials which are not the desired target nucleons and which cause external Bremsstrahlung radiation, resulting in a loss of energy. This also leads to changes in the measured kinematics. At 29 GeV, typical energy loss due to

ionization within the external materials is 3 MeV–negligible at E143 kinematics.

To obtain the Born cross sections that we set out to measure, internal and external radiative processes had to be corrected for. Both the unpolarized cross-sections and the cross-section asymmetries had to be corrected. Since the coupling constant for Quantum Electrodynamics (QED), which describes the interaction, is small, very precise perturbative calculations to low order can be done to obtain the Born term. However, the measured cross-sections and the measured asymmetries were needed as inputs to the correction program, so an iterative process was used. The goal was to obtain the Born quantities (σ_o, A_o) from the measured quantities (σ_m, A_m) using the relations:

$$\sigma_o = R_c * \sigma_m \qquad R_c = \sigma_u / \sigma_r \qquad (5.23)$$

where σ_u and σ_r are the un-radiated and radiated quantities respectively. For the first pass we used the measured quantities as the un-radiated quantities, and then calculated the radiative correction. This correction was multiplied by the measured cross section and then passed to the program again as the un-radiated quantities and new corrections were found, and so on, iterating until convergence within errors so that

$$R_c = \frac{\sigma_u}{\sigma_r} = \frac{\sigma_o}{\sigma_m}. \qquad (5.24)$$

The unpolarized radiative corrections were needed for the dilution factor and the nitrogen correction. A target model was made that described the thicknesses and densities of each material that the electrons encountered, and a separate radiative correction was calculated for each. The carbon and empty target cross-sections and corrections were calculated separately as well. A multiplicative factor was found for each cross-section that included both internal and external radiative corrections. The code used was based on formulas from

Tsai [127]. Then the factors for each material were combined into one factor C such that:

$$\sigma_{actual} = C * \sigma_{Born}. \quad (5.25)$$

The asymmetries were corrected in a similar manner. Because the measured asymmetries were used as input to the calculations, a Q^2 -dependent fit to the 29, 16.2, and 9.7 GeV A_1 data was used assuming $g_2 = g_2^{WW}$ (see Equation 2.61) for the transverse asymmetry contribution. For the internal corrections, formulas were taken from Reference [128], and included terms for lepton vacuum polarization, electron vertex corrections, and internal bremsstrahlung. For the bremsstrahlung processes, the corrections were different for polarized and unpolarized cross-sections, so all the cross sections were written as a combination of polarized (p) and unpolarized (a) parts:

$$\sigma^{\downarrow\uparrow} = \sigma^a + \sigma^p \quad \sigma^{\uparrow\uparrow} = \sigma^a - \sigma^p \quad (5.26)$$

so that the asymmetry $A_{||}$ is

$$A_{||} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}} = \frac{\sigma^p}{\sigma^a}. \quad (5.27)$$

Then the Born cross sections could be written as:

$$\sigma_{int}^p = \sigma_{Born}^p (1 + \delta_V + \delta_{el}^p + \delta_{inel}^p) \quad (5.28)$$

$$\sigma_{int}^a = \sigma_{Born}^a (1 + \delta_V + \delta_{el}^a + \delta_{inel}^a). \quad (5.29)$$

Here the last two terms in each are corrections for the elastic and inelastic tails from internal bremsstrahlung processes. δ_V is the correction for the virtual photon processes, vacuum polarization and vertex, and is the same for polarized and unpolarized cross-sections.

After the cross-sections were corrected for internal radiative processes, they

Figure 5.16: Top: Radiative Corrections A_{RC} to the asymmetry $A_{||}^p$ for both spectrometers. Systematic errors are plotted. Bottom: Multiplicative correction factor f_{RC} where the asymmetry error $dA \rightarrow dA/f$.

were then corrected for external bremsstrahlung using the formula:

$$\sigma_{ext}(t_b, t_a, E_o, E_f) = \int \int I(E_o, E, t_b) \sigma_{int}(E, E') I(E', E_f, t_a) dE dE' \quad (5.30)$$

where E_o is the initial beam energy, E'_f is the final energy of the scattered electron, and t_a, t_b are the thicknesses of the material before and after the deep inelastic scattering occurs. $I(E_1, E_2, t)$ is the probability that a particle with initial energy E_1 has energy E_2 after passing through a radiator of thickness t . Then the radiated asymmetry becomes

$$A_{rad} = \frac{\sigma_{ext}^p}{\sigma_{ext}^a}. \quad (5.31)$$

The radiative correction to the radiated asymmetry can be applied as a multiplicative correction. Yet because the difference between the radiated and Born asymmetries is often large, the ratio can be very sensitive to the errors in the calculated asymmetries, an additive correction was used so that $A_{RC} = A_{Born} - A_{rad}$. Finally, the measured asymmetry is corrected as follows:

$$A_m^{\parallel} \pm dA_m^{\parallel} \rightarrow A_m^{\parallel} + A_{RC} \pm \frac{dA_m^{\parallel}}{f_{RC}} \quad (5.32)$$

Here it is assumed that the elastic tails are background that can be treated as a dilution effect $1/f_{RC}$. Figure 5.16 shows the radiative corrections as a function of x .

5.4 Asymmetry Results

5.4.1 Sign Change for $\cos \phi$ in $A_{\perp}$

As shown in Equation 2.28, the asymmetry for the transverse target position is defined by

$$A_{\perp} = \frac{\sigma^{\downarrow\leftarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\downarrow\leftarrow} + \sigma^{\uparrow\leftarrow}}. \quad (5.33)$$

Here the first indicates the beam helicity and the second, that of the target. The second arrow pointing to the left indicates that the target helicity is perpendicular to that of the beam, and by convention, on the *same side* as the scattered electron. However, since our 7° spectrometer was on the opposite side as the scattered electron in our coordinate system, we had to correct the sign in the asymmetries. This corresponds to $\cos \phi = -1$ in Equation 2.27. Therefore, we multiplied our results for the 7° $A_{\perp}$ by a factor of -1 before adding the radiative corrections. This was equivalent to moving the 7° spectrometer to the same side of the beam line as the 4.5° .

Figure 5.17: Raw longitudinal proton asymmetries $A_{\parallel}^p$ for the 4.5° and 7.0° spectrometers at 29 GeV.

Figure 5.18: Raw transverse proton asymmetries $A_{\perp}^p$ for the 4.5° and 7.0° spectrometers at 29 GeV.

Figure 5.19: Raw longitudinal deuteron asymmetries $A_{\parallel}^d$ for the 4.5° and 7.0° spectrometers at 29 GeV.

Figure 5.20: Raw transverse deuteron asymmetries $A_{\perp}^d$ for the 4.5° and 7.0° spectrometers at 29 GeV.

5.4.2 Raw Asymmetries

After finding all the appropriate corrections to the cross section asymmetries, we can form the proper, corrected asymmetries using equation 5.2. Figure 5.17 shows $A_{\parallel}^p$ for the proton 29 GeV longitudinal events. The transverse asymmetry $A_{\perp}^p$ for the proton is shown in Figure 5.18 at the same energy.

The deuteron asymmetries are likewise shown in Figure 5.19 and Figure 5.20 at 29 GeV. Error bars on these plots are statistical errors only. Using these asymmetries, we can reconstruct both the structure function g_1^p and g_1^d as well as the asymmetries A_1^p and A_1^d , described in Chapter 2. These will be shown in the next chapter.

5.4.3 Systematic Effects on Asymmetries

An important check to see if there is a systematic trend in our measured asymmetries due to experimental setup is to see if there is evidence of false asymmetries. During the experiment, data were taken with the target B-field pointing in two directions, east and west. For each direction, two different target polarization enhancement fields were used, parallel or antiparallel to the B-field. We then had four different field configurations. For our checks, we separated each of the different subsets of data corresponding to the four configurations, and compared the asymmetries by calculating the χ^2 of the data using:

$$\chi^2 = A(x)^2 / \sigma(x)^2. \quad (5.34)$$

Here $A(x)$ is the difference of the asymmetries from two of the configurations, and $\sigma(x)$ is the statistical error of $A(x)$ calculated by

$$\sigma(x) = \sqrt{\sigma_{pos}(x)^2 + \sigma_{neg}(x)^2}, \quad (5.35)$$

where σ_{pos} and σ_{neg} are the respective statistical errors on the asymmetries with positive or negative polarization or magnetic field. For the proton, the χ^2 distributions were all nicely centered at 1 and there were no points further than χ^2 of 2. In addition, when taking the mean values of the differences in the asymmetries, the mean values were almost all within one standard deviation from zero, and well within two. For the deuteron the results were slightly farther apart, yet still very reasonable. The χ^2 distributions of the asymmetry differences were centered around one, with few points greater than two, and the mean values of the differences of the asymmetries for each configuration were zero to within two standard deviations. We conclude that there were no significant systematic effects on the asymmetry due to changes in target B-field or target enhancement field directions.

Another check for systematic differences was done by dividing the target cell into horizontal strips, and calculating the asymmetry in each section of the target. While the integral Γ_1^p was found to be stable within statistical errors for each of the different horizontal sections, the integral Γ_1^d showed larger fluctuations, yet still remained within error. Nevertheless, Γ_1^p showed a slight decreasing trend from the bottom to the top of the target. Yet the trend was small enough that we still conclude that there are no significant systematic effects on the asymmetries due to the location of the scattering within the target.

Chapter 6

Spin Structure Results

6.1 Asymmetries to Structure Functions

6.1.1 Virtual Photon-Absorption Asymmetries

After the raw asymmetries $A_{\parallel}$ and $A_{\perp}$ have been extracted, they can be used to obtain the virtual photon-absorption asymmetries (Equation 2.48):

$$\begin{aligned} A_1 &= \frac{1}{D'} \left[A_{\parallel} (1 + \gamma^2 y/2) - A_{\perp} \frac{\gamma^2 y}{2 \tan(\theta/2)} \right], \\ A_2 &= \frac{\gamma(2-y)}{2D'} \left[A_{\parallel} + A_{\perp} \frac{y(1 + \gamma^2 y/2)}{(1-y) \sin \theta} \right]. \end{aligned} \tag{6.1}$$

Figure 6.1 shows the proton asymmetry A_1^p at 29 GeV as measured by each spectrometer. While the values of x reached by the two spectrometers overlap, the Q^2 values for the data in each can differ by more than a factor of two. Comparing the two plots, we can see that A_1^p is approximately independent of Q^2 . The transverse asymmetry A_2^p is displayed in Figure 6.2 for the proton at 29 GeV. We see that within the errors of the data, A_2^p is likewise approximately

Figure 6.1: The virtual photon-absorption asymmetry A_1^p for the proton at 29 GeV is shown here for each spectrometer.

independent of Q^2 .

Combining Data from Each Spectrometer

At the kinematics of E143, the structure of the nucleons exhibit scaling to a large degree. As discussed in Section 2.1.1, this implies that the structure functions are independent of Q^2 . Historically, to compare data from different experiments, and in this case, from our two spectrometers, which cover different kinematical regions, the data were averaged together within the x -bins assuming that the asymmetries A_1 and A_2 are independent of Q^2 . This has proven to be a good approximation to reality. Therefore, to combine the E143 data on the virtual photon cross-sections and compare them to the world data, the data were simply averaged together without taking into account the Q^2 of the events.

In general, for each x -bin within the spectrometer data set, the average x

Figure 6.2: The virtual photon-absorption asymmetry A_2^p for the proton at 29 GeV is shown here for each spectrometer.

and Q^2 values were found by adding all of the values and dividing by the number of events in that bin. When combining the data from the two spectrometers, however, the Q^2 values were averaged together weighted by the errors on the asymmetries $dA_1^p(x)$ and $dA_2^p(x)$. This was done because the kinematical factors were different for each spectrometer giving different weights to the number of events at different Q^2 values. The asymmetry values were combined similarly— weighted by the errors on the asymmetry values.

Figure 6.3 shows the overall asymmetries A_1^p and A_2^p for the proton with the E143 data combined. For completeness, the combined virtual photon-absorption asymmetries for the deuteron are shown in Figure 6.4 as well.

6.1.2 The Proton Structure Function g_1^p

We can obtain the longitudinal spin structure function g_1 of the proton using Equation 2.32 from Section 2.3.2:

$$g_1(x, Q^2) = \frac{F_1(x, Q^2)}{D'} \left[A_{\parallel} + \tan \frac{\theta}{2} A_{\perp} \right]. \quad (6.2)$$

Figure 6.3: The virtual photon-absorption asymmetries A_1^p and A_2^p for the proton at 29 GeV are shown here for both spectrometers combined.

Figure 6.4: The virtual photon-absorption asymmetries A_1^d and A_2^d for the deuteron at 29 GeV are shown here for both spectrometers combined.

Figure 6.5: The proton spin structure function g_1^p for each spectrometer plotted at the experimental x and Q^2 values.

The E143 results on g_1^p for both the 4.5° and the 7.0° spectrometers are shown in Figure 6.5. These data are at their x and Q^2 values as measured in the experiment.

Evolving the Data to Constant Q^2

To compare the data to those from other experiments, as well as to use g_1^p to evaluate the sum rules described in Chapter 2, the data need to be at a constant Q^2 . There are differing opinions on what assumptions can be made in order to evolve the data in Q^2 . Up until E143, experimental measurements were given by evolving the data using the assumption that A_1 and A_2 are independent of Q^2 . This is the same assumption that was made in combining the different Q^2 values of the two spectrometers for the virtual photon-absorption asymmetries in Figure 6.3. However, the precision of the E143 structure function measurements is such that such assumptions can only now make a difference in the outcome of the measurements.

Since scaling approximately holds at the E143 kinematics, another option

in the data evolution is to assume that g_1/F_1 is independent of Q^2 . The motivation for this is that this quantity is much less sensitive to contributions from $A_\perp$, which has not been measured as precisely as necessary to be confident in its behavior with Q^2 . However, in reality both assumptions are incorrect, since scaling is only approximate, so we would like to choose the method that employs the *best* assumption of the two.

As discussed in Section 2.4.1, the asymmetry A_2 has the positivity constraint that $|A_2| < \sqrt{R}$. As we can see in Figure 6.3, and as is discussed in References [19] [106], our data on A_2^p were not consistent with zero. At infinite Q^2 , however, A_2 must approach zero since R goes to zero. This requires that A_2 changes with Q^2 , and could possibly have the small Q^2 -dependence $A_2^p/\sqrt{Q^2}$ [129]. Although it is currently not completely clear which method of evolving the data is better, we choose to assume that g_1/F_1 is independent of Q^2 , and we make this assumption both in merging the two data sets from the two spectrometers and in evolving the data in Q^2 .

$g_1^p(x)$ at Common Q^2

Figure 6.6 shows the combined E143 data at a common $Q_o^2 = 3$. To obtain these data, the quantity g_1/F_1 at the experimental kinematics was multiplied by $F_1(x, Q_o^2)$. This is the method used in the published E143 results [21] as well. Because the unpolarized structure function F_1 increases at low x , a small low- x asymmetry $A_1 \sim g_1/F_1$ turns into a large structure function value at low x , as can be seen in the plot.

It is common in the literature to see the data displayed in the form xg_1^p versus x as well. Multiplying g_1 by x not only decreases the error bars at low x so that the structure is more visible, it also has the feature that if plotted versus $\log x$, the area under the xg_1 curve is proportional to the integral over x in the data region. The quantity is displayed in Figure 6.7.

Figure 6.6: The proton spin structure function g_1^p for the 29 GeV data combined. The data have been evolved to the mean Q^2 of E143, $Q_o^2 = 3 \text{ GeV}^2$. The systematic error on g_1^p as a function of x is indicated by a band at the bottom of the plot.

Figure 6.7: The proton spin structure function g_1^p multiplied by x for the 29 GeV data combined. The data are at $Q_o^2 = 3 \text{ GeV}^2$. The area under the data is proportional to the integral over x .

6.2 The First Moment of $g_1(x)$

The integral $\int_0^1 g_1 dx$, also known as the first moment of the longitudinal proton spin structure function g_1 can be used to compare to the theoretical sum rules presented in Section 2.11. To find the integral, it is necessary to integrate our data on g_1^p and g_1^d over the x -range obtained in the experiment, and then to extrapolate g_1 at high- x to $x = 1$ and at low- x to $x = 0$. Then we add together the area under the data curve with the area under both extrapolation regions to obtain the value of the integral.

6.2.1 Data Region

The integration over the data region was done for the values $0.029 < x < 0.800$. The value I calculated for the first moment in the measured range is

$$\Gamma_1^p(data) = \int_{0.029}^{0.80} g_1^p(x, 3) dx = 0.123 \pm 0.003 \quad (6.3)$$

$$\Gamma_1^d(data) = \int_{0.029}^{0.80} g_1^d(x, 3) dx = 0.042 \pm 0.003 \quad (6.4)$$

for the average E143 $Q_o^2 = 3 \text{ GeV}^2$.

6.2.2 Low- x Extrapolation

The extrapolation of g_1 from the measured region down to $x = 0$ is not well understood. Various theoretical models exist for the behavior of g_1 at low x , but there is no agreement on which one, if any, is correct. Since it is not clear how much twist-2 and twist-3 corrections contribute at low x , but it is expected that they *do* contribute some amount (see Section 2.8.2), for the analysis we choose to vary the method of extrapolation and then to assign an error based

on the variation of the values this gives for the resulting low- x integral.

One of the favorite models that is currently considered for g_1 at low x uses Regge-type behavior, and is of the form [130] [131] [132]:

$$g_1 = ax^\alpha, \quad (6.5)$$

with $0.05 \geq \alpha \geq 0$. We considered this the standard fit for the extrapolation. However, it is important to look at how far up in x this functional form is valid. The structure function F_2 has been fit with this Regge form and was found to be stable up to a fitting limit of $x \approx 0.1$ [41]. For g_1 , I fit the data up to $x \approx 0.10$, or 11 bins. The data were fitted with both $\alpha = 0.5$ and $\alpha = 0$, and then the integral over the region below $x = 0.029$ was calculated for each fit. The two results, $I(\alpha = .5)$ and $I(\alpha = 0)$ were then averaged to obtain $\bar{I}$, and that was the value taken for the central value of the low- x integral.

The statistical error on the extrapolation came from the error on the fitting parameter “ a ”, and turned out to be very small for E143 data. For the total integral, it was added in quadrature with the statistical errors from the data and the high- x region.

The error due to the fit was a little more complicated. The first fit error was set to be $|I_0 - \bar{I}| = |I_{0.5} - \bar{I}|$, the difference between the median and extreme values of the functional fit. The second fit error was determined by varying the number of low- x data points (from 4 to 13), and re-fitting g_1 each time, using the value for α that corresponds to the exponent that would produce $\bar{I}$. Then the largest difference between $\bar{I}$ and these integral values is taken to be the second fit error. The two fit errors were combined in quadrature to obtain an overall fit error.

The systematic error was an estimate of the effect of the systematic errors of the data points used in the extrapolation. The relative systematic error on the data points within the fit was multiplied by the integral over the extrapolation

region, and this became the systematic error for the low- x extrapolation.

The result for the integral over the fitted low- x region was $\bar{I} = 0.0064 \pm 0.0010 \pm 0.0010 \pm 0.0030$, where the first error is statistical, the second systematic, and the third is the fit error. The value for α that corresponds to this integral value was $\alpha = 0.19$.

Alternative Low- x Extrapolation Forms

There have been other suggested methods to extrapolate to $x = 0$ from the last data points. First, it is useful to look at the above Regge-type extrapolation $g_1 = ax^\alpha$ with $\alpha \in [x, y]$ and examine the results if a different number of data points are used for the fit. Fitting the lowest 6 data points, rather than the 11 above, corresponds to fitting g_1 up to $x \approx 0.06$. The integral obtained is $\bar{I} = 0.0078 \pm 0.0010 \pm 0.0010 \pm 0.0030$, corresponding to $\alpha = 0.20$.

The SMC publications [11] fit their data to a constant in g_1 , corresponding to the same Regge fit as above but with $\alpha = 0$. In addition, low- x behavior of the form $g_1 = a \ln(1/x)$ has been suggested [132] [133].

The results of the first extrapolation, as well as these alternative methods are presented in Table 6.1.

Integral Over Low- x Region

As can be seen in Table 6.1, the results for the low- x integral from the various low- x fits to the form $g_1 = ax^\alpha$ are close in value. However, the results from the fits to the form $g_1 = a \ln(1/x)$ can be almost twice the size as the Regge fits. Since the best extrapolation is not yet known, and there is more agreement on the Regge-type low- x behavior than on the log behavior, for the low- x contribution to the integral I take the average of the four fits, using 6 or 11

Table 6.1: Values for $\int_0^{0.03} g_1(x, Q_0^2) dx$ at $Q_0^2 = 3 \text{ GeV}^2$ using various models for the low- x behavior of g_1 . Fits were done both to E143 data up to $x = 0.06$ and $x = 0.10$. The fits using α were done with the functional Regge form $g_1 = ax^\alpha$.

	x_{cut}	$\alpha : [0, 0.5]$	$\alpha = 0$	$\ln(1/x)$
Proton	0.10	0.0064	0.0091	0.0112
Deuteron	0.10	0.0014	0.0020	0.0033
Proton	0.06	0.0078	0.0103	0.0152
Deuteron	0.06	0.0008	0.0011	0.0017

low- x data points, and using a constant fit ($\alpha = 0$) or a fit with $\alpha \in [0, 0.5]$ as described above. The errors again include the range of values potentially obtained by varying the fitting range and the value of α . The result becomes

$$\Gamma_1^p = \int_0^{0.03} g_1^p(x, 3) dx = 0.0082 \pm 0.0010 \pm 0.0010 \pm 0.0030 \quad (6.6)$$

$$\Gamma_1^d = \int_0^{0.03} g_1^d(x, 3) dx = 0.0013 \pm 0.0010 \pm 0.0010 \pm 0.0020 \quad (6.7)$$

where again the first error is statistical, the second systematic, and the third is due to fitting.

6.2.3 High- x Extrapolation

The region in x from where the data ends to $x = 1$ contains resonances that are not well measured. For this reason, we have to use a theoretical model to approximate how the data on g_1 would behave so that we can obtain the integral over the entire region from $0 \leq x \leq 1$. The principle of duality [134] suggests that to extrapolate from the deep-inelastic region into the resonance region, we essentially need to smooth out resonance peaks into smooth curves, while not losing information for the integration.

We know from pQCD that at high x , more quarks are aligned parallel to the nucleon helicity than antiparallel. The difference is a factor of $(1 - x)^2$ [135], an effect called *helicity retention*. Let q^+ (q^-) be the quark distribution with helicity parallel (antiparallel) to the nucleon helicity. Then $q^+ = q^-(x)/(1 - x)^2$. When the two distributions are decoupled, we find that as x approaches one,

$$q^+ \approx (1 - x)^3 \quad (6.8)$$

$$q^- \approx (1 - x)^5 \quad (6.9)$$

Furthermore, the strange quark distribution follows the form $(1 - x)^5$ or higher as $x \rightarrow 1$.

The extrapolation to $x = 1$ was therefore done by fitting the three highest data points using the form

$$g_1(x) = A(1 - x)^3. \quad (6.10)$$

This fit was then used to find the integral of g_1 up to $x = 1$.

The extrapolation error is once again divided into a statistical part, a systematic part, and an error due to fitting. The systematic part of the error was due to the systematic errors on the three g_1 data points used in the fit. All of these errors were very small for the E143 results.

As in the high- x extrapolation, the statistical error was basically the error on the fitting parameter A . This was later combined quadratically with the statistical errors from the low- x extrapolation and from the data region.

The fitting error was estimated, as in the low- x region, based on how the results change when the fit was varied. In particular, the number of highest data points used in the fit was varied from two to six, and the integral was determined using each fit. The largest difference between these integrals and

the integral using three data points was taken to be the fit error.

The systematic error was estimated by examining how the extrapolation results would change if the three highest data points were raised or lowered by their systematic errors. For this, the integral over the high- x region was then multiplied by the relative systematic error on the points.

Integral Over High- x Region

The results of the integration over the high region of x are as follows, indicating that the high x region contributes very little to the integral of g_1 :

$$\Gamma_1^p(\text{high}) = \int_{0.8}^1 g_1^p(x) dx = 0.001 \pm 0.001 \pm 0.001 \pm 0.000 \quad (6.11)$$

$$\Gamma_1^d(\text{high}) = \int_{0.8}^1 g_1^d(x) dx = 0.000 \pm 0.001 \pm 0.000 \pm 0.000. \quad (6.12)$$

.

6.3 Systematic Errors

The systematic error was calculated for both A_1 and g_1 . For the high energy data, although $A_\perp$ was measured and used for the calculation of g_1 , only the systematic error due to $A_\parallel$ was included, since for $A_\perp$ the systematic error was negligible compared to the statistical error. Some of the contributions to the systematic error were multiplicative, and were the same for each point in x . However, others were dependent upon the value of x at which the data was taken.

6.3.1 Multiplicative Errors

- The **beam polarization** P_b was determined to be $\pm 2.5\%$. This is approximately ± 0.022 , relative, since the average beam polarization was around 85%. The error includes a contribution of $\pm 0.8\%$ to account for non-statistical fluctuations in the data; $\pm 1.6\%$ contribution from the single arm and coincidence Moller systematic errors, and a $\pm 1.7\%$ contribution from the uncertainty in the target foil polarization.

- The **target polarization** P_t had a relative systematic error for protons of 0.025, while for deuterons it was 0.04. The errors for each of the runs were treated as 100% correlated, since the error was determined by the spread in thermal equilibrium (TE) measurements (see Section 3.4.3), and the TE measurements were the same for large groups of runs. Included in the error is the 3% correction for the contribution due to the polarization of the torlon target cups, as well as corrections due to inhomogeneities in target polarization from local beam heating effects (see Section 3.4.4).

- The **nitrogen correction** for the proton contributed 0.004 relative to the systematic error, since the correction factor C_p (see Section 5.3.3) was stable at around 0.02 and the error on C_p itself was about 0.2 relative.

6.3.2 x -Dependent Errors

- The **dilution factor** error came from several different sources. The amount of ammonia in the target cell, and hence the density, was known to 4% for both targets. The cross-section σ_n/σ_p contributed an error of 1% [136]. It was one of the dominant errors for the NH_3 target, but did not contribute to the error on ND_3 . The EMC effect, or the ratio of nuclear to deuterium cross-sections was another large source of error, contributing 1.5% relative. It contains a 1% overall normalization error and a 1% uncorrelated error [64]. The mass of the

NMR coil was only known to 20%, but since it is so small it did not contribute to the total error significantly. The total error on the dilution factor f is then 2% for NH_3 and 1.5% for ND_3 .

- The **nitrogen correction** was applied using two constants C_1 and C_2 (see again Section 5.3.3). The factor C_1 was small and stable, so its error was neglected. However, the factor C_2 , which is applied to ND_3 only, contained the proton asymmetry, so the error had to be calculated for each x -bin using the measured proton asymmetry and its errors. The factor C_2 itself was determined for each run by averaging the C_2 values over the run weighted by the statistical error in each x -bin. For the systematic error, then, the average C_2 was multiplied by the relative error on the proton asymmetry to arrive at the systematic error.

- The error from the **radiative corrections** was calculated for each x -bin by varying the input models used. See Figure 5.16.

- The **structure function** $R(x, Q^2)$ contributed to g_1/F_1 differently than to g_1 due to the relationship between F_1 and F_2 . The effect of R is negligible for g_1 , whereas it is a significant error for g_1/F_1 , where R shows up in the kinematical depolarization factor D' . The error was taken directly from Reference [42] and ranged from 3% to 7.5%.

- For the g_1 calculation from g_1/F_1 , the systematic error due the ratio F_1/D' was needed. This turns out to be essentially the systematic error on on the **unpolarized structure function** F_2 . References [67] and [68] contain only the central value F_2^{NMC} , and no errors. For this reason, a subroutine based on Reference [137] was used, which provided the central value, F_2^{glob} , plus both a statistical error δF_2^{stat} and a systematic error. In addition, an overall normalization uncertainty was given for each target: $n = 0.021$ (proton) and

$n = 0.017$ (deuteron). The systematic error was determined using

$$\frac{\delta F_2}{F_2} = \sqrt{\left(\frac{\delta F_2^{stat}}{F_2^{glob}}\right)^2 + n^2 + \left(1 - \frac{F_2^{glob}}{F_2^{NMC}}\right)^2}. \quad (6.13)$$

The high- x error was larger due to discrepancies between F_2^{glob} and F_2^{NMC} . At $x < 0.08$, only the NMC data are used due to their superior accuracy for F_2 . In this region, the error was taken to be $\delta F_2/F_2 = 0.04$, obtained by the spread of the experimental data.

6.3.3 Merging Errors

In the region of overlap in x for the 4.5° and 7.0° spectrometers, the errors were combined weighted by the errors on g_1/F_1 or on g_1 . If the errors were correlated between the two spectrometers, they were linearly combine using

$$s = \frac{\frac{s_4}{\delta_4^2} + \frac{s_7}{\delta_7^2}}{\frac{1}{\delta_4^2} + \frac{1}{\delta_7^2}}. \quad (6.14)$$

Here s is being used for systematic errors and δ is being used for statistical. If the errors were uncorrelated, they were quadratically combined:

$$s^2 = \frac{\left(\frac{s_4}{\delta_4^2}\right)^2 + \left(\frac{s_7}{\delta_7^2}\right)^2}{\frac{1}{\delta_4^2} + \frac{1}{\delta_7^2}}. \quad (6.15)$$

The sole uncorrelated error was that due to the nitrogen correction on the deuteron, since the statistical errors on the proton asymmetry from which the nitrogen correction was calculated were uncorrelated in the two spectrometers.

Table 6.2: Listing of the major sources of systematic error and their contributions to the error on the integrals over the data region. The nitrogen error was negligible for g_1^n and for $g_1^p - g_1^n$.

	Γ_1^p	Γ_1^d	Γ_1^n	$\Gamma_1^p - \Gamma_1^n$
Rad. Corr.	0.0021	0.0024	0.0056	0.0067
Dil. Fact.	0.0037	0.0012	0.0046	0.0073
F_1/D'	0.0034	0.0010	0.0023	0.0055
Beam Pol.	0.0029	0.0011	0.0005	0.0033
Targ. Pol.	0.0030	0.0018	0.0052	0.0074
Nit. Corr.	0.0005	0.0001	—	—
Combined	0.0068	0.0036	0.0092	0.0139

For the systematic error on the neutron structure function and on the difference $g_1^p - g_1^n$, the errors due to the beam polarization, due to the unpolarized cross-sections in the dilution factor, and due to the normalization uncertainty (Equation 6.13) were taken to be 100% correlated. The other errors were all assumed to be uncorrelated.

For the integrals, the systematic error was found assuming all errors between the x bins were 100% correlated, except for the above-mentioned deuteron nitrogen correction. The systematic errors for the low- x and high- x extrapolations were added together along with that of the data region. This sum was quadratically combined with the fit errors for the two extrapolations to obtain the total systematic error on the integral. Table 6.2 shows the largest contributions to the systematic error for the various integrals over the data region.

6.4 Integrals of $g_1^p(x)$ and $g_1^d(x)$

Here we can combine the integrals over the data region, the high x region, and the low x region. The largest theoretical uncertainty comes from the lack of

Table 6.3: Results for the integration over x of g_1^p and g_1^d . The first error is statistical, the second systematic, and the third is the error due to fitting. The fitting error is combined with the systematic error for the final integral.

	Γ_1^p	Γ_1^d
Low x	$0.008 \pm 0.001 \pm 0.001 \pm 0.003$	$0.001 \pm 0.001 \pm 0.001 \pm 0.002$
Data Range	$0.123 \pm 0.003 \pm 0.007$	$0.042 \pm 0.003 \pm 0.004$
High x	$0.001 \pm 0.001 \pm 0.001 \pm 0.000$	$0.000 \pm 0.001 \pm 0.001 \pm 0.000$
Sum 0 to 1	$0.130 \pm 0.003 \pm 0.008$	$0.044 \pm 0.003 \pm 0.004$

information available about the behavior of the structure function g_1 at low x . Table 6.3 shows the contributions to each region of the integral as well as the full integral for the proton and deuteron. In the next chapter, we will extract the neutron structure function, and look at the E143 results compared to the Bjorken and the Ellis-Jaffe Sum Rules. We will also be able to extract the quark contribution to the helicity of the nucleons. In addition, the low-energy data sets will be presented, giving insight into the evolution of the structure functions with Q^2 .

6.5 Extraction of Neutron Information A_1^n and g_1^n

Since the deuteron is essentially a bound proton and neutron, we are able to extract the neutron information by basically subtracting the proton out in the proper way. The relationship between the deuteron asymmetries and structure function and those of the neutron is detailed in Section 2.7. The knowledge of deuteron spin states allows us to obtain both A_1^n and g_1^n (as well as it's first moment) using Equations 2.71 and 2.72. The asymmetry A_1^n is plotted in Figure 6.8 together with the data from E142. In Figure 6.9 the E143 g_1^n results are plotted at the average experimental Q^2 of 3 GeV². In

Figure 6.8: The neutron virtual photon asymmetry A_1^n from E143 high energy data and from E142. E143 bins have been averaged for clarity.

Figure 6.9: The neutron spin structure function g_1^n as measured by E143 at 29 GeV, by SMC, and by E142. The E143 bins are averaged, and two low- x SMC data points are not shown. The E143 systematic error is shown in the lower band.

addition, the results from E142 are plotted at the average experimental value $Q^2 = 2 \text{ GeV}^2$, and the SMC neutron results are plotted at $Q^2 = 10 \text{ GeV}^2$. For the E143 data, each point is the average of 3 bins from the original binning, except that the highest point contains 4 bins. This was done to clarify the plots. In Figure 6.9 two SMC data points below $x = 0.01$ were left off the plot.

Chapter 7

Discussion and Future

7.1 The Sum Rules

The integration of g_1^p and g_1^d (or g_1^n) over Bjorken x gives the first moments, Γ_1 , of the spin structure functions of the nucleons. To compare these values with those predicted by the sum rules, the sum rules have to be corrected. For this analysis, the sum rules are corrected to third-order in α_s [75]. At $Q^2 = 3 \text{ GeV}^2$, the average Q^2 of E143 data points, the value for α_s has been found to be $\alpha_s = 0.35 \pm 0.05$ [138], which is the value that has been used.

7.1.1 Test of Ellis-Jaffe Sum Rules

The integral of g_1^p over $x = [0, 1]$ for E143 was determined to be $\Gamma_1^p = 0.130 \pm 0.003 \pm 0.008$ [21] where the first error is statistical and the second error is a combination of systematic and fitting errors. The structure function was taken to be at the average experimental value of $Q^2 = 3 \text{ GeV}^2$. The Ellis-Jaffe Sum Rule corrected to third order in α_s at the same Q^2 is $\Gamma_1^p = 0.160 \pm 0.007$,

Table 7.1: The first moments of the proton, deuteron, and neutron as measured by E143 at $Q^2 = 3 \text{ GeV}^2$, SMC at $Q^2 = 10 \text{ GeV}^2$, and E142 at $Q^2 = 2 \text{ GeV}^2$. These are shown together with the prediction for the Ellis-Jaffe Sum Rule for $Q^2 = 3 \text{ GeV}^2$.

	Γ_1^p	Γ_1^d	Γ_1^n
E143	$0.130 \pm 0.003 \pm 0.008$	$0.044 \pm 0.003 \pm 0.004$	$-0.035 \pm 0.007 \pm 0.010$
SMC	$0.120 \pm 0.005 \pm 0.015$	$0.019 \pm 0.006 \pm 0.013$	$-0.078 \pm 0.016 \pm 0.020$
E142	—	—	$-0.031 \pm 0.006 \pm 0.009$
Pred.	0.161 ± 0.006	0.070 ± 0.004	-0.011 ± 0.006

however, which is three standard deviations from our experimental result. Furthermore, the value obtained by SMC is $\Gamma_1^p = 0.120 \pm 0.005 \pm 0.015$ [14] at $Q^2 = 10 \text{ GeV}^2$ can be compared to the Ellis-Jaffe prediction of $\Gamma_1^p = 0.171 \pm 0.005$. Again, the Sum Rule is violated by about three standard deviations.

The integral for the deuteron Γ_1^d for E143 was found to be $\Gamma_1^d = 0.044 \pm 0.003 \pm 0.008$ compared to the Ellis-Jaffe Sum Rule value of $\Gamma_1^d = 0.070 \pm 0.004$ at $Q^2 = 3 \text{ GeV}^2$. SMC obtained $\Gamma_1^d = 0.019 \pm 0.006 \pm 0.013$ at $Q^2 = 10 \text{ GeV}^2$ whereas the Ellis-Jaffe Sum Rule predicts $\Gamma_1^d = 0.071 \pm 0.004$. The Sum Rule is again violated, this time by more than 5 standard deviations for our E143 result.

Finally, the E143 value for Γ_1^n at $Q^2 = 3 \text{ GeV}^2$ is $\Gamma_1^n = -0.035 \pm 0.007 \pm 0.010$, while the Ellis-Jaffe Sum Rule value is $\Gamma_1^n = -0.011 \pm 0.006$. E142 found $\Gamma_1^n = -0.031 \pm 0.006 \pm 0.009$ [15] at $Q^2 = 2 \text{ GeV}^2$, and SMC obtained $\Gamma_1^n = -0.078 \pm 0.016 \pm 0.020$ at $Q^2 = 2 \text{ GeV}^2$ (extracted from the reported value for Γ_1^p and for $\Gamma_1^{NS} = \Gamma_1^p - \Gamma_1^n$). Table 7.1 shows the various values obtained for the first moment of g_1 .

Table 7.2: Contributions to the Bjorken Sum Rule for different regions in x as well as the total integral for the E143 result. The first error is statistical, the next systematic, and the third is a fit error, if appropriate. The Bjorken Sum Rule prediction for $Q^2 = 3$ is shown at the bottom for comparison.

x Region	$\Gamma_1^p - \Gamma_1^n$
Low x	$0.010 \pm 0.001 \pm 0.001 \pm 0.005$
Data Range	$0.155 \pm 0.009 \pm 0.014$
High x	$0.001 \pm 0.001 \pm 0.000 \pm 0.001$
0 to 1	$0.165 \pm 0.009 \pm 0.016$
Bj Pred.	0.171 ± 0.009

7.1.2 Test of Bjorken Sum Rule

If one takes the flavor non-singlet combination of the nucleon spin-dependent structure functions, $g_1^{NS} = g_1^p - g_1^n$, and integrate it with respect to x_{BJ} from 0 to 1, one arrives at the Bjorken Sum Rule as discussed in Section 2.11. Evaluating our E143 data at $Q^2 = 3 \text{ GeV}^2$, we obtain $\Gamma_1^p - \Gamma_1^n = \int_0^1 dx(g_1^p - g_1^n) = 0.165 \pm 0.009 \pm 0.016$. This can be compared with the Sum Rule prediction of $\Gamma_1^p - \Gamma_1^n = 0.171 \pm 0.009$ for the same Q^2 . At $Q^2 = 10$, SMC obtains the value $\Gamma_1^p - \Gamma_1^n = 0.198 \pm 0.023$ [14]. All of these values agree within errors. Table 7.2 shows the contributions from the data region and the extrapolation regions to the E143 Bjorken Sum Rule result.

7.2 Constituent Quark Model

As discussed in Section 2.11.3, after obtaining the integrals Γ_1 of the nucleon spin structure functions we are able to extract the contribution of the quarks Δq to the nucleon helicity. For the E143 proton, the quarks contribute $\Delta q = 0.28 \pm 0.09(\text{stat. and syst.})$. The value obtained by SMC [11] is $\Delta q = 0.22 \pm 0.10(\text{stat}) \pm 0.10(\text{syst.})$. My result for E143 includes third-order

Figure 7.1: Comparison of Δs and Δq from various experiments as compared to the Ellis-Jaffe prediction, which makes the assumption that $\Delta s = 0$.

leading twist QCD corrections, so that $Q^2 = \infty$ approximately holds. Yet higher order leading twist corrections and higher twist corrections have not been included, and there has been speculation that the higher twist contributions could be non-negligible, especially at the kinematics of E143. Assuming SU(3) flavor symmetry and assuming that the contribution from the strange quark $\Delta s = 0$, as was done for the Ellis-Jaffe Sum Rule, we would expect $\Delta q = 0.58$. However, in E143 we determined the strange sea contribution to the helicity of the **proton** to be $\Delta s = -0.10 \pm 0.03$, which agrees with the SMC [11] value of $\Delta s = -0.12 \pm 0.04(\text{stat}) \pm 0.04(\text{syst})$.

For the **deuteron**, this analysis found that the contribution to the helicity by the quarks is $\Delta q = 0.32 \pm 0.05$. We can compare this to the less-precise value found by SMC [11]: $\Delta q = 0.20 \pm 0.11$. The two are nevertheless in agreement. For the strange sea quark contribution, E143 finds $\Delta s = -0.09 \pm 0.02$, which is consistent with the SMC value of $\Delta s = -0.12 \pm 0.04$. The various results from the SLAC and CERN experiments as compared to the Ellis-Jaffe Sum

Rule are shown in Figure 7.1.

7.3 Results from the Lower Energy Data

The data that were taken at 9.7 GeV and 16.2 GeV were analyzed in much the same way as the 29 GeV data. The difference was that because we did not collect transverse data to obtain $A_\perp$ at these energies, we evaluated A_1 and g_1 assuming that the contribution from $A_\perp$ was negligible, and set $A_\perp = 0$. We could then obtain A_1 from Equation 2.48 and g_1 using Equation 2.32. The virtual photon-absorption asymmetry A_1 for the proton and deuteron is shown in Figure 7.2 for both the 9.7 GeV and 16.2 GeV data sets. Notice that a good portion of the 9.7 GeV data in the high- x region has been cut because of the resonances in that kinematical region. Figure 7.2 shows the structure function g_1 for both the proton and the deuteron at both energies. The deuteron results are less statistically precise than those of the proton for both energies.

The data from these sets were taken at relatively low Q^2 values, as indicated in Table 7.3. This implies that much of these data are from scattering off of nucleons and not single quarks. Furthermore, at low values of Q^2 , higher twist contributions are expected to become important, so that interpreting the meaning of the structure functions and how they relate to quark content becomes difficult. Both because of the opacity of the interpretation of the data, and because extrapolations necessarily become larger due to the smaller kinematical coverage of the data, integration of g_1 to obtain the first moment makes less sense than with the 29 GeV data. Nevertheless, the results of the integrals are presented in Table 7.3 along with the associated statistical errors.

Figure 7.2: The virtual photon-absorption asymmetry A_1 and the spin structure function g_1 for the proton and deuteron at 16.2 and 9.7 GeV.

Table 7.3: Kinematics at which each data set was taken as well as the integral of g_1 over the data region. Errors are statistical only.

Beam Energy	Range in x	Range in Q^2	$\Gamma_1^p(\text{data})$	$\Gamma_1^d(\text{data})$
29 GeV	0.029-0.800	1.27-9.53	0.123 ± 0.003	0.042 ± 0.003
16 GeV	0.020-0.500	0.47-3.02	0.101 ± 0.003	0.042 ± 0.006
9 GeV	0.033-0.245	0.31-1.03	0.043 ± 0.003	0.012 ± 0.004

7.4 Q^2 -Dependence of the Asymmetries

As mentioned earlier, in order to combine data taken at different values of Q^2 , it is necessary to make some assumptions as to the dependence on Q^2 of either the asymmetry A_1 or the quantity g_1/F_1 . Note that Equation 2.49 shows that for low x and high Q^2 , where $\gamma^2 \ll 1$, the two quantities are almost equivalent: $A_1 \approx g_1/F_1$. For the analysis presented in this thesis (as well as in the E143 publications), in which the first moments are determined with the data at a common $Q^2 = Q_o^2$, we used the assumption that g_1/F_1 is independent of Q^2 in general. However, it is a good idea to check these assumptions to see how valid they are.

7.4.1 Analysis

Because for the 9.7 GeV and the 16.2 GeV data sets there are no data for the transverse mode, one can set $A_\perp = 0$ as above, or one can set $g_2 = 0$ or $g_2 = g_2^{WW}$. Both of these models give very similar results [21]. In order to use a model for g_2 , Equation 2.47 and Equation 2.48 can be re-expressed as:

$$\begin{aligned}
 A_1 &= \frac{A_\parallel}{D'} \left[1 + \frac{xMy}{E + E' \cos \theta} \right] - \frac{g_2}{F_1} \left[\frac{4xME' \cos^2(\theta/2)}{\nu(E + E' \cos \theta)} \right] \\
 g_1 &= \frac{A_\parallel F_1}{D'} \left[\frac{E + E'}{E + E' \cos \theta} \right] + g_2 \left[\frac{2Mx}{E + E' \cos \theta} \right].
 \end{aligned} \tag{7.1}$$

These equations can then be used to obtain the asymmetry A_1 and the quantity g_1/F_1 for each of the two low energy data sets using the $A_{||}$ data and a g_2 model.

Since we used the assumption that g_1/F_1 is independent of Q^2 , we checked to see that within a certain range in x , the values of g_1/F_1 agree regardless of the momentum-transfer Q^2 at which the measurement is made. Figure 7.3 and Figure 7.4 show the proton and deuteron low energy data, obtained using the model $g_2 = g_2^{WW}$, as well as the 29.1 GeV data obtained using the measure $A_{\perp}$. In addition, the measurements from EMC [8], SMC [11, 12, 13, 9], SLAC E80 [6], and SLAC E130 [7] are included in the plot. All of the data, including the SMC data at $Q^2 \leq 1$ (GeV/c)², were fit to the parameterization

$$\frac{g_1}{F_1} \sim ax^{\alpha}(1 + bx + cx^2)[1 + Cf(Q^2)]. \quad (7.2)$$

The fits included three different models for the form of the Q^2 -dependence assumed:

- $f(Q^2) = 0$: No Q^2 -dependence.
- $f(Q^2) = 1/Q^2$: Higher twist behavior.
- $f(Q^2) = \ln(1/Q^2)$: pQCD behavior.

The coefficients of the fits are shown in Table 7.4. Also shown are the minimum Q^2 cuts in column 2. The χ^2/df is close to unity for the proton data in all of the fits except for fit I. Since this fit didn't include a Q^2 -dependence, this implies that there is a dependence on Q^2 for $Q^2 \leq 1$ (GeV/c)². The data are not strong enough to determine what the functional form of the dependence on Q^2 is. However, fit II shows that for $Q^2 \geq 1$ (GeV/c)², a straight-line fit in Q^2 is consistent with the data. Since our deep-inelastic scattering cut-off is at $Q^2 = 1$, we see that evolving the data to $Q^2 = Q_0^2$ assuming that g_1/F_1 is independent of Q^2 is a valid method, at least for the current experimental precision of the data.

For the deuteron data, the fits are not as good. Again we see, though, the the data agree more with the need for a Q^2 -dependence when extending the

Figure 7.3: g_1^p / F_1^p as a function of Q^2 for 8 different x -bins. Included are data from this experiment (solid circle), SMC [11] [13] (open circle), EMC [8] (squares), SLAC E80 [6] (triangles), and SLAC E130 [7] (diamonds). The dashed curves correspond to global fits II in Table 7.4 with no Q^2 -dependence, and the solid curves correspond to global fits III which contains a Q^2 -dependence. The dot-dashed curve shows the E154 NLO pQCD fit [25].

Figure 7.4: g_1^d/F_1^d as a function of Q^2 for 8 different x -bins. Included are data from this experiment (solid circle), and from SMC [11] [13] (open circle). The dashed curves correspond to global fits II in Table 7.4 with no Q^2 -dependence, and the solid curves correspond to global fits III which contains a Q^2 -dependence. The dot-dashed curve shows the E154 NLO pQCD fit [25].

Table 7.4: Fit parameters for the data shown in Figures 7.3 and 7.4. Data was fit to the form $ax^\alpha(1 + bx + cx^2)[1 + Cf(Q^2)]$ with different inputs for the dependence on Q^2 . The χ^2 for the fits as well as the number of degrees of freedom *d.f.* are shown in the last columns. Fits use statistical errors only. Fits V are to A_1 rather than g_1/F_1 .

Fit To:	Q_{min}^2	$f(Q^2)$	α	a	b	c	C	χ^2	<i>d.f.</i>
I. g_1^p/F_1^p	0.3	none	0.62	0.641	2.231	-2.666	0.000	145	117
II. g_1^p/F_1^p	1.0	none	0.64	0.749	1.466	-1.982	0.000	112	106
III. g_1^p/F_1^p	0.3	$1/Q^2$	0.62	0.762	1.434	-1.917	-0.160	116	116
IV. g_1^p/F_1^p	0.3	$-\ln(Q^2)$	0.66	0.728	0.850	-1.384	-0.100	120	116
V. A_1^p	0.3	$1/Q^2$	0.66	0.898	0.595	-0.371	-0.180	118	116
I. g_1^d/F_1^d	0.3	none	1.52	2.439	-1.718	0.867	0.000	122	111
II. g_1^d/F_1^d	1.0	none	1.46	2.222	-1.666	0.829	0.000	115	100
III. g_1^d/F_1^d	0.3	$1/Q^2$	1.44	2.342	-1.724	0.902	-0.260	119	110
IV. g_1^d/F_1^d	0.3	$-\ln(Q^2)$	1.48	2.030	-1.812	0.979	-0.100	120	110
V. A_1^d	0.3	$1/Q^2$	1.46	2.493	-1.915	1.376	-0.260	119	110

allowed range down to $Q^2 = 3$ (GeV/c)². Fit V, with a minimum Q^2 of 3 (GeV/c)², was used to iterate the radiative corrections and to evaluate g_2^{WW} .

Also shown in Figures 7.3 and 7.4 in the dot-dash lines are the leading order pQCD evolution fits to the world data, including earlier results from E143, done by SLAC E154 [25]. Although data below $Q^2 = 1$ were not used in the pQCD fit, it is in fairly good agreement with the data in this region. Nevertheless, at high x the Q^2 -dependence is a bit exaggerated for the proton.

7.5 Conclusions

The results of E143 at SLAC have increased our knowledge of the nucleon spin structure dramatically within the kinematical range $0.03 < x < 0.8$ and $Q^2 > 1$. The high statistics measurements were of great precision compared to previous experiments. Yet E143 had several other positive features in addition

to the high statistics:

- E143 made the first useful measurement of the transverse asymmetry $A_{\perp}$, which allowed the extraction of g_1 from the 29 GeV data without any systematic error due to assumptions on $A_{\perp}$.
- Since both the deuteron and proton structure functions were measured in the same experiment, systematic errors were minimized for the extraction of the neutron structure function from these data.
- Data were taken at three different incident energies, and with two spectrometers, giving us up to 6 independent data points within a given x -bin, each with a different value of Q^2 . This allowed us to perform the first meaningful study of the Q^2 evolution of the asymmetry A_1 and the ratio g_1/F_1 .

Our result on the Bjorken sum rule, $\Gamma_1^p - \Gamma_1^n$, was found to be less than one standard deviation away from the theoretical value, and was therefore in good agreement with it. This confirmed the validity of isospin symmetry, the only assumption made in the sum rule. In addition, since the Bjorken sum rule contains QCD corrections up to $\mathcal{O}(\alpha_s)^3$ in order to correct for the measurement being at finite Q^2 , this also confirmed that pQCD describes the scaling violation of the spin structure functions well.

However, the results of Γ_1^p and Γ_1^n separately showed a violation of the Ellis-Jaffe sum rule. This violation is in agreement with previous experiments. However, since the Ellis-Jaffe sum rule was developed under the assumptions of SU(3) symmetry and no strange-sea polarization, it has now been accepted within the community that these assumptions are likely invalid. In particular, the E143 measurement of the strange sea polarization gives $\Delta s \sim -10\%$ for the proton, and $\Delta s \sim -9\%$ for the deuteron, rather large and negative values, and inconsistent with the Ellis-Jaffe assumption of $\Delta s = 0$.

Furthermore, we have found that the total contribution to the spin of the

nucleon due to the quarks, Δq ($\equiv \Delta \Sigma$), is about 30%. This is very different from the 100% value predicted by the naive QPM, as well as from other theories, such as the relativistic bag model [139], which predicts that close to 65% of the nucleon spin comes from the quarks. Our result, which is in agreement with other experiments, indicates that it is necessary to look to other models of nucleon spin, which include contributions from gluon polarization Δg , and from orbital angular momentum. In addition, the possibility of SU(3) symmetry breaking, and the resulting implications, should be explored.

Various other theoretical models explaining these constituent quark results exist. One lattice QCD calculation [140] has obtained a value for Δq of 18% with a 10% error, and another such calculation [141] has found 25% with an error of 12%. Both are consistent with world experimental results. The “Skyrme” model is under discussion as well. It contains zero quark polarization and massless quarks, and attributes all of the nucleon spin to orbital motion of the quarks. The necessary quark mass correction, as well as other features of the Skyrme model, have allowed [142] a quark contribution of $\leq 30\%$ of the nucleon spin, and is also in agreement with experimental results. Another suggested model, which would salvage the naive QPM, allows for a large gluon polarization [39], which then cancels with the orbital angular momentum to conserve angular momentum of the nucleon. In such a model, the total quark polarization goes to $\Delta q \sim 0.6$, with $\Delta s \sim 0$, consistent with the relativistic bag model. This model is also in agreement with the Bjorken sum rule.

It is clear that more experiments are necessary to further understand the spin structure of the nucleon. A measurement of the gluon polarization is key to developing a good model for nucleon spin. Also, as the experiments have become more precise, the error due to theoretical uncertainties have become important, such as in the extrapolations to low- and high- x for the integrals, as well as the assumptions of the behavior of the asymmetries with changing Q^2 . E143 has found that the assumptions commonly used— that either A_1 or g_1/F_1 is independent of Q^2 — are valid within error for the region $Q^2 \geq 1$. Yet

the behavior could be very different when extending into kinematical regions of low Q^2 , as suggested by the E143 data at $Q^2 < 1$. In addition, information at low- x is needed in order to determine the possible higher-twist contributions, so that the integral region can be extended to $x = 0$ with less uncertainty.

At SLAC, E154 and E155 have provided supplemental data on the proton, neutron, and deuteron spin, and have slightly extended the covered kinematical regions. Results from E155 are expected in early 1999. E155X, to run in early 1999, will also give us the best measurement of $A_\perp$, A_2 and g_2 to date. At CERN, SMC has just published a final paper with a significantly improved reanalysis of all their results [14], extending the world data to lower x than achievable in the SLAC experiments. The HERMES experiment at DESY in Germany has also published first results [30], and will continue to run with the intent of adding valuable knowledge to the field of nucleon structure.

To make progress beyond the reach of these experiments, the most important future measurements will be of the gluon polarization and its contribution to nucleon spin. Several upcoming experiments will measure Δg via quark-gluon fusion by studying open-charm processes, $\gamma g \rightarrow c\bar{c} \rightarrow \text{mesons}$. The COMPASS experiment at CERN will make such a measurement of Δg , as will HERMES, although the HERMES rates will be low and the error bars large. In addition, the RHIC (Relativistic Hadron and Ion Collider) machine at Brookhaven (BNL) is planning a spin program in which polarized hadron-hadron collisions will be used to measure the gluon polarization in the nucleon.

Clearly there is much more information to look forward to in the near future, from both experiment and theory, that will add to our understanding of the structure of the nucleon and its spin. $\square$

Appendix A

Tables of Results: Proton

A.1 Proton 4.5° $A_{\parallel}$ and $A_{\perp}$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\perp}$	$A_{\parallel}(\text{uncorr})$	$A_{\perp}(\text{uncorr})$
0.031	1.27	0.048 ± 0.026	0.086 ± 0.066	0.034 ± 0.021	0.082 ± 0.052
0.035	1.40	0.091 ± 0.019	0.038 ± 0.040	0.077 ± 0.015	0.034 ± 0.033
0.039	1.52	0.060 ± 0.016	-0.018 ± 0.027	0.046 ± 0.014	-0.022 ± 0.023
0.044	1.65	0.076 ± 0.015	0.015 ± 0.021	0.064 ± 0.013	-0.003 ± 0.019
0.050	1.79	0.083 ± 0.014	0.015 ± 0.019	0.072 ± 0.012	0.010 ± 0.017
0.056	1.92	0.082 ± 0.013	0.000 ± 0.017	0.072 ± 0.011	-0.004 ± 0.016
0.063	2.07	0.082 ± 0.012	0.010 ± 0.016	0.074 ± 0.011	0.005 ± 0.015
0.070	2.22	0.086 ± 0.011	0.010 ± 0.015	0.078 ± 0.011	0.005 ± 0.014
0.079	2.38	0.102 ± 0.011	-0.017 ± 0.014	0.096 ± 0.011	-0.022 ± 0.014
0.090	2.53	0.082 ± 0.012	0.003 ± 0.015	0.076 ± 0.012	-0.002 ± 0.014
0.101	2.69	0.114 ± 0.012	0.001 ± 0.015	0.109 ± 0.012	-0.004 ± 0.015
0.113	2.84	0.108 ± 0.012	-0.015 ± 0.015	0.104 ± 0.012	-0.020 ± 0.015
0.128	3.00	0.097 ± 0.013	0.001 ± 0.016	0.094 ± 0.013	-0.004 ± 0.016
0.144	3.15	0.086 ± 0.013	-0.020 ± 0.016	0.083 ± 0.013	-0.025 ± 0.016
0.162	3.30	0.113 ± 0.013	0.013 ± 0.016	0.110 ± 0.013	0.008 ± 0.016
0.182	3.45	0.110 ± 0.014	-0.006 ± 0.017	0.107 ± 0.014	-0.011 ± 0.017
0.205	3.59	0.097 ± 0.014	0.040 ± 0.017	0.094 ± 0.014	0.036 ± 0.017
0.231	3.73	0.118 ± 0.015	-0.004 ± 0.018	0.115 ± 0.015	-0.008 ± 0.018
0.259	3.85	0.107 ± 0.016	0.012 ± 0.019	0.105 ± 0.016	0.008 ± 0.019
0.292	3.98	0.096 ± 0.017	-0.033 ± 0.021	0.094 ± 0.017	-0.037 ± 0.021
0.329	4.09	0.110 ± 0.018	-0.038 ± 0.023	0.109 ± 0.018	-0.042 ± 0.023
0.370	4.20	0.081 ± 0.020	0.040 ± 0.025	0.079 ± 0.020	0.036 ± 0.025
0.416	4.30	0.141 ± 0.021	-0.012 ± 0.028	0.139 ± 0.021	-0.016 ± 0.028
0.468	4.39	0.140 ± 0.026	-0.057 ± 0.033	0.138 ± 0.026	-0.060 ± 0.033
0.526	4.47	0.139 ± 0.031	-0.032 ± 0.039	0.131 ± 0.030	-0.035 ± 0.038
0.593	4.55	0.066 ± 0.037	-0.053 ± 0.047	0.063 ± 0.036	-0.055 ± 0.046

Table A.1: Results for the proton at 29 GeV: $A_{\parallel}$ and $A_{\perp}$ for 4.5° spectrometer, both with and without radiative corrections. Statistical errors shown.

A.2 Proton 7.0° $A_{\parallel}$ and $A_{\perp}$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\perp}$	$A_{\parallel}(\text{uncorr})$	$A_{\perp}(\text{uncorr})$
0.079	3.17	0.159 ± 0.043	-0.070 ± 0.074	0.144 ± 0.040	-0.077 ± 0.067
0.090	3.48	0.118 ± 0.029	0.027 ± 0.042	0.102 ± 0.028	0.020 ± 0.039
0.101	3.79	0.143 ± 0.024	0.019 ± 0.031	0.133 ± 0.023	0.013 ± 0.030
0.113	4.11	0.158 ± 0.022	-0.010 ± 0.027	0.150 ± 0.021	-0.017 ± 0.026
0.128	4.43	0.164 ± 0.021	-0.037 ± 0.025	0.157 ± 0.020	-0.044 ± 0.024
0.144	4.78	0.159 ± 0.020	0.003 ± 0.023	0.154 ± 0.019	-0.003 ± 0.023
0.162	5.13	0.171 ± 0.019	0.013 ± 0.022	0.167 ± 0.019	0.007 ± 0.022
0.182	5.50	0.192 ± 0.019	0.039 ± 0.022	0.189 ± 0.019	0.032 ± 0.022
0.205	5.86	0.215 ± 0.019	0.019 ± 0.022	0.213 ± 0.019	0.013 ± 0.022
0.231	6.24	0.151 ± 0.019	-0.001 ± 0.023	0.148 ± 0.019	-0.007 ± 0.022
0.259	6.60	0.250 ± 0.020	0.006 ± 0.023	0.248 ± 0.020	-0.000 ± 0.023
0.292	6.97	0.197 ± 0.021	-0.017 ± 0.024	0.195 ± 0.021	-0.023 ± 0.024
0.329	7.34	0.195 ± 0.022	-0.029 ± 0.026	0.194 ± 0.022	-0.034 ± 0.026
0.370	7.69	0.190 ± 0.024	0.019 ± 0.028	0.188 ± 0.024	0.014 ± 0.028
0.416	8.04	0.244 ± 0.026	-0.053 ± 0.031	0.243 ± 0.026	-0.058 ± 0.031
0.468	8.37	0.223 ± 0.030	0.007 ± 0.035	0.221 ± 0.030	0.003 ± 0.035
0.526	8.68	0.233 ± 0.034	0.003 ± 0.040	0.232 ± 0.034	-0.000 ± 0.040
0.593	8.99	0.224 ± 0.041	-0.020 ± 0.048	0.222 ± 0.041	-0.023 ± 0.048
0.666	9.26	0.155 ± 0.051	-0.029 ± 0.060	0.152 ± 0.051	-0.032 ± 0.060
0.749	9.53	0.224 ± 0.069	-0.069 ± 0.081	0.218 ± 0.069	-0.070 ± 0.080

Table A.2: Results for the proton at 29 GeV: $A_{\parallel}$ and $A_{\perp}$ for 7.0° spectrometer, both with and without radiative corrections. Statistical errors shown.

A.3 Proton g_1/F_1 , A_1 and A_2 at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	A_2
0.031	1.27	$0.068 \pm 0.034 \pm 0.007$	0.061 ± 0.034	0.143 ± 0.109
0.035	1.40	$0.125 \pm 0.025 \pm 0.008$	0.122 ± 0.025	0.068 ± 0.066
0.039	1.52	$0.082 \pm 0.023 \pm 0.007$	0.084 ± 0.023	-0.027 ± 0.046
0.044	1.65	$0.110 \pm 0.021 \pm 0.007$	0.110 ± 0.021	0.007 ± 0.036
0.050	1.78	$0.126 \pm 0.020 \pm 0.008$	0.124 ± 0.020	0.031 ± 0.033
0.056	1.92	$0.129 \pm 0.020 \pm 0.008$	0.129 ± 0.020	0.008 ± 0.030
0.063	2.07	$0.137 \pm 0.020 \pm 0.008$	0.136 ± 0.020	0.026 ± 0.029
0.070	2.22	$0.150 \pm 0.020 \pm 0.009$	0.149 ± 0.020	0.028 ± 0.028
0.079	2.47	$0.190 \pm 0.020 \pm 0.018$	0.194 ± 0.020	-0.026 ± 0.028
0.090	2.77	$0.158 \pm 0.020 \pm 0.014$	0.157 ± 0.020	0.024 ± 0.028
0.101	3.10	$0.223 \pm 0.020 \pm 0.019$	0.222 ± 0.020	0.029 ± 0.028
0.113	3.40	$0.232 \pm 0.021 \pm 0.020$	0.236 ± 0.021	-0.007 ± 0.028
0.128	3.71	$0.233 \pm 0.021 \pm 0.020$	0.237 ± 0.022	-0.006 ± 0.028
0.144	4.03	$0.229 \pm 0.022 \pm 0.020$	0.234 ± 0.022	0.001 ± 0.029
0.162	4.38	$0.285 \pm 0.023 \pm 0.024$	0.282 ± 0.023	0.060 ± 0.030
0.182	4.72	$0.317 \pm 0.024 \pm 0.026$	0.314 ± 0.025	0.075 ± 0.031
0.205	5.06	$0.347 \pm 0.026 \pm 0.027$	0.337 ± 0.027	0.113 ± 0.033
0.231	5.39	$0.311 \pm 0.028 \pm 0.027$	0.314 ± 0.029	0.042 ± 0.036
0.259	5.72	$0.447 \pm 0.031 \pm 0.035$	0.447 ± 0.032	0.095 ± 0.039
0.292	6.04	$0.386 \pm 0.035 \pm 0.031$	0.405 ± 0.036	0.008 ± 0.044
0.329	6.39	$0.426 \pm 0.040 \pm 0.035$	0.454 ± 0.042	-0.002 ± 0.050
0.370	6.72	$0.417 \pm 0.047 \pm 0.033$	0.400 ± 0.049	0.172 ± 0.058
0.416	7.06	$0.611 \pm 0.055 \pm 0.052$	0.651 ± 0.059	0.041 ± 0.069
0.468	7.38	$0.615 \pm 0.068 \pm 0.054$	0.645 ± 0.072	0.127 ± 0.085
0.526	7.66	$0.673 \pm 0.083 \pm 0.058$	0.698 ± 0.091	0.183 ± 0.106
0.593	7.90	$0.596 \pm 0.107 \pm 0.048$	0.670 ± 0.118	0.092 ± 0.137
0.666	9.26	$0.490 \pm 0.164 \pm 0.042$	0.540 ± 0.181	0.080 ± 0.206
0.750	9.53	$0.749 \pm 0.237 \pm 0.070$	0.877 ± 0.265	0.062 ± 0.301

Table A.3: Results for the proton at 29 GeV: g_1/F_1 , A_1 and A_2 for both spectrometers combined. Statistical errors shown first, systematic last.

A.4 Proton 4.5° and 7.0° $g_1(x, Q^2)$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$g_1(Q^2)$ 4.5°	$\langle Q^2 \rangle$	$g_1(Q^2)$ 7.0°
0.031	1.27	0.261 ± 0.132	-	—————
0.035	1.40	0.438 ± 0.089	-	—————
0.039	1.52	0.263 ± 0.073	-	—————
0.044	1.65	0.320 ± 0.062	-	—————
0.050	1.79	0.331 ± 0.054	-	—————
0.056	1.92	0.308 ± 0.048	-	—————
0.063	2.07	0.294 ± 0.042	-	—————
0.070	2.22	0.290 ± 0.039	-	—————
0.079	2.38	0.326 ± 0.037	3.17	0.369 ± 0.104
0.090	2.53	0.249 ± 0.036	3.48	0.255 ± 0.065
0.101	2.69	0.333 ± 0.036	3.79	0.294 ± 0.049
0.113	2.84	0.302 ± 0.035	4.11	0.298 ± 0.042
0.128	3.00	0.261 ± 0.034	4.43	0.284 ± 0.037
0.144	3.15	0.220 ± 0.034	4.78	0.261 ± 0.032
0.162	3.30	0.280 ± 0.033	5.13	0.262 ± 0.029
0.182	3.45	0.258 ± 0.033	5.49	0.273 ± 0.026
0.205	3.59	0.221 ± 0.032	5.86	0.280 ± 0.024
0.231	3.73	0.250 ± 0.031	6.24	0.178 ± 0.023
0.259	3.85	0.214 ± 0.031	6.60	0.268 ± 0.021
0.292	3.98	0.174 ± 0.030	6.97	0.187 ± 0.020
0.329	4.09	0.181 ± 0.030	7.34	0.161 ± 0.019
0.370	4.20	0.120 ± 0.029	7.69	0.135 ± 0.017
0.416	4.30	0.175 ± 0.028	8.04	0.141 ± 0.015
0.468	4.39	0.142 ± 0.027	8.37	0.102 ± 0.014
0.526	4.47	0.108 ± 0.025	8.68	0.079 ± 0.012
0.593	4.55	0.039 ± 0.022	8.99	0.051 ± 0.009
0.666	—	—————	9.26	0.021 ± 0.007
0.750	—	—————	9.53	0.015 ± 0.005

Table A.4: Results for the proton structure function g_1 at 29 GeV for the 4.5° and 7.0° spectrometers at the experimental kinematics.

A.5 Proton $g_1(x, Q^2 = 3 \text{ GeV}^2)$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$g_1(3 \text{ GeV}^2)$
0.031	1.27	0.313 ± 0.158
0.035	1.40	0.512 ± 0.104
0.039	1.52	0.299 ± 0.083
0.044	1.65	0.357 ± 0.069
0.050	1.79	0.362 ± 0.059
0.056	1.92	0.331 ± 0.051
0.063	2.07	0.311 ± 0.045
0.070	2.22	0.303 ± 0.040
0.079	2.47	0.341 ± 0.036
0.090	2.77	0.253 ± 0.032
0.101	3.10	0.318 ± 0.029
0.113	3.40	0.295 ± 0.026
0.128	3.71	0.265 ± 0.024
0.144	4.03	0.234 ± 0.023
0.162	4.38	0.259 ± 0.021
0.182	4.72	0.257 ± 0.020
0.205	5.06	0.250 ± 0.019
0.231	5.39	0.198 ± 0.018
0.259	5.72	0.249 ± 0.018
0.292	6.04	0.185 ± 0.017
0.329	6.39	0.173 ± 0.016
0.370	6.72	0.142 ± 0.016
0.416	7.06	0.169 ± 0.015
0.468	7.38	0.135 ± 0.015
0.526	7.66	0.115 ± 0.014
0.593	7.90	0.078 ± 0.014
0.666	9.26	0.047 ± 0.016
0.750	9.53	0.046 ± 0.015

Table A.5: Results for the proton structure function g_1 at 29 GeV for both spectrometers at the average experimental Q^2 of 3 GeV². The Q^2 -evolution was performed with the assumption that g_1/F_1 is independent of Q^2 .

Appendix B

Tables of Results: Deuteron

B.1 Deuteron 4.5° $A_{\parallel}$ and $A_{\perp}$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\perp}$	$A_{\parallel}(\text{uncorr})$	$A_{\perp}(\text{uncorr})$
0.031	1.27	0.035 ± 0.030	-0.015 ± 0.083	0.039 ± 0.026	-0.017 ± 0.071
0.035	1.40	0.004 ± 0.021	-0.092 ± 0.055	0.007 ± 0.019	-0.095 ± 0.048
0.039	1.52	0.043 ± 0.019	0.092 ± 0.048	0.047 ± 0.017	0.090 ± 0.043
0.044	1.65	-0.011 ± 0.017	0.047 ± 0.040	-0.008 ± 0.016	0.045 ± 0.038
0.050	1.79	-0.009 ± 0.016	0.014 ± 0.035	-0.006 ± 0.015	0.011 ± 0.032
0.056	1.92	0.012 ± 0.015	0.032 ± 0.032	0.015 ± 0.014	0.030 ± 0.030
0.063	2.07	0.010 ± 0.014	-0.029 ± 0.030	0.013 ± 0.013	-0.032 ± 0.028
0.070	2.22	0.014 ± 0.013	-0.035 ± 0.028	0.017 ± 0.013	-0.037 ± 0.027
0.079	2.38	0.023 ± 0.013	0.017 ± 0.028	0.026 ± 0.013	0.015 ± 0.027
0.090	2.53	0.038 ± 0.014	0.005 ± 0.029	0.041 ± 0.014	0.002 ± 0.028
0.101	2.69	0.028 ± 0.014	0.039 ± 0.030	0.031 ± 0.014	0.037 ± 0.029
0.113	2.84	0.037 ± 0.015	-0.015 ± 0.031	0.039 ± 0.015	-0.018 ± 0.030
0.128	3.00	0.079 ± 0.015	-0.023 ± 0.031	0.081 ± 0.015	-0.025 ± 0.031
0.144	3.15	0.053 ± 0.016	0.011 ± 0.032	0.055 ± 0.016	0.008 ± 0.032
0.162	3.30	0.046 ± 0.016	-0.022 ± 0.034	0.047 ± 0.016	-0.024 ± 0.033
0.182	3.45	0.054 ± 0.017	-0.000 ± 0.035	0.055 ± 0.017	-0.003 ± 0.035
0.205	3.59	0.049 ± 0.017	0.004 ± 0.036	0.049 ± 0.017	0.002 ± 0.036
0.231	3.73	0.020 ± 0.019	-0.025 ± 0.038	0.020 ± 0.019	-0.028 ± 0.038
0.259	3.85	0.021 ± 0.020	0.127 ± 0.041	0.021 ± 0.020	0.125 ± 0.041
0.292	3.98	0.054 ± 0.021	-0.029 ± 0.044	0.053 ± 0.021	-0.030 ± 0.044
0.329	4.09	0.078 ± 0.023	0.027 ± 0.049	0.077 ± 0.023	0.026 ± 0.049
0.370	4.20	0.072 ± 0.026	0.047 ± 0.055	0.071 ± 0.026	0.045 ± 0.055
0.416	4.30	0.063 ± 0.030	-0.080 ± 0.064	0.062 ± 0.030	-0.082 ± 0.064
0.468	4.39	0.010 ± 0.036	-0.003 ± 0.075	0.009 ± 0.036	-0.005 ± 0.075
0.526	4.47	0.065 ± 0.043	-0.020 ± 0.090	0.064 ± 0.042	-0.022 ± 0.089
0.593	4.55	0.057 ± 0.052	-0.073 ± 0.111	0.057 ± 0.052	-0.075 ± 0.109

Table B.1: Results for the deuteron at 29 GeV: $A_{\parallel}$ and $A_{\perp}$ for 4.5° spectrometer, both with and without radiative corrections. Statistical errors shown.

B.2 Deuteron 7.0° $A_{\parallel}$ and $A_{\perp}$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\perp}$	$A_{\parallel}(\text{uncorr})$	$A_{\perp}(\text{uncorr})$
0.079	3.17	-0.020 ± 0.049	-0.112 ± 0.118	-0.016 ± 0.046	-0.115 ± 0.111
0.090	3.48	0.044 ± 0.033	0.086 ± 0.072	0.048 ± 0.032	0.082 ± 0.069
0.101	3.79	0.044 ± 0.027	0.026 ± 0.055	0.047 ± 0.026	0.023 ± 0.053
0.113	4.11	0.023 ± 0.025	-0.034 ± 0.049	0.026 ± 0.024	-0.037 ± 0.048
0.128	4.43	0.053 ± 0.024	-0.027 ± 0.045	0.057 ± 0.023	-0.030 ± 0.044
0.144	4.78	0.110 ± 0.023	0.008 ± 0.043	0.112 ± 0.022	0.005 ± 0.042
0.162	5.13	0.051 ± 0.022	-0.034 ± 0.042	0.053 ± 0.022	-0.037 ± 0.041
0.182	5.50	0.133 ± 0.022	0.034 ± 0.041	0.135 ± 0.022	0.031 ± 0.041
0.205	5.86	0.067 ± 0.022	-0.024 ± 0.042	0.069 ± 0.022	-0.027 ± 0.042
0.231	6.24	0.088 ± 0.023	-0.053 ± 0.043	0.089 ± 0.023	-0.056 ± 0.043
0.259	6.60	0.051 ± 0.024	0.013 ± 0.045	0.051 ± 0.024	0.010 ± 0.045
0.292	6.97	0.102 ± 0.026	-0.036 ± 0.048	0.101 ± 0.026	-0.039 ± 0.048
0.329	7.34	0.108 ± 0.028	-0.020 ± 0.052	0.107 ± 0.028	-0.023 ± 0.052
0.370	7.69	0.143 ± 0.030	-0.024 ± 0.057	0.142 ± 0.030	-0.026 ± 0.057
0.416	8.04	0.089 ± 0.034	0.040 ± 0.063	0.089 ± 0.034	0.038 ± 0.063
0.468	8.37	0.125 ± 0.039	-0.018 ± 0.073	0.125 ± 0.039	-0.020 ± 0.073
0.526	8.68	0.172 ± 0.046	-0.020 ± 0.085	0.172 ± 0.046	-0.022 ± 0.085
0.593	8.99	0.094 ± 0.056	0.160 ± 0.103	0.095 ± 0.056	0.157 ± 0.103
0.666	9.26	0.086 ± 0.070	-0.149 ± 0.131	0.087 ± 0.070	-0.151 ± 0.131
0.749	9.53	0.193 ± 0.096	-0.061 ± 0.176	0.193 ± 0.096	-0.063 ± 0.175

Table B.2: Results for the deuteron at 29 GeV: $A_{\parallel}$ and $A_{\perp}$ for 7.0° spectrometer, both with and without radiative corrections. Statistical errors shown.

B.3 Deuteron g_1/F_1 , A_1 and A_2 at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	A_2
0.031	1.27	$0.046 \pm 0.040 \pm 0.007$	0.046 ± 0.040	-0.023 ± 0.136
0.035	1.40	$-0.000 \pm 0.028 \pm 0.007$	-0.000 ± 0.028	-0.152 ± 0.091
0.039	1.52	$0.065 \pm 0.026 \pm 0.006$	0.065 ± 0.026	0.156 ± 0.080
0.044	1.65	$-0.013 \pm 0.025 \pm 0.006$	-0.013 ± 0.025	0.080 ± 0.068
0.050	1.78	$-0.013 \pm 0.024 \pm 0.005$	-0.013 ± 0.024	0.023 ± 0.060
0.056	1.92	$0.020 \pm 0.023 \pm 0.004$	0.020 ± 0.023	0.058 ± 0.057
0.063	2.07	$0.015 \pm 0.023 \pm 0.004$	0.015 ± 0.023	-0.053 ± 0.055
0.070	2.22	$0.022 \pm 0.023 \pm 0.004$	0.022 ± 0.023	-0.065 ± 0.053
0.079	2.47	$0.034 \pm 0.023 \pm 0.004$	0.034 ± 0.023	0.020 ± 0.054
0.090	2.77	$0.073 \pm 0.023 \pm 0.005$	0.073 ± 0.023	0.042 ± 0.054
0.101	3.10	$0.062 \pm 0.024 \pm 0.005$	0.062 ± 0.024	0.076 ± 0.053
0.113	3.40	$0.057 \pm 0.024 \pm 0.006$	0.057 ± 0.024	-0.039 ± 0.053
0.128	3.71	$0.128 \pm 0.025 \pm 0.007$	0.128 ± 0.025	-0.037 ± 0.054
0.144	4.03	$0.153 \pm 0.026 \pm 0.008$	0.153 ± 0.026	0.035 ± 0.055
0.162	4.38	$0.094 \pm 0.027 \pm 0.009$	0.094 ± 0.027	-0.048 ± 0.057
0.182	4.72	$0.198 \pm 0.029 \pm 0.010$	0.198 ± 0.029	0.063 ± 0.060
0.205	5.06	$0.126 \pm 0.031 \pm 0.009$	0.126 ± 0.031	-0.009 ± 0.064
0.231	5.39	$0.124 \pm 0.034 \pm 0.010$	0.124 ± 0.034	-0.079 ± 0.070
0.259	5.72	$0.097 \pm 0.039 \pm 0.011$	0.097 ± 0.039	0.160 ± 0.079
0.292	6.04	$0.202 \pm 0.044 \pm 0.015$	0.202 ± 0.044	-0.048 ± 0.089
0.329	6.39	$0.255 \pm 0.051 \pm 0.017$	0.255 ± 0.051	0.045 ± 0.102
0.370	6.72	$0.326 \pm 0.060 \pm 0.021$	0.326 ± 0.060	0.077 ± 0.120
0.416	7.06	$0.241 \pm 0.072 \pm 0.018$	0.241 ± 0.072	0.059 ± 0.144
0.468	7.38	$0.261 \pm 0.089 \pm 0.021$	0.261 ± 0.089	0.029 ± 0.178
0.526	7.66	$0.450 \pm 0.113 \pm 0.028$	0.450 ± 0.113	0.072 ± 0.225
0.593	7.90	$0.313 \pm 0.148 \pm 0.032$	0.313 ± 0.148	0.425 ± 0.294
0.666	9.26	$0.246 \pm 0.226 \pm 0.031$	0.246 ± 0.226	-0.396 ± 0.443
0.750	9.53	$0.648 \pm 0.330 \pm 0.040$	0.648 ± 0.330	0.052 ± 0.636

Table B.3: Results for the deuteron at 29 GeV: g_1/F_1 , A_1 and A_2 for both spectrometers combined. Statistical errors shown first, systematic last.

B.4 Deuteron 4.5° and 7.0° $g_1(x, Q^2)$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$g_1(Q^2)$ 4.5°	$\langle Q^2 \rangle$	$g_1(Q^2)$ 7.0°
0.031	1.27	0.169 ± 0.145	—	—
0.035	1.40	-0.001 ± 0.096	—	—
0.039	1.52	0.200 ± 0.080	—	—
0.044	1.65	-0.037 ± 0.068	—	—
0.050	1.79	-0.033 ± 0.059	—	—
0.056	1.92	0.046 ± 0.052	—	—
0.063	2.07	0.030 ± 0.046	—	—
0.070	2.22	0.040 ± 0.042	—	—
0.079	2.38	0.070 ± 0.040	3.17	-0.059 ± 0.110
0.090	2.53	0.109 ± 0.039	3.48	0.100 ± 0.069
0.101	2.69	0.080 ± 0.039	3.79	0.085 ± 0.051
0.113	2.84	0.093 ± 0.038	4.11	0.035 ± 0.043
0.128	3.00	0.189 ± 0.037	4.43	0.082 ± 0.038
0.144	3.15	0.124 ± 0.036	4.78	0.161 ± 0.033
0.162	3.30	0.098 ± 0.036	5.13	0.065 ± 0.030
0.182	3.45	0.112 ± 0.035	5.49	0.165 ± 0.027
0.205	3.59	0.096 ± 0.034	5.86	0.073 ± 0.025
0.231	3.73	0.034 ± 0.034	6.24	0.085 ± 0.023
0.259	3.85	0.044 ± 0.033	6.60	0.046 ± 0.022
0.292	3.98	0.081 ± 0.032	6.97	0.078 ± 0.020
0.329	4.09	0.107 ± 0.032	7.34	0.072 ± 0.019
0.370	4.20	0.088 ± 0.031	7.69	0.080 ± 0.017
0.416	4.30	0.060 ± 0.030	8.04	0.042 ± 0.015
0.468	4.39	0.008 ± 0.029	8.37	0.043 ± 0.014
0.526	4.47	0.039 ± 0.026	8.68	0.043 ± 0.012
0.593	4.55	0.024 ± 0.023	8.99	0.017 ± 0.009
0.666	—	—	9.26	0.008 ± 0.007
0.750	—	—	9.53	0.009 ± 0.005

Table B.4: Results for the deuteron structure function g_1 at 29 GeV for the 4.5° and 7.0° spectrometers at the experimental kinematics.

B.5 Deuteron $g_1(x, Q^2 = 3 \text{ GeV}^2)$ at 29 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$g_1(3 \text{ GeV}^2)$
0.031	1.27	0.224 ± 0.193
0.035	1.40	-0.001 ± 0.123
0.039	1.52	0.251 ± 0.100
0.044	1.65	-0.045 ± 0.083
0.050	1.79	-0.040 ± 0.071
0.056	1.92	0.054 ± 0.061
0.063	2.07	0.034 ± 0.053
0.070	2.22	0.045 ± 0.048
0.079	2.47	0.061 ± 0.042
0.090	2.77	0.117 ± 0.037
0.101	3.10	0.088 ± 0.033
0.113	3.40	0.071 ± 0.030
0.128	3.71	0.141 ± 0.028
0.144	4.03	0.148 ± 0.025
0.162	4.38	0.080 ± 0.023
0.182	4.72	0.146 ± 0.021
0.205	5.06	0.081 ± 0.020
0.231	5.39	0.069 ± 0.019
0.259	5.72	0.046 ± 0.018
0.292	6.04	0.079 ± 0.017
0.329	6.39	0.083 ± 0.017
0.370	6.72	0.085 ± 0.016
0.416	7.06	0.048 ± 0.015
0.468	7.38	0.039 ± 0.013
0.526	7.66	0.048 ± 0.012
0.593	7.90	0.023 ± 0.011
0.666	9.26	0.011 ± 0.010
0.750	9.53	0.016 ± 0.008

Table B.5: Results for the deuteron structure function g_1 at 29 GeV for both spectrometers at the average experimental Q^2 of 3 GeV². The Q^2 -evolution was performed with the assumption that g_1/F_1 is independent of Q^2 .

Appendix C

Tables of Results: Neutron

C.1 Neutron A_1 , g_1/F_1 , and g_1 at 29 GeV

$\langle x \rangle$	A_1	g_1/F_1	g_1
0.031	0.041 ± 0.097	0.030 ± 0.097	0.172 ± 0.446
0.035	-0.115 ± 0.070	-0.138 ± 0.070	-0.513 ± 0.286
0.039	0.035 ± 0.064	0.058 ± 0.064	0.243 ± 0.232
0.044	-0.163 ± 0.061	-0.152 ± 0.061	-0.454 ± 0.193
0.050	-0.172 ± 0.058	-0.170 ± 0.058	-0.449 ± 0.164
0.056	-0.108 ± 0.058	-0.098 ± 0.058	-0.215 ± 0.142
0.063	-0.109 ± 0.057	-0.121 ± 0.057	-0.237 ± 0.124
0.070	-0.105 ± 0.058	-0.120 ± 0.058	-0.206 ± 0.111
0.079	-0.148 ± 0.059	-0.140 ± 0.058	-0.208 ± 0.097
0.090	-0.023 ± 0.060	-0.015 ± 0.059	-0.001 ± 0.087
0.101	-0.134 ± 0.061	-0.118 ± 0.061	-0.128 ± 0.078
0.113	-0.138 ± 0.064	-0.145 ± 0.063	-0.142 ± 0.071
0.128	0.030 ± 0.067	0.021 ± 0.066	0.039 ± 0.064
0.144	0.075 ± 0.071	0.085 ± 0.070	0.087 ± 0.059
0.162	-0.112 ± 0.076	-0.137 ± 0.074	-0.087 ± 0.054
0.182	0.071 ± 0.082	0.075 ± 0.081	0.059 ± 0.051
0.205	-0.131 ± 0.091	-0.157 ± 0.088	-0.075 ± 0.047
0.231	-0.068 ± 0.103	-0.120 ± 0.100	-0.050 ± 0.045
0.259	-0.493 ± 0.119	-0.404 ± 0.115	-0.150 ± 0.043
0.292	-0.011 ± 0.141	-0.042 ± 0.134	-0.014 ± 0.041
0.329	-0.012 ± 0.170	0.034 ± 0.159	0.005 ± 0.039
0.370	0.271 ± 0.209	0.244 ± 0.193	0.041 ± 0.037
0.416	-0.405 ± 0.263	-0.365 ± 0.239	-0.064 ± 0.035
0.468	-0.302 ± 0.345	-0.347 ± 0.307	-0.051 ± 0.033
0.526	0.190 ± 0.471	0.109 ± 0.407	-0.011 ± 0.030
0.593	-0.760 ± 0.669	-0.226 ± 0.560	-0.030 ± 0.027
0.666	0.365 ± 1.062	-0.251 ± 0.892	-0.023 ± 0.027
0.750	0.696 ± 1.538	0.592 ± 1.251	-0.011 ± 0.023

Table C.1: Results for the neutron at 29 GeV: A_1 , g_1/F_1 , and g_1 . Statistical errors shown.

Appendix D

Tables of Results: Proton, 16 GeV and 9 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{ }$	$A_{ }(\text{uncorr})$
0.056	1.16	0.091 ± 0.043	0.079 ± 0.036
0.063	1.26	0.034 ± 0.031	0.023 ± 0.027
0.071	1.37	0.092 ± 0.026	0.082 ± 0.023
0.080	1.47	0.082 ± 0.023	0.074 ± 0.021
0.090	1.58	0.113 ± 0.022	0.105 ± 0.020
0.101	1.69	0.101 ± 0.021	0.094 ± 0.019
0.113	1.80	0.108 ± 0.019	0.102 ± 0.018
0.128	1.91	0.115 ± 0.018	0.110 ± 0.018
0.144	2.03	0.120 ± 0.018	0.115 ± 0.017
0.162	2.14	0.103 ± 0.017	0.100 ± 0.017
0.182	2.26	0.106 ± 0.018	0.102 ± 0.017
0.205	2.36	0.116 ± 0.018	0.113 ± 0.018
0.231	2.47	0.136 ± 0.018	0.133 ± 0.018
0.260	2.57	0.165 ± 0.019	0.163 ± 0.019
0.292	2.67	0.159 ± 0.019	0.157 ± 0.019
0.329	2.76	0.127 ± 0.020	0.124 ± 0.020
0.370	2.85	0.157 ± 0.022	0.154 ± 0.021
0.416	2.94	0.142 ± 0.023	0.139 ± 0.023
0.468	3.02	0.150 ± 0.025	0.146 ± 0.025
0.080	0.69	0.076 ± 0.025	0.073 ± 0.022
0.090	0.74	0.064 ± 0.019	0.060 ± 0.017
0.101	0.78	0.057 ± 0.017	0.053 ± 0.015
0.113	0.82	0.073 ± 0.015	0.070 ± 0.014
0.128	0.86	0.048 ± 0.015	0.045 ± 0.014
0.144	0.90	0.047 ± 0.014	0.043 ± 0.013
0.162	0.93	0.065 ± 0.013	0.062 ± 0.013
0.182	0.97	0.087 ± 0.013	0.084 ± 0.012
0.205	1.00	0.068 ± 0.013	0.064 ± 0.012
0.231	1.03	0.084 ± 0.014	0.080 ± 0.012

Table D.1: Results for $A_{||}$ of the proton at 16 GeV(top) and 9 GeV(bottom) for the 7.0° spectrometer, both with and without radiative corrections. Statistical errors shown.

APPENDIX D. TABLES OF RESULTS: PROTON, 16 GEV AND 9 GEV 241

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\parallel}(\text{uncorr})$
0.022	0.47	0.023 ± 0.024	0.018 ± 0.019
0.024	0.51	0.066 ± 0.019	0.061 ± 0.015
0.027	0.55	0.058 ± 0.018	0.054 ± 0.014
0.031	0.59	0.054 ± 0.016	0.049 ± 0.013
0.035	0.64	0.036 ± 0.015	0.031 ± 0.012
0.039	0.68	0.041 ± 0.014	0.036 ± 0.012
0.044	0.73	0.051 ± 0.013	0.046 ± 0.011
0.050	0.78	0.074 ± 0.012	0.069 ± 0.011
0.056	0.83	0.052 ± 0.012	0.047 ± 0.010
0.063	0.88	0.068 ± 0.012	0.064 ± 0.011
0.071	0.92	0.043 ± 0.012	0.039 ± 0.011
0.080	0.97	0.046 ± 0.012	0.042 ± 0.011
0.090	1.02	0.062 ± 0.012	0.058 ± 0.011
0.101	1.06	0.071 ± 0.012	0.067 ± 0.011
0.113	1.10	0.050 ± 0.012	0.046 ± 0.012
0.128	1.15	0.060 ± 0.012	0.057 ± 0.012
0.144	1.18	0.056 ± 0.012	0.053 ± 0.012
0.162	1.22	0.069 ± 0.013	0.065 ± 0.012
0.182	1.26	0.051 ± 0.013	0.048 ± 0.012
0.205	1.29	0.060 ± 0.013	0.058 ± 0.013
0.231	1.32	0.047 ± 0.014	0.044 ± 0.013
0.260	1.35	0.041 ± 0.014	0.038 ± 0.014
0.292	1.38	0.055 ± 0.015	0.051 ± 0.014
0.035	0.31	0.048 ± 0.019	0.046 ± 0.014
0.039	0.33	0.011 ± 0.017	0.009 ± 0.013
0.044	0.35	0.046 ± 0.016	0.044 ± 0.013
0.050	0.36	0.043 ± 0.016	0.042 ± 0.013
0.056	0.38	0.040 ± 0.015	0.039 ± 0.013
0.063	0.40	0.026 ± 0.015	0.025 ± 0.012
0.071	0.41	0.030 ± 0.015	0.029 ± 0.013
0.080	0.43	0.032 ± 0.015	0.031 ± 0.013
0.090	0.44	0.018 ± 0.014	0.017 ± 0.012
0.101	0.45	0.024 ± 0.014	0.023 ± 0.012
0.113	0.47	0.041 ± 0.013	0.040 ± 0.011
0.128	0.48	0.002 ± 0.013	0.000 ± 0.011

Table D.2: Results for $A_{\parallel}$ of the proton at 16 and 9 GeV for the 4.5° spectrometer, both with and without radiative corrections. Statistical errors shown.

APPENDIX D. TABLES OF RESULTS: PROTON, 16 GEV AND 9 GEV242

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	g_1
0.056	1.16	0.131 ± 0.062	0.131 ± 0.063	0.285 ± 0.136
0.063	1.26	0.051 ± 0.046	0.051 ± 0.046	0.101 ± 0.091
0.071	1.37	0.143 ± 0.041	0.144 ± 0.041	0.257 ± 0.073
0.080	1.47	0.134 ± 0.038	0.135 ± 0.039	0.219 ± 0.062
0.090	1.58	0.194 ± 0.037	0.195 ± 0.038	0.286 ± 0.055
0.101	1.69	0.183 ± 0.037	0.184 ± 0.037	0.244 ± 0.050
0.113	1.80	0.207 ± 0.037	0.208 ± 0.037	0.249 ± 0.044
0.128	1.91	0.233 ± 0.037	0.235 ± 0.037	0.253 ± 0.040
0.144	2.03	0.258 ± 0.038	0.260 ± 0.038	0.253 ± 0.037
0.162	2.14	0.236 ± 0.040	0.239 ± 0.040	0.209 ± 0.035
0.182	2.26	0.256 ± 0.043	0.259 ± 0.043	0.204 ± 0.034
0.205	2.36	0.300 ± 0.046	0.304 ± 0.047	0.213 ± 0.033
0.231	2.47	0.375 ± 0.050	0.380 ± 0.051	0.237 ± 0.032
0.260	2.57	0.486 ± 0.055	0.493 ± 0.056	0.269 ± 0.030
0.292	2.67	0.499 ± 0.061	0.507 ± 0.062	0.240 ± 0.029
0.329	2.76	0.422 ± 0.068	0.430 ± 0.069	0.172 ± 0.028
0.370	2.85	0.557 ± 0.077	0.569 ± 0.078	0.190 ± 0.026
0.416	2.94	0.535 ± 0.087	0.548 ± 0.089	0.148 ± 0.024
0.468	3.02	0.598 ± 0.101	0.614 ± 0.104	0.131 ± 0.022
0.080	0.69	0.162 ± 0.053	0.164 ± 0.054	0.239 ± 0.078
0.090	0.74	0.143 ± 0.043	0.145 ± 0.043	0.193 ± 0.058
0.101	0.78	0.136 ± 0.040	0.137 ± 0.040	0.168 ± 0.049
0.113	0.82	0.189 ± 0.039	0.191 ± 0.040	0.213 ± 0.044
0.128	0.86	0.134 ± 0.041	0.135 ± 0.041	0.137 ± 0.042
0.144	0.90	0.139 ± 0.042	0.141 ± 0.042	0.130 ± 0.039
0.162	0.93	0.210 ± 0.043	0.213 ± 0.044	0.180 ± 0.037
0.182	0.97	0.301 ± 0.045	0.306 ± 0.046	0.235 ± 0.035
0.205	1.00	0.252 ± 0.048	0.257 ± 0.049	0.179 ± 0.034
0.231	1.03	0.331 ± 0.050	0.339 ± 0.051	0.212 ± 0.032

Table D.3: Proton results for g_1/F_1 , A_1 and g_1 at 16 GeV(top) and 9 GeV(bottom) for the 7.0° spectrometer.

APPENDIX D. TABLES OF RESULTS: PROTON, 16 GEV AND 9 GEV243

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	g_1
0.022	0.47	0.031 ± 0.033	0.031 ± 0.033	0.132 ± 0.138
0.024	0.51	0.093 ± 0.027	0.093 ± 0.027	0.363 ± 0.105
0.027	0.55	0.086 ± 0.026	0.086 ± 0.026	0.311 ± 0.093
0.031	0.59	0.083 ± 0.025	0.084 ± 0.025	0.276 ± 0.084
0.035	0.64	0.059 ± 0.024	0.059 ± 0.024	0.176 ± 0.073
0.039	0.68	0.072 ± 0.024	0.072 ± 0.024	0.197 ± 0.066
0.044	0.73	0.096 ± 0.023	0.096 ± 0.023	0.239 ± 0.058
0.050	0.78	0.146 ± 0.024	0.147 ± 0.024	0.332 ± 0.054
0.056	0.83	0.109 ± 0.025	0.110 ± 0.025	0.226 ± 0.051
0.063	0.88	0.156 ± 0.027	0.156 ± 0.027	0.292 ± 0.050
0.071	0.92	0.106 ± 0.029	0.107 ± 0.029	0.180 ± 0.048
0.080	0.97	0.122 ± 0.031	0.122 ± 0.031	0.187 ± 0.048
0.090	1.01	0.177 ± 0.034	0.178 ± 0.034	0.247 ± 0.048
0.101	1.06	0.219 ± 0.037	0.221 ± 0.038	0.278 ± 0.047
0.113	1.10	0.167 ± 0.041	0.168 ± 0.041	0.192 ± 0.047
0.128	1.14	0.219 ± 0.045	0.221 ± 0.045	0.229 ± 0.047
0.144	1.18	0.221 ± 0.049	0.223 ± 0.049	0.210 ± 0.046
0.162	1.22	0.292 ± 0.054	0.295 ± 0.054	0.251 ± 0.046
0.182	1.26	0.238 ± 0.060	0.240 ± 0.060	0.185 ± 0.047
0.205	1.29	0.304 ± 0.066	0.307 ± 0.067	0.214 ± 0.046
0.231	1.32	0.255 ± 0.073	0.259 ± 0.074	0.161 ± 0.046
0.260	1.35	0.237 ± 0.082	0.241 ± 0.083	0.134 ± 0.046
0.292	1.38	0.341 ± 0.091	0.347 ± 0.092	0.170 ± 0.045
0.035	0.31	0.095 ± 0.037	0.095 ± 0.037	0.237 ± 0.093
0.039	0.33	0.024 ± 0.037	0.024 ± 0.037	0.055 ± 0.085
0.044	0.35	0.105 ± 0.037	0.106 ± 0.037	0.225 ± 0.080
0.050	0.36	0.107 ± 0.039	0.107 ± 0.039	0.210 ± 0.077
0.056	0.38	0.107 ± 0.041	0.108 ± 0.041	0.193 ± 0.074
0.063	0.40	0.076 ± 0.043	0.076 ± 0.044	0.126 ± 0.072
0.071	0.41	0.095 ± 0.049	0.095 ± 0.049	0.144 ± 0.074
0.080	0.43	0.111 ± 0.053	0.112 ± 0.054	0.154 ± 0.074
0.090	0.44	0.067 ± 0.055	0.068 ± 0.055	0.086 ± 0.070
0.101	0.45	0.100 ± 0.057	0.101 ± 0.057	0.118 ± 0.067
0.113	0.47	0.183 ± 0.059	0.185 ± 0.060	0.199 ± 0.065
0.128	0.48	0.008 ± 0.064	0.008 ± 0.064	0.008 ± 0.064

Table D.4: Proton results for g_1/F_1 , A_1 and g_1 at 16 GeV(top) and 9 GeV(bottom) for the 4.5° spectrometer.

Appendix E

Tables of Results: Deuteron, 16 GeV and 9 GeV

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{ }$	$A_{ }(\text{uncorr})$
0.056	1.16	0.062 ± 0.106	0.067 ± 0.093
0.063	1.26	-0.003 ± 0.075	0.001 ± 0.067
0.071	1.37	0.054 ± 0.061	0.058 ± 0.055
0.080	1.47	-0.073 ± 0.054	-0.069 ± 0.050
0.090	1.58	0.121 ± 0.049	0.124 ± 0.047
0.101	1.69	0.054 ± 0.046	0.057 ± 0.044
0.113	1.80	0.027 ± 0.041	0.029 ± 0.039
0.128	1.91	0.018 ± 0.038	0.020 ± 0.037
0.144	2.03	0.039 ± 0.036	0.041 ± 0.035
0.162	2.14	0.060 ± 0.036	0.061 ± 0.035
0.182	2.26	0.097 ± 0.036	0.098 ± 0.035
0.205	2.36	0.045 ± 0.036	0.045 ± 0.036
0.231	2.47	0.041 ± 0.038	0.041 ± 0.037
0.260	2.57	0.041 ± 0.039	0.041 ± 0.038
0.292	2.67	0.115 ± 0.040	0.114 ± 0.040
0.329	2.76	0.160 ± 0.042	0.159 ± 0.042
0.370	2.85	0.039 ± 0.045	0.037 ± 0.045
0.416	2.94	0.055 ± 0.049	0.054 ± 0.049
0.468	3.02	0.168 ± 0.054	0.167 ± 0.054
0.080	0.69	-0.013 ± 0.037	0.017 ± 0.034
0.090	0.74	0.013 ± 0.028	-0.010 ± 0.026
0.101	0.78	0.016 ± 0.024	0.019 ± 0.023
0.113	0.82	0.024 ± 0.022	0.027 ± 0.021
0.128	0.86	0.052 ± 0.021	0.054 ± 0.020
0.144	0.90	0.012 ± 0.020	0.013 ± 0.019
0.162	0.93	0.036 ± 0.020	0.037 ± 0.019
0.182	0.97	0.028 ± 0.019	0.028 ± 0.018
0.205	1.00	0.005 ± 0.019	0.004 ± 0.018
0.231	1.03	0.044 ± 0.019	0.043 ± 0.018

Table E.1: Results for $A_{||}$ of the deuteron for the 7.0° spectrometer at 16 GeV(top) and 9 GeV(bottom), both with and without radiative corrections. Statistical errors shown.

APPENDIX E. TABLES OF RESULTS: DEUTERON, 16 GEV AND 9 GEV 245

$\langle x \rangle$	$\langle Q^2 \rangle$	$A_{\parallel}$	$A_{\parallel}(\text{uncorr})$
0.022	0.47	-0.018 ± 0.064	-0.014 ± 0.054
0.024	0.51	-0.035 ± 0.049	-0.031 ± 0.042
0.027	0.55	-0.012 ± 0.045	-0.008 ± 0.038
0.031	0.59	0.012 ± 0.042	0.017 ± 0.035
0.035	0.64	-0.009 ± 0.038	-0.005 ± 0.033
0.039	0.68	0.056 ± 0.034	0.060 ± 0.031
0.044	0.73	-0.021 ± 0.028	-0.017 ± 0.025
0.050	0.78	0.046 ± 0.026	0.049 ± 0.024
0.056	0.83	0.003 ± 0.025	0.006 ± 0.024
0.063	0.88	0.021 ± 0.025	0.024 ± 0.023
0.071	0.92	0.026 ± 0.025	0.028 ± 0.023
0.080	0.97	-0.016 ± 0.025	-0.014 ± 0.024
0.090	1.02	0.038 ± 0.025	0.040 ± 0.024
0.101	1.06	0.019 ± 0.025	0.020 ± 0.024
0.113	1.10	0.016 ± 0.025	0.017 ± 0.024
0.128	1.15	0.063 ± 0.025	0.064 ± 0.024
0.144	1.18	0.050 ± 0.025	0.051 ± 0.024
0.162	1.22	-0.025 ± 0.025	-0.024 ± 0.024
0.182	1.26	0.043 ± 0.026	0.043 ± 0.025
0.205	1.29	0.042 ± 0.027	0.042 ± 0.026
0.231	1.32	0.016 ± 0.028	0.015 ± 0.027
0.260	1.35	0.022 ± 0.029	0.021 ± 0.028
0.292	1.38	0.062 ± 0.031	0.060 ± 0.030
0.035	0.31	-0.014 ± 0.028	-0.009 ± 0.023
0.039	0.33	0.017 ± 0.025	0.021 ± 0.021
0.044	0.35	0.001 ± 0.024	0.005 ± 0.020
0.050	0.36	0.022 ± 0.023	0.026 ± 0.020
0.056	0.38	-0.017 ± 0.022	-0.013 ± 0.019
0.063	0.40	-0.002 ± 0.021	0.001 ± 0.019
0.071	0.41	0.032 ± 0.022	0.035 ± 0.020
0.080	0.43	-0.004 ± 0.022	-0.002 ± 0.019
0.090	0.44	0.008 ± 0.021	0.010 ± 0.018
0.101	0.45	0.038 ± 0.020	0.040 ± 0.018
0.113	0.47	-0.002 ± 0.019	-0.000 ± 0.017
0.128	0.48	-0.017 ± 0.019	-0.016 ± 0.016

Table E.2: Results for $A_{\parallel}$ of the deuteron for the 4.5° spectrometer at 16 GeV(top) and 9 GeV(bottom), both with and without radiative corrections. Statistical errors shown.

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	g_1
0.056	1.16	0.090 ± 0.153	0.091 ± 0.154	0.186 ± 0.316
0.063	1.26	-0.005 ± 0.111	-0.005 ± 0.112	-0.008 ± 0.208
0.071	1.37	0.084 ± 0.094	0.085 ± 0.095	0.143 ± 0.159
0.080	1.47	-0.119 ± 0.088	-0.120 ± 0.088	-0.181 ± 0.134
0.090	1.58	0.208 ± 0.085	0.209 ± 0.085	0.286 ± 0.117
0.101	1.69	0.097 ± 0.083	0.098 ± 0.083	0.120 ± 0.102
0.113	1.80	0.051 ± 0.078	0.052 ± 0.079	0.057 ± 0.086
0.128	1.91	0.037 ± 0.076	0.037 ± 0.077	0.036 ± 0.076
0.144	2.03	0.085 ± 0.077	0.086 ± 0.078	0.075 ± 0.068
0.162	2.14	0.137 ± 0.081	0.138 ± 0.082	0.107 ± 0.064
0.182	2.26	0.235 ± 0.086	0.238 ± 0.087	0.164 ± 0.060
0.205	2.36	0.116 ± 0.094	0.117 ± 0.095	0.071 ± 0.058
0.231	2.47	0.113 ± 0.104	0.114 ± 0.105	0.061 ± 0.056
0.260	2.57	0.122 ± 0.114	0.123 ± 0.116	0.057 ± 0.054
0.292	2.67	0.360 ± 0.126	0.366 ± 0.128	0.145 ± 0.051
0.329	2.76	0.533 ± 0.142	0.543 ± 0.144	0.180 ± 0.048
0.370	2.85	0.137 ± 0.160	0.140 ± 0.164	0.038 ± 0.044
0.416	2.94	0.207 ± 0.185	0.213 ± 0.189	0.046 ± 0.041
0.468	3.02	0.671 ± 0.216	0.689 ± 0.221	0.116 ± 0.037
0.080	0.69	0.029 ± 0.078	0.029 ± 0.079	0.040 ± 0.108
0.090	0.74	-0.029 ± 0.062	-0.029 ± 0.063	-0.036 ± 0.078
0.101	0.78	0.038 ± 0.058	0.039 ± 0.059	0.044 ± 0.067
0.113	0.82	0.063 ± 0.057	0.063 ± 0.058	0.065 ± 0.059
0.128	0.86	0.145 ± 0.059	0.147 ± 0.059	0.136 ± 0.055
0.144	0.90	0.035 ± 0.060	0.035 ± 0.061	0.030 ± 0.051
0.162	0.93	0.117 ± 0.063	0.118 ± 0.064	0.090 ± 0.049
0.182	0.97	0.095 ± 0.067	0.097 ± 0.068	0.066 ± 0.047
0.205	1.00	0.017 ± 0.071	0.017 ± 0.072	0.011 ± 0.044
0.231	1.03	0.175 ± 0.075	0.179 ± 0.077	0.098 ± 0.042

Table E.3: Proton results for g_1/F_1 , A_1 and g_1 at 16 GeV(top) and 9 GeV(bottom) for the 7.0° spectrometer.

$\langle x \rangle$	$\langle Q^2 \rangle$	g_1/F_1	A_1	g_1
0.022	0.47	-0.025 ± 0.087	-0.025 ± 0.087	-0.101 ± 0.355
0.024	0.51	-0.050 ± 0.070	-0.050 ± 0.070	-0.186 ± 0.261
0.027	0.55	-0.018 ± 0.066	-0.018 ± 0.066	-0.062 ± 0.228
0.031	0.59	0.019 ± 0.064	0.019 ± 0.065	0.060 ± 0.203
0.035	0.64	-0.015 ± 0.062	-0.015 ± 0.062	-0.044 ± 0.177
0.039	0.68	0.098 ± 0.060	0.099 ± 0.060	0.257 ± 0.157
0.044	0.73	-0.038 ± 0.052	-0.038 ± 0.052	-0.091 ± 0.124
0.050	0.78	0.092 ± 0.052	0.092 ± 0.052	0.198 ± 0.112
0.056	0.83	0.007 ± 0.054	0.007 ± 0.054	0.014 ± 0.106
0.063	0.88	0.048 ± 0.057	0.049 ± 0.057	0.086 ± 0.101
0.071	0.92	0.068 ± 0.061	0.064 ± 0.061	0.101 ± 0.097
0.080	0.97	-0.043 ± 0.066	-0.043 ± 0.066	-0.062 ± 0.095
0.090	1.01	0.110 ± 0.071	0.110 ± 0.071	0.143 ± 0.092
0.101	1.06	0.058 ± 0.077	0.058 ± 0.077	0.068 ± 0.090
0.113	1.10	0.054 ± 0.083	0.054 ± 0.084	0.057 ± 0.088
0.128	1.14	0.229 ± 0.090	0.231 ± 0.091	0.219 ± 0.086
0.144	1.18	0.199 ± 0.099	0.201 ± 0.099	0.171 ± 0.085
0.162	1.22	-0.106 ± 0.108	-0.107 ± 0.109	-0.082 ± 0.084
0.182	1.26	0.200 ± 0.121	0.202 ± 0.122	0.138 ± 0.083
0.205	1.29	0.211 ± 0.134	0.214 ± 0.136	0.131 ± 0.083
0.231	1.32	0.086 ± 0.150	0.087 ± 0.152	0.047 ± 0.082
0.260	1.35	0.131 ± 0.169	0.133 ± 0.171	0.064 ± 0.082
0.292	1.38	0.385 ± 0.192	0.392 ± 0.195	0.164 ± 0.082
0.035	0.31	-0.027 ± 0.055	-0.027 ± 0.055	-0.064 ± 0.130
0.039	0.33	0.035 ± 0.053	0.035 ± 0.053	0.077 ± 0.116
0.044	0.35	0.003 ± 0.054	0.003 ± 0.054	0.006 ± 0.110
0.050	0.36	0.055 ± 0.057	0.055 ± 0.058	0.102 ± 0.107
0.056	0.38	-0.045 ± 0.059	-0.045 ± 0.060	-0.077 ± 0.101
0.063	0.40	-0.006 ± 0.062	-0.007 ± 0.062	-0.010 ± 0.097
0.071	0.41	0.104 ± 0.070	0.104 ± 0.071	0.148 ± 0.101
0.080	0.43	-0.014 ± 0.076	-0.014 ± 0.076	-0.018 ± 0.099
0.090	0.44	0.030 ± 0.078	0.031 ± 0.079	0.036 ± 0.093
0.101	0.45	0.157 ± 0.081	0.159 ± 0.082	0.173 ± 0.089
0.113	0.47	-0.008 ± 0.085	-0.008 ± 0.086	-0.008 ± 0.086
0.128	0.48	-0.083 ± 0.092	-0.084 ± 0.093	-0.076 ± 0.085

Table E.4: Proton results for g_1/F_1 , A_1 and g_1 at 16 GeV(top) and 9 GeV(bottom) for the 4.5° spectrometer.

Appendix F

Other E143 Theses

- **Philippe Grenier**, *Study of Nucleon Spin Structure Functions: Experiment E143 at SLAC*, University of Clermont-Ferrand, France (September 1995). Advisor: Yves Terrien. [DAPNIA-SPHN-95-03-T.]
- **Todd Denson Averett**, *Measurement of the Spin Structure Function $g_2(x, Q^2)$ for the Proton and Deuteron*, University of Virginia, Department of Physics (August 1995). Advisor: Ralph C. Minehart.
- **Tingjun Liu**, *Measurements of Spin Structure Function g_1^p and g_1^d for Proton and Deuteron at SLAC E143*, University of Virginia, Department of Physics (January 1996). Advisor: James S. McCarthy. [UMI-96-16004-mc (microfiche).]
- **Masao Kuriki**, *Study of Deep Inelastic Scattering of Polarized Electrons Off Polarized Deuterons*, Tohoku University, Department of Physics (March 1996). Advisor: Haruo Yuta. [SLAC-REPORT-482.]
- **Johannes Maria Bauer**, *Measurement of the Longitudinal Deuteron Spin-Structure Function in Deep-Inelastic Scattering*, University of Massachusetts, Amherst, Department of Physics and Astronomy (September 1996). Advisor: Janice Button-Shafer. [SLAC-REPORT-492.]
- **Paul Edgar Raines**, *Measurement of Spin Structure Function g_1 for the Proton and Deuteron in the Resonance Region*, University of Pennsylvania, Department of Physics (September 1996). Advisor: Keith A. Griffioen. [UMI-97-12989-mc (microfiche).]
- **Philipp Steiner**

Appendix G

The E143 Collaboration

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