Emergent Gauge Bosons*

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Abstract

The old idea that the photon is a Goldstone boson emergent from a spontaneously broken theory of interacting fermions is revisited. It is conjectured that the gauge-potential condensate has a vacuum expectation value which is very large, perhaps the GUT/Planck momentum scale, but that the magnitude of the effective potential which generates it is very small, so small that in the limit of vanishing cosmological constant it would vanish. In this way, the threat of unacceptably large observable, noncovariant residual effects is mitigated. The linkage of these ideas to other speculative ideas involving black holes and parametrizations of Standard-Model coupling constants is also described.

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1 Introduction

It is a pleasure to write this note on the occasion of Holger Bech Nielsen's 60th birthday celebration. It will deal with very speculative matters, most of which down through the years have benefitted by interactions with him. Holger Bech is one of few with whom I find it fruitful to even discuss such things. Not only has he been always receptive, but most importantly he has reacted thoughtfully and critically, instead of the commonplace reaction of indifference.

The subject matter discussed below is actually of common interest: whether gauge bosons such as the photon may be regarded as emergent degrees of freedom. The word “emergent” is used in the condensed matter community to describe collective modes, appropriate for a low energy description of a quantum fluid (or other medium) built of more fundamental degrees of freedom. I will further specify that the emergent gauge bosons be considered as Goldstone modes, associated with a spontaneous breakdown of Lorentz symmetry. This is an idea I entertained long ago [1], one which clearly has a lot to do with Holger's viewpoints as well [2].

Recently I have played with the notion of relating Standard Model parameters to gravitational parameters [3], and this has prompted me to revisit the old Goldstone-photon ideas again. What follows will be a status report on what I have found out. In the next section I will sketch out a modernized version of ancient work on Goldstone photons. Section 3 contains the speculations regarding Standard Model parameters, and how those ideas mesh with the Goldstone-photon description. Section 4 deals with potentially observable effects of Lorentz-covariance violation, something which is a central problem of any approach based upon emergence. Section 5 attempts to summarize what is done in somewhat more general language, and to connect it with some recent work of Holger Bech and his collaborators [4]. In Section 6 we look at some of the future problems and opportunities presented by this approach.

2 The Goldstone Photon

Shortly after the work of Nambu and Jona-Lasinio [5] on the “Goldstone pion”, I explored whether their method might be applicable to the description of the photon as a Goldstone mode. A Fermi current-current contact interaction between fermions was posited, and then the Nambu-Jona-Lasinio methods were imitated. A nonvanishing condensate carrying a (timelike) vacuum current was assumed, determined by a gap equation even more obscure than the quadratically divergent one they considered for hadronic matter. The properties of the current-current correlation function, however, became constrained by the gap equation in such a way as to yield essentially a photon propagator in temporal gauge. The reasoning was analogous to their arguments for a massless-pion propagator. A challenge remained, however. It appeared that there were physically observable manifestations of Lorentz noncovariance occurring in higher order effects. These were concentrated in nonlinear, gauge noncovariant,

photon self-interactions. To suppress them, the Fermi constant in the original current-current interaction, as well as the magnitude of the vacuum current, had to be taken as very large. The physics of these assumptions, as well as that of the gap equation, was left up in the air.

We now review these results, using the modern language of path integrals and effective potentials. From the standard path integral we introduce the vector field A_μ which will mediate the four-fermi interaction and in fact will allow the fermion fields to be “integrated out”. Starting with the original Lagrangian density,

$$\mathcal{L} = \bar{\psi}(i \not{\nabla} - m)\psi + \frac{G}{2} (\bar{\psi}\gamma_\mu\psi)^2, \quad (1)$$

we write the path integral as

$$\begin{aligned} W(J) &= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp i \int d^4x [\mathcal{L} - \bar{\psi}\gamma_\mu\psi J^\mu] \\ &= N \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \exp i \int d^4x \left[\mathcal{L} - \bar{\psi}\gamma_\mu\psi J^\mu - \frac{1}{2G} (A_\mu - G\bar{\psi}\gamma_\mu\psi)^2 \right]. \end{aligned} \quad (2)$$

The auxiliary Gaussian factor is designed to eliminate the four-fermi interaction, allowing us to integrate out the fermions. We find, using standard techniques

$$\begin{aligned} W(J) &= N' \int \mathcal{D}A \exp -i \int d^4x \left[\frac{A^2}{2G} - V(A - J) \right] \\ &= N' \int \mathcal{D}A \exp -i \int d^4x \left[\frac{(A + J)^2}{2G} - V(A) \right] \end{aligned} \quad (3)$$

with $V(A)$ the one-loop effective potential, formally given by

$$\begin{aligned} i \int d^4x V(A) &= \ell n \det(i \not{\nabla} + \not{A} - m) - \ell n \det(i \not{\nabla} - m) \\ &= \text{Tr} \ell n \frac{1}{i \not{\nabla} - m} (i \not{\nabla} + \not{A} - m) \\ &= \int d^4x \left[\text{tr} \not{A}(x) S_F(x, x) \right. \\ &\quad \left. - \frac{1}{2} \text{tr} \int d^4y \not{A}(x) S_F(x, y) \not{A}(y) S_F(y, x) + \dots \right] \end{aligned} \quad (4)$$

and with tr implying a Dirac trace.

The formal steps up to this point are straightforward. But the physics to be applied is less clear. We here assume that, despite the presence of ultraviolet cutoffs, the form of the effective potential obeys Lorentz covariance, or at least that the corrections thereto will not disturb the line of argument to follow. Thus V is assumed to be a function only of the invariant $A_\mu A^\mu \equiv A^2$. We also assume that, in the limit

of J_μ being an infinitesimal constant timelike external source, there is a classical condensate A_μ which remains proportional to J_μ and nonvanishing as J_μ is turned off. That is, spontaneous Lorentz-symmetry breaking is assumed. The equilibrium value of A_μ will be determined in the usual way by a saddle-point integration over the fields A .

We need to evaluate the important contributions to $V(A)$, all of which are lowest order fermion closed loops. We expand in powers of A , keeping for now only quadratic and quartic terms. For the quadratic piece we make a derivative expansion. The term with no derivatives renormalizes the explicit quadratic term in the action, proportional to G^{-1} . We also clearly need a kinetic term in the action, which is quadratic in derivatives and quadratic in A . In the ancient work it was, in momentum space,

$$\begin{aligned} \int d^4x \Delta V^{(2)}(x) &= \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \tilde{A}_\mu(q) \tilde{A}_\nu(q) (q_\mu q_\nu - g_{\mu\nu} q^2) \frac{1}{e^2(q^2)} \\ &\approx \frac{1}{4e^2(0)} \int d^4x F_{\mu\nu}(x) F^{\mu\nu}(x) \end{aligned} \quad (5)$$

with $1/e^2$ given by the usual vacuum polarization integral

$$\frac{1}{e^2(0)} = \frac{1}{12\pi^2} \ell n \frac{\Lambda^2}{m^2} \Rightarrow \frac{1}{12\pi^2} \int_{4m^2}^{\Lambda^2} R(s) \frac{ds}{s} \quad (6)$$

and Λ some momentum-space cutoff. With this definition, the running coupling constant

$$\frac{1}{e^2(q^2)} \cong \frac{1}{12\pi^2} \ell n \frac{\Lambda^2}{q^2} \Rightarrow \frac{1}{12\pi^2} \int_{q^2}^{\Lambda^2} R(s) \frac{ds}{s} \quad (7)$$

vanishes as $q^2 \rightarrow \Lambda^2$, corresponding to the compositeness condition $Z_3 = 0$, which implies infinite bare charge at the highest energy scale Λ .[†]

But we here choose to add an additional contribution $1/e_0^2$, not present in the ancient work, to account for the fact that even at the GUT unification scale the running gauge coupling constants are finite. That is, we expect that there must be additional contributions from physics beyond the Standard Model. We shall return to this question in the Section 5. Putting all this together yields the terms in the effective action which are quadratic in A_μ :

$$S^{(2)} = \int d^4x \mathcal{L}^{(2)}(x) = \int d^4x \left[\frac{A^2}{2G} + \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right]. \quad (8)$$

There must be quartic, and possibly higher, terms in $V(A)$ as well, in order to generate the familiar Mexican hat structure of the potential and induce spontaneous symmetry

[†]In those ancient times, there was no electroweak unification. In what follows we make believe, with eyes wide open, that this is still the case.

breaking. At zero momentum transfer, the conventional contributions to the quartic fermion loop vanish by current conservation. But, just as we assumed an extra piece in the vacuum polarization term, we also assume that the quartic term is nonvanishing:

$$S^{(4)} = - \int d^4x \frac{\lambda}{4} (A^2)^2 . \quad (9)$$

Clearly this leads to a minimum value of the potential at

$$A_{cl}^2 \equiv M^2 = \frac{1}{G\lambda} . \quad (10)$$

We may now do the path integral over the field A_0 , leaving the classical term in the exponent. In this short note we will drop the determinantal prefactor, because it corresponds to photon closed loop contributions beyond the scope of these considerations. One can eventually do better in this regard.

What is left is an effective action similar to the effective action of chiral perturbation theory, to be considered in tree approximation. In the case of pions, there are many ways to express the effective action, depending upon how one parametrizes the fields. This is also true in this case, and an especially convenient way is to eliminate the time component A_0 in terms of the space components.

$$A_0 = \sqrt{M^2 + \vec{A}^2} = M + \frac{\vec{A}^2}{2M} + \cdots . \quad (11)$$

This is analogous to the elimination of the sigma component of the field in terms of the pions in the effective action via

$$\sigma = \sqrt{f^2 - \vec{\pi}^2} = f - \frac{\vec{\pi}^2}{2f} + \cdots . \quad (12)$$

Such a choice provides a very efficient way of deriving the structure of pion-pion scattering at second order in momentum.

In our case the elimination of A_0 in terms of the space components leads to the effective Lagrangian

$$\mathcal{L} = \frac{1}{2e^2} [E^2 - B^2] - V(A_{cl}) = \frac{1}{2e^2} \left[\left(\dot{\vec{A}} + \vec{\nabla} A_0 \right)^2 - \left(\vec{\nabla} \times \vec{A} \right)^2 \right] - V(A_{cl}) \quad (13)$$

with A_0 expressed in terms of $\vec{A}$ by Eq. (11). Note that in tree approximation, to which we have limited ourselves, there is no dependence left on the nature of the potential V . The complications have been transferred into the kinetic energy portion of the Lagrangian. In the approximation of a quadratic action, we reduce to free fields in temporal gauge. When going beyond that approximation, there is a nonlinear term in the effective Lagrangian containing the time derivative of $\vec{A}$. Such correction

terms are small but serious, since they appear to violate Lorentz covariance and gauge invariance. We will return to this in Section 4, after consideration of the orders of magnitude of the terms we have introduced.

As we already mentioned, in the ancient work it was necessary to assume that the vacuum value of A_μ was very large

$$\langle A_\mu \rangle \equiv \eta_\mu M \quad (14)$$

with η_μ a unit timelike vector and M very large. We may naturally implement this by assuming

$$\lambda = \frac{\mu}{M} \quad (15)$$

with μ a very small mass. Then the overall effective potential is

$$V_{\text{eff}} \equiv \frac{-A^2}{2G} + V(A) = \frac{-A^2}{2G} + \frac{\mu}{4M} A^4 \equiv \frac{\mu M^3}{4} \left[-2 \left(\frac{A^2}{M^2} \right) + \left(\frac{A^2}{M^2} \right)^2 \right] \quad (16)$$

where we identify

$$G \equiv \frac{1}{\mu M} . \quad (17)$$

The general form of this overall effective potential

$$V_{\text{eff}} = \mu M^3 F \left(\frac{A^2}{M^2} \right) \quad (18)$$

may well be preserved even in the presence of radiatively induced terms of higher order, such as A^6 , A^8 , *etc.* In the next section the parameter μ will be identified with the cosmological-constant scale and M with the GUT or Planck scale. The small expansion parameter in the ancient work, characterizing the importance of noncovariant physical effects, was associated with a hypothetical momentum-space cutoff Λ

$$\langle A_\mu \rangle = \eta_\mu M \sim G \Lambda^3 = \frac{\Lambda^3}{M \mu} . \quad (19)$$

Higher order effects were characterized by powers of the dimensionless parameter

$$\frac{1}{G \Lambda^2} = \left(\frac{\mu}{M} \right)^{1/3} \quad (20)$$

which evidently is small, perhaps small enough to escape experimental constraints. However, there is unfinished business in relating the scheme presented in the ancient work to the language in this note, where the expansion is in integer powers of $\mu/M \sim 10^{-30}$. Some of this unfinished business appears to involve real physics, not mere formalism.

3 Standard Model Parameters

For a long time I have enjoyed contemplating the 20-parameter Standard Model in the formal, “gaugeless” limit of vanishing gauge coupling constants [6]. Originally the motivation was to exhibit that, unlike the claims made all too often and too sloppily, it is not true that all known Standard Model interactions are gauge interactions. The proof is to look at the Standard Model in the gaugeless limit. The electron becomes unstable, decaying into a neutrino and massless longitudinal W (the Goldstone mode, which does not decouple in the limit). Evidently there has to be an extra, nongauge force present for this to happen, and in the standard picture it comes from the Higgs sector.

Now I think we all believe that the formal gaugeless limit is artificial, and that in a better theory the other parameters will not stay the same while gauge couplings are varied. This belief became motivation for trying to guess how they might change. And without going into details, I have found a scenario which relates all the major Standard Model parameters to the Planck/GUT scale M and the cosmological constant scale μ (μ^4 is the usual cosmological constant) [3]. The first guess is that

$$\frac{1}{g^2} \sim \frac{1}{h^2} \sim \frac{1}{\lambda} \sim \frac{1}{4\pi^2} \log \frac{M^2}{\mu^2} \quad (21)$$

where g is the gauge coupling constant, h the top quark coupling constant, and λ is the Higgs quartic coupling, all evaluated at the GUT scale. The large logarithm explains why these couplings are small.

The second guess is that the relation of the Higgs condensate value $\langle v \rangle$ to the gravitational parameters is, up to coefficients which may contain the above large logarithm, just the geometric mean of the cosmological-constant and Planck/GUT scales

$$\langle v \rangle^2 \sim M\mu . \quad (22)$$

It is very unlikely that such simple-minded relations should exist. But if the chance is nonvanishing, then it is extremely attractive to pursue the idea further, because it is most likely that the relations will only hold if the underlying dynamics at and above the electroweak scale is austere and simple, instead of being the very rich and complicated scenario more typically presumed to be the case. The most viable option for simplicity is something like the “desert” scenario, with little if any new physics this side of the short distance frontier at the GUT/Planck scales. Note that the usual hierarchy-problem argument against this scenario is inoperative. From the form of the above expression, Eq. (22), for the Higgs $\langle v \rangle$, it is clear that the explanation for its value would be as deep as the explanation for the value of the cosmological constant, and probably correlated.

I cannot resist going one step further at this point, although it is a little off the main track of this subject matter. Recently there has been a proposal that black hole interiors exist in a different vacuum phase than their Schwarzschild exteriors, in

particular that they are nonsingular and described by a static deSitter metric [7]. This means that the cosmological constant in the interior of a black hole is different from its exterior value, and therefore serves as an order parameter which distinguishes the two phases. The magnitude of the cosmological constant scales as the inverse square of the radius of the black hole:

$$\mu^4 \sim \frac{M^2}{R^2} . \quad (23)$$

Consequently the Higgs $\langle v \rangle$ and other Standard Model parameters also are different in a black hole interior. For example, in the interior of the black hole at the center of our galaxy, the electroweak $\langle v \rangle$ would increase by about 4 orders of magnitude, while the ratio of electron to proton mass would presumably diminish by about a factor 20, and the ratio of pion mass to proton mass by a factor 5. In general, within an infinitely large black hole, the Standard Model would become noninteracting and trivial, while for a Planck-scale black hole it would become strongly interacting. This scenario has very interesting implications for the evolutionary cosmology advocated by Smolin [8].

The implications of this picture for our model of the Goldstone photon are also interesting, and that is the main reason for this digression. We have chosen our notation with this in mind: the parameters M and μ in this section are (provisionally) to be identified with those in the previous section. Especially interesting is the form of the effective potential in Eq. (18). In the limit of vanishing cosmological constant it also vanishes. This is quite consistent with the behavior of the rest of the Standard Model dynamics which we have described.

A simple model which might place these ideas in more concrete terms would be to assume our universe to be the (static DeSitter) interior of a Reissner-Nordstrom black hole, with an approximately critical amount of charge on the horizon, created somehow via spontaneous breakdown. The electrostatic potential in the DeSitter interior is then of the same order of magnitude M as the estimate inferred in the previous section. I have not yet worked this out in any detail. But no matter how it turns out, I do not think this is a very realistic model. Surface charge on a black hole horizon is expected to be screened out. And the electroweak gauge bosons, as well as the QCD gluons, deserve a similar Goldstone treatment, and the details of their emergence will certainly interact with those of the Goldstone photon, leading to textures in spacetime and in momentum space of greater richness and complexity.

4 Lorentz Noncovariance

The spontaneous Lorentz symmetry breaking probably leaves in its wake residual noncovariant effects, along with possible violations of gauge invariance. There clearly exists a preferred reference frame in the formalism, naturally identified with the frame for which the cosmic background radiation is locally at rest. Most insidious is the violation of gauge invariance. Our emergent QED is defined in temporal gauge, and

in that gauge the potentials are in principle observable. This is hazardous, because there is no energetic reason in the gauge-invariance approximation for gauge potentials to be small. Therefore small coefficients in front of gauge-dependent terms in the Hamiltonian or Lagrangian may not be sufficient to keep their physical effects small.

We have not attempted a thorough study of these effects, and here only sketch out some of the issues. We recall that the QED effective Lagrangian we arrived at looks completely standard

$$\mathcal{L} = \frac{1}{e^2} \left[\frac{1}{2}(E^2 - B^2) - j_\mu A^\mu \right] + \dots \quad (24)$$

where irrelevant terms have been dropped, and where

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla} \phi \quad \vec{B} = \vec{\nabla} \times \vec{A} \quad j_\mu = \frac{e^2 J_\mu}{G} . \quad (25)$$

However the definition of the electric field is non-standard because the scalar potential has been eliminated in terms of a nonlinear function of the vector potential $\vec{A}$

$$\phi(\vec{A}) = \sqrt{M^2 + \vec{A}^2} - M = \frac{A^2}{2M} + \dots . \quad (26)$$

As a consequence, the equations of motion contain extra terms, with coefficients which are inverse powers of the GUT/Planck mass. A short calculation gives for the equation of motion

$$\dot{\vec{E}} = \vec{j} + (\vec{\nabla} \times \vec{B}) + \Gamma \vec{\beta} . \quad (27)$$

In the above expression, the correction-term field β is defined as

$$\vec{\beta} = \frac{\vec{A}}{\sqrt{M^2 + A^2}} . \quad (28)$$

Note that its magnitude is bounded above by unity. Note also that the quantity

$$\Gamma = \vec{\nabla} \cdot \vec{E} - j_0 \quad (29)$$

which we may call the Gauss-law field, multiplies this new correction term.

This equation of motion must account for two of the four Maxwell equations, the other two source-free equations following automatically from the definition of the fields. The Gauss'-law piece follows from taking the divergence of Eq. (27):

$$\frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{j} + \vec{\nabla} \cdot [\vec{\beta} \Gamma] = \frac{\partial j_0}{\partial t} + \vec{\nabla} \cdot [\vec{\beta} \Gamma] . \quad (30)$$

This is more conveniently written

$$\frac{\partial \Gamma}{\partial t} = \vec{\nabla} \cdot [\vec{\beta} \Gamma] . \quad (31)$$

Without the correction term, this implies that the Gauss-law field Γ is time independent. In canonically quantized QED, this is implemented as a constraint equation on the Hilbert space. Only states that are annihilated by the Gauss-law field are deemed physically admissible. With the correction term, this situation still persists, albeit marginally. If the Gauss-law field Γ vanishes at some initial time, then Eq. (31) shows that this will be true at subsequent times. The result is that, given this constraint, all the Maxwell equations survive without correction. The effects of the nonlinear correction terms are limited to complicating the relationship of the gauge potentials $\vec{A}$ to field strengths $\vec{E}$ and $\vec{B}$.

This is encouraging, but hardly the end of the story. Loop corrections might change this situation. And there seems to be an inherent instability or at least metastability in the structure of this effective action. In addition, the generating functional $W(J)$ obtained by integrating out all fields A_μ appears to contain nonlinear terms, leading *e.g.* to small anomalous N -body ($N \geq 3$) Coulomb-like interactions between charged sources. And even in the gauge invariant terms, possible noncovariant corrections generated by loops must be carefully examined. Especially important are the noncovariant corrections to the $(E^2 - B^2)$ term, which are sensitive to the ultraviolet and which are experimentally bounded to better than one part in 10^{31} [9].

5 Summary

The Goldstone photon discussed above was motivated initially by the Nambu-Jona-Lasinio (NJL) Goldstone pion. In the intervening forty years a great deal of progress has been made in the strong interactions, and our perspective on what the NJL model means has matured. To this day it still plays a rather prominent, albeit controversial, role in chiral QCD. The work of Holger Bech's colleague Mitya Diakonov [10] and his collaborators provides an especially sharp example of the role of the NJL picture within QCD. In brief, starting from the basic QCD action expressed in terms of quarks and gluons, they first integrate out the gluons, assuming that the dominant configurations are instanton-induced. This leaves behind an effective action describing interacting quarks not very dissimilar to the NJL interaction, with occurrence of chiral symmetry breaking via instanton zero modes. After introducing $(\pi$ and $\sigma)$ bosonic fields built from quark bilinears, the quarks and σ field can also be integrated out, leaving behind a nonlinear chiral effective action describing the residual pionic Goldstone modes.

While the details of the procedure, such as the assumed dominance of instanton configurations, may be debatable [11], the bottom line is still the emergent effective action describing the Goldstone pions. This effective action contains essentially an infinite number of terms. Only a handful are operators of dimension less than or equal four, essentially a kinetic energy term and a derivative-free effective potential. The remainder will be of negligible importance at sufficiently low energy scales. Some coefficients of those terms are determined by symmetry considerations. but in general

their sizes are determined by dimensional analysis; the only scale in the problem (in the limit of massless up and down quarks) is Λ_{QCD} (perhaps multiplied by 4π). To view this physics from low energy upward is to see an effective theory of massless, almost noninteracting pions, (trivially) accurate over the many orders of magnitude of energy scales below the natural QCD energy scale [12]. It of course becomes rapidly inoperative as that scale is approached.

Now let us consider the analogous situation for the Standard Model gauge theories. The Standard Model action is composed of a finite number of terms, all of dimension less than or equal four. Higher dimension terms are assumed to be strictly absent. If the gauge bosons are indeed emergent, then one must expect that an infinite number of terms in the action will be present, with a strength scaled by the size of the ultraviolet cutoff M , which we have taken to be of order GUT/Planck size. The terms of dimension greater than four will be extremely small at present energy scales. In practice they will be not very relevant, provided they do not break symmetries such as Lorentz covariance or gauge invariance. However, this may not be the case for terms violating gauge invariance, so special care should be taken in searching phenomenologically for the possible presence of such terms [9].

Most of the above conclusions are not especially novel, because any attempt, such as string theory, to incorporate gravity within the Standard Model is likely to also generate an effective action for the Standard-Model fields, described by a power series in the gravitational constant. However, what we described in the previous sections for the Goldstone photon goes somewhat further. First of all there is the aforementioned issue of noncovariance, which may not need to be addressed in theories with increasingly large symmetries at increasingly large energy scales, but almost certainly must be addressed in theories with emergent degrees of freedom (in the sense implied by analogy to condensed-matter systems), where low-energy symmetries are increasingly broken as the energy scale increases [2].

In addition, the structure of terms in the action with dimension less than or equal four is in our case different. The terms with no derivatives of the gauge fields, which comprise an overall effective potential, are in the Standard Model action forbidden to be present. And this almost remains true in the picture we have described. Unlike the case of the chiral effective action, there is for these terms an assumed suppression factor of μ/M , with μ the cosmological constant scale and M the GUT/Planck scale. This suppression is not gratuitous, but introduced to protect the phenomenology from gross Lorentz-noncovariant effects. Even in the Higgs sector the suppression factor exists for the Higgs mass term, assumed (*cf.* Eq. (22)) to be of order μ/M . However, the quartic Higgs boson self-coupling is for some reason not suppressed; this is a way of re-expressing the hierarchy problem in this language. In addition, it has been assumed that all terms involving spacetime derivatives of A_μ obey current conservation (other than the triangle anomaly). In particular this means that the dimension-four term $(\partial_\mu A^\mu)^2$ is absent or strongly suppressed. In summary, we must in general assume

1. Fermion loop terms allowed in standard gauge theory are present, with typically a coefficient which is enhanced relative to the standard radiative-correction contribution by a factor $\sim \ell n(M/\mu)$.
2. Terms disallowed in standard gauge theory, such as A^2 , A^4 , $(\partial_\mu A^\mu)^2$, are suppressed by at least one power of μ/M .
3. Mass terms in general (in particular the Higgs mass term) are suppressed by at least one power of μ/M .
4. If for some reason there are intrinsically noncovariant contributions, they too are suppressed by positive powers of μ/M .

The actual power of μ/M which multiplies the leading contribution to the effective potential need not be unity; arguments can easily be found for a quadratic or even a quartic dependence. But of course it will be necessary to go further to see whether any such pattern of suppressions can be motivated by real physics.

At this point we have arrived at a situation very close to what has been recently discussed by Holger Bech and his collaborators [4]. They argue that spontaneously broken Lorentz covariance plus strict non-observability of noncovariant effects requires the effective action of the spontaneously broken theory to be identical to the gauge-invariant Standard-Model action. We arrive at their results in the limit $\mu \rightarrow 0$, but prefer to not quite go to the limit. No matter what choice is made, what is really needed are concrete examples of the mechanism of spontaneous symmetry breakdown.

We also assumed that the Standard Model coupling strengths are connected to the gravitational parameters. Here we note that the behavior which was assumed can be established in a simple and elegant way. One multiplies the entire Standard Model Lagrangian density (expressed at the GUT/Planck scale) by a factor $\ell n M^2/\mu^2$, sets the interior coupling constants to values of order unity, and then rescales all fields, fermionic as well as bosonic, to create properly normalized kinetic energy terms. The behavior of the coupling constants then follows Eq. (21), implying that as the cosmological constant μ^4 tends to zero, all the Standard Model interactions vanish.

6 Outlook

These considerations are not worth much unless the underlying physics, if any, of the spontaneous breakdown is delineated. I personally am attracted to the ideas of Volovik [13], which of course rely on earlier work of Holger Bech in a major way. Particularly attractive is the argument for a nearly vanishing cosmological constant: if the vacuum is analogous to a droplet of quantum liquid in equilibrium at very low temperature, the pressure vanishes, up to boundary-surface corrections. Vanishing vacuum pressure is the same statement as vanishing cosmological constant. This simple argument in my mind creates a new setting of the cosmological-constant problem:

it becomes the understanding of Standard Model particles as collective excitations of this purported quantum liquid. The original problem does not go away, but is restated in terms of why the collective modes of such a quantum liquid so faithfully respect gauge invariance and Lorentz covariance, as well as general covariance for the emergent graviton. Very important in the Volovik viewpoint is the structure and topology of momentum space. It may be there that we may find the textures needed for understanding the existence of not just the Goldstone photon but of all twelve Goldstone gauge bosons. The condensed-matter phenomenon of “Fermi points” in momentum space, where for dynamical reasons the fermionic excitations in their neighborhood take on Dirac-like or Weyl-like character, may be an important analog [14]. Perhaps there is an array of such Fermi points in the vacuum momentum space that can account for the large number of fermion species and related gauge excitations.

The problem of the graviton as an emergent degree of freedom must also be addressed. It presumably must be faced at a more fundamental level, *e.g.* higher energy scale, than the gauge sector. There probably needs to be a condensate associated with it as well. If one does not choose to invent new degrees of freedom, about the only possibility seems to be to somehow exploit the right-handed, gauge-singlet, Majorana neutrino condensates present in the see-saw mechanism of neutrino mass generation.

All of this is a big order. The QCD example of the remote relationship of fundamental theory to the chiral effective theory, as well as a host of similar examples from condensed matter theory, does not provide encouragement that one can comprehend the underlying mechanisms, if any, of emergence. On the other hand, the phenomenology of the Standard Model, including the behavior of its parameters that we have assumed, is simpler than those examples. There is a lot more symmetry. There are many excitations but only one limiting velocity of propagation for them. And the dynamic range over which the effective theory is valid is huge. The many Standard-Model parameters of mass and mixing must serve as valuable, albeit enigmatic clues. A skeptic may argue that these differences from historical examples indicate that the emergence idea is inapplicable. But it just might be possible, however unlikely, that the situation to be faced is in fact simpler and more comprehensible than those examples. Given any nonvanishing chance that this is true, this provides more than enough motivation for an optimist to vigorously pursue this program.

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