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PION AND NUCLEON TWIST-4 STRUCTURE FUNCTIONS IN THE LIMIT $x \rightarrow 1^*$

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1. INTRODUCTION

Twist-2 effects in inclusive and semi-inclusive processes depend on only a few non-perturbative functions of a single variable, the quark and gluon longitudinal momentum distribution or decay functions. It has been possible to determine these functions phenomenologically in a small number of processes for subsequent use in analyzing other processes. In contrast, the estimation of twist-4 effects requires knowledge of several non-perturbative functions which depend on two or three variables. It is probably impractical to pursue the strategy followed in twist-2 analyses as a result. Accordingly, it seems to be necessary to organize twist-4 effects in such a way that the non-perturbative features can be estimated roughly, especially in the kinematic regions where higher twist may be significant. The description of twist-4 in terms of multi-constituent longitudinal momentum distributions [1,2,3] is well-suited to meet such a requirement. This assertion will be supported here by a calculation of the $x \rightarrow 1$ behavior of the twist-4 pion and nucleon structure functions to lowest non-trivial order in g within the framework of the operator product expansion (OPE).

In the language of the OPE, longitudinal momentum distributions correspond to single-particle forward matrix elements of "longitudinal" ("canonical") operators, totally symmetric operators with no contracted derivatives. The evaluation of such matrix elements probes only the wave function of the hadron if the system is quantized on the light-cone. In particular, matrix elements of operators of high spin receive large, and probably dominant, contributions from pieces of the wave function which are accessible to perturbative analysis. [The large n

(high spin) behavior in the moment analysis fixes the $x \rightarrow 1$ behavior of the structure function.] In twist-2, the appropriate use of perturbation theory to describe the high-momentum tail in the longitudinal variable of hadronic wave functions led to the phenomenologically successful spectator counting rules.[4] The work described here is an effort to extend these ideas to twist-4: in a longitudinal basis the form of the twist-4 structure functions as $x \rightarrow 1$ can be determined by power counting arguments alone in lowest order, just as for twist-2.

For simplicity, in the remainder of this article emphasis will be placed on the pion structure function. As an illustration of the method used to extract the $x \rightarrow 1$ behavior and in order to clarify the method's limitations, the pion's twist-2 structure function will be considered in sect. 2. The extension to twist-4 is greatly facilitated through a convenient re-expression of the tree-level twist-4 coefficient functions in a longitudinal basis [5], which is given at the beginning of sect. 3. To check the consistency of a direct graphical approach with the OPE analysis where the correspondence is unambiguous in twist-4, a relatively simple twist-4 contribution to the structure function will be studied directly as well as by using the method illustrated in sect. 2. In this instance the leading twist contribution is twist-4. The extraction of twist-4 effects from even the simplest diagrams with twist-2 pieces is more difficult. The OPE provides one technique to make the separation; a more general graphical technique now exists in the literature [6]. The method of sect. 2 is used to compute the leading n -dependence of the twist-4, spin- n matrix elements

corresponding to such subleading effects. The effects are large for a meson target, and the ratio of the twist-4 transverse and longitudinal structure functions is computed in the weak binding approximation. In lowest order, the power counting arguments, when applied to the case of a nucleon target, lead to the result that the transverse twist-4 structure function is enhanced relative to the twist-2 transverse structure function as $x \rightarrow 1$ by only one power of $1/(1-x)$, while the longitudinal piece is not enhanced at all. Conclusions are then given. The appendix contains some technical details of the bound-state formalism used in the body of the paper.

2. MOMENTS OF THE TWIST-2 PION STRUCTURE FUNCTION

From the perspective of the OPE, the importance of twist-4 effects depends both on the coefficient functions and the matrix elements of the spin- n operators. The calculation of the matrix elements, of course, requires non-perturbative information. There is reason to believe, however, that the leading n -dependence of the matrix elements can be determined through perturbative methods. The same situation arises in twist-2. The usual perturbative prediction is that the pion twist-2 transverse structure function falls as $(1-x)^2$ as $x \rightarrow 1$. [7] It is instructive to see how this result can be obtained through the OPE.

It is probably sufficient in both the twist-2 and -4 calculations to use the tree-level coefficient functions only. To this order.

$$-i \int d^4x e^{iq \cdot x} T[J^\mu(x) J^\nu(0)] = X_{T=2}^{\mu\nu} + X_{T=4}^{\mu\nu} + Y_{T=4}^{\mu\nu} \quad , \quad (1)$$

where $X^{\mu\nu}(Y^{\mu\nu})$ is calculated from the graphs of Fig. 1(2). An elementary calculation gives

$$X_{T=2}^{\mu\nu} = \frac{4}{q^4} \sum_{\substack{n=2 \\ \text{(even)}}}^{\infty} T_{\mu_1 \mu_2}^{\mu\nu} \left[\frac{2q_{\mu_3}}{q^2} \right] \dots \left[\frac{2q_{\mu_n}}{q^2} \right] S[\bar{\psi} Q^2 \gamma^{\mu_1} d^{\mu_2} \dots d^{\mu_n} \psi] \quad (2)$$

where

$$T_{\mu_1 \mu_2}^{\mu\nu} = q^2 g_{\mu_1}^{\mu} g_{\mu_2}^{\nu} - (g_{\mu_1}^{\mu} q^{\nu} + g_{\mu_1}^{\nu} q^{\mu}) q_{\mu_2} + g^{\mu\nu} q_{\mu_1} q_{\mu_2} \quad , \quad (3)$$

$d^\mu = i D^\mu$, and S denotes symmetrization and the subtraction of traces.

Q is the charge matrix of the up, down, and strange quarks. In deep inelastic scattering, one needs

$$\langle p | S[\bar{\psi} \gamma^{\mu_1} d^{\mu_2} \dots d^{\mu_n} \psi] | p \rangle = A_n (p^{\mu_1} \dots p^{\mu_n} - \text{traces}) \quad . \quad (4)$$

A simple form for A_n is

$$A_n = \frac{1}{(p^+)^n} \langle p | S[\bar{\psi} Q^2 \gamma^+ (d^+)^{n-1} \psi] | p \rangle \quad . \quad (5)$$

Given this form, it is most natural to view the system as quantized on the light-cone, and work in $A^+ = 0$ gauge. The trace terms in Eq. (4) lead to the kinematic target mass corrections handled through the Nachtmann moment analysis [8], a subtlety which will be ignored hereafter. In any case, the target mass, M , should not be set to zero in the evaluation of A_n . Combining results gives

$$\langle p | X_{T=2}^{\mu\nu} | p \rangle = \frac{4}{q^2} T_{\alpha\beta}^{\mu\nu} p^\alpha p^\beta \sum_{\substack{n=2 \\ (\text{even})}} \left(\frac{2p \cdot q}{q^2} \right)^{n-2} X_{n,T=2} \quad (6)$$

where

$$X_{n,T=2} = \left\{ \frac{1}{n!} \left(\frac{\partial}{\partial \alpha} \right)^n \left[\frac{\alpha}{p^+} \langle p | \bar{\psi} Q^2 \gamma^+ \vec{g}(\alpha) \psi | p \rangle \right] \right\} \Big|_{\alpha=0}$$

and $\vec{g}(\alpha) = (1 - i\alpha \vec{D}^+ / p^+)^{-1}$.

The claim which must be justified is that perturbation theory can supply the leading n -dependence of $X_{n,T=2}$. It is certainly not possible to prove this statement, but it can be made plausible through a graphical examination. Consider the graph of fig. 3, which contributes only one of the terms with the dominant behavior in n . Old-fashioned perturbation theory on the light-cone (OFPT $_{\omega}$) [9,10] is most convenient for the calculation. The bound state formalism used throughout the paper is discussed in the appendix. The expression for the graph is

$$2^{10} \cdot \frac{g^4}{2(2\pi)^3} \cdot 3 \cdot c_3^2 \int dy \, d^2 k_{\perp} \, dy' \, d^2 k'_{\perp} \, \psi_p^{(-)}(yp^+, \vec{k}_{\perp}) \psi_p^{(-)*}(y'p^+, \vec{k}'_{\perp})$$

$$\langle Q^2 \rangle \int dx \, d^2 \ell_{\perp} \frac{1}{(x-y)^2} \frac{1}{(x-y')^2} x^{n-1} \left[\frac{1}{M^2 - \frac{\vec{\ell}_{\perp}^2 + m^2}{x(1-x)} + i\epsilon} \right]^2$$

The graph should be a reasonable representation of a contribution to the matrix element only in certain regions of the loop integration space.

The measure of the "off-shellness" of an intermediate state in OFPT_∞ is the size of its energy denominator. In this particular example the energy denominator is $M^2 - (\vec{\ell}_{\perp}^2 + m^2)/x(1-x) + i\epsilon$. For $\vec{\ell}_{\perp}^2 \gg M^2$ the off-shellness is large and space-like, where perturbation theory should be valid. The real part of the energy denominator vanishes at some point as $\ell_{\perp}^2$ decreases provided that $M^2 > 4m^2$. The matrix element acquires an imaginary part when the real part of the denominator vanishes, which it should not as the operator is hermitian and the target is stable under strong interactions. The problem arises because such a conventional description of a stable composite state with mass greater than the sum of the masses of its constituents is not completely satisfactory. It does not make sense to interpret the quark masses as constituent rather than current masses. Therefore, one must introduce an ad hoc infrared cutoff, κ^2 , on the order of 1 GeV², on the $\vec{k}_{\perp}^2$ integration.

Perturbative analysis suggests that interactions beyond the minimal number produce subleading effects in $(1-x)$, or dress the leading low order result with logarithms. It will be assumed that the non-perturbative interactions occurring below κ^2 also produce subleading effects. Note that from this perspective the large n behavior of the

moments is independent of the up and down quark masses. A lower cutoff, x_0 , should be introduced in the x integration as well. Other components of the hadron wave function besides those with the minimal number of constituents contribute significantly when x is less than x_0 . It is misleading to include the valence configuration only. Furthermore, the non-perturbative contributions to the wave function, $\psi(y p^+, \vec{k}_\perp)$, must be assumed to decrease rapidly as $y \rightarrow 1$. In general the integrals over $\vec{k}_\perp$ and $\vec{x}_\perp$ will not factorize, and it will be necessary to assume as well that $\psi(y p^+, \vec{k}_\perp)$ decreases rapidly when $k_\perp^2$ exceeds some intrinsic scale, ρ^2 , on the order of $0.5 \text{ GeV}^2 < x^2$.

After making these assumptions it is straightforward to count the powers of $(1-x)$ which occur. Note that in OFPT $_\infty$ $\vec{g}(\alpha)$ simply became $1/(1-\alpha x)$. The operation

$$\left[\frac{1}{n!} \left(\frac{\partial}{\partial \alpha} \right)^n \right] \Big|_{\alpha=0}$$

led to a factor of x^{n-1} . Consequently, the integration region in x which contributes significantly as n becomes large is strongly peaked near $x=1$, where the perturbative treatment is probably valid. The leading behavior of the final result is proportional to

$$\int_{x_0}^1 dx x^n (1-x)^2 .$$

as expected. The constant of proportionality is real and positive, because the $i\epsilon$ prescriptions can be ignored. The graphs which give rise to this form are in one-to-one correspondence with the graphs in OFPT $_\infty$ which lead to the form $(1-x)^2$ in a direct calculation of the pion structure function.[7,12]

3. TWIST-4 EFFECTS

The construction of a well-defined procedure for obtaining the leading n -dependence of the twist-4 matrix elements is made possible by a convenient re-expression of the tree-level coefficient functions [5] in terms of $g(\alpha)$, when working in $A^+ = 0$ gauge.¹ In the notation of Eq. (1),

$$\langle p | Y_{T=4}^{\mu\nu} | p \rangle = \frac{-2g^2}{q^6} T_{\alpha\beta}^{\mu\nu} p^\alpha p^\beta \sum_{\substack{n=2 \\ \text{(even)}}}^{\infty} \left(\frac{2p \cdot q}{q^2} \right)^{n-2}$$

$$[Y_{n,RR}^{(1)} + Y_{n,RR}^{(2)} - Y_{n,RL}^{(3)} - Y_{n,RL}^{(4)}] + (R \leftrightarrow L) \quad (7)$$

$Y_n^{(k)}$ can be written in terms of a generating function as

$$Y_n^{(k)} = \left[\frac{1}{n!} \left(\frac{\partial}{\partial \alpha} \right)^n Y^{(k)}(\alpha) \right] \Big|_{\alpha=0}$$

where

$$Y_{RR}^{(1)}(\alpha) = \frac{\alpha^2}{(p^+)^2} \langle p | [\vec{g}(\alpha)(\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(\alpha) \psi_R)] (\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(-\alpha) \psi_R) | p \rangle \quad , \quad (8)$$

$$Y_{RR}^{(2)}(\alpha) = \frac{\alpha^2}{(p^+)^2} \langle p | [\vec{g}(\alpha)(\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(\alpha) \psi_R)] (\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(-\alpha) \psi_R) | p \rangle \quad , \quad (9)$$

$$Y_{RL}^{(3)}(\alpha) = \frac{\alpha^2}{(p^+)^2} \langle p | [\vec{g}(\alpha)(\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(\alpha) \psi_R)] (\bar{\psi}_L Q \tau_a \gamma^+ \vec{g}(-\alpha) \psi_L) | p \rangle \quad , \quad (10)$$

¹ I thank R. L. Jaffe for repeatedly stressing to me the usefulness of the following form for the coefficient functions.

$$Y_{RL}^{(4)}(\alpha) = \frac{\alpha^2}{(p^+)^2} \langle p | [\vec{g}(\alpha) (\bar{\psi}_R Q \tau_a \gamma^+ \vec{g}(\alpha) \psi_R)] (\bar{\psi}_L Q \tau_a \gamma^+ \vec{g}(-\alpha) \psi_L) | p \rangle \quad (11)$$

Also,

$$\begin{aligned} \langle p | X_{T=4}^{\mu\nu} | p \rangle &= \frac{-2g}{q^2} \sum_{\substack{n=2 \\ \text{(even)}}}^{\infty} \left[\frac{-2p \cdot q}{q^2} \right]^{n-2} \frac{1}{(n+2)^2} \\ &\quad \left[\left[\frac{2p \cdot q}{q^2} \right]^2 \left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right] \right. \\ &\quad \left. \left\{ -(n+1)X_n^{(1)} - 2(n+1)[X_n^{(2)} - X_n^{(3)}] - 4X_n^{(4)} + 2X_n^{(5)} \right\} \right. \\ &\quad \left. + \frac{4}{q^4} \tau_{\alpha\beta}^{\mu\nu} p^\alpha p^\beta \right. \\ &\quad \left. \left\{ \frac{n(n-1)}{4} X_n^{(1)} - \left[\frac{5n+4}{2} \right] [X_n^{(2)} - X_n^{(3)}] - (n+8)X_n^{(4)} \right. \right. \\ &\quad \left. \left. + \left[\frac{n+8}{2} \right] X_n^{(5)} \right\} \right] \quad (12) \end{aligned}$$

Again, in terms of its generating function,

$$X_n^{(k)} = \left[\frac{1}{n!} \left(\frac{\partial}{\partial \alpha} \right)^n X^{(k)}(\alpha) \right] \Big|_{\alpha=0}$$

where

$$X^{(1)}(\alpha) = \frac{\alpha}{p^+} \langle p | \bar{\psi} Q^2 g(\alpha) * F^{+\beta} \gamma_\beta \gamma_5 g(\alpha) \psi | p \rangle \quad (13)$$

$$X^{(2)}(\alpha) = \frac{\alpha^3}{(p^+)^3} g \langle p | \bar{\psi} Q^2 g(\alpha) F^{+\beta} g^2(\alpha) F^{+\beta} g(\alpha) \gamma^+ \psi | p \rangle \quad (14)$$

$$X^{(3)}(\alpha) = \frac{\alpha^3}{(p^+)^3} \frac{ig}{2} \langle p | \bar{\psi} Q^2 g(\alpha) * F^{+B} g^2(\alpha) F^{+B} g(\alpha) \gamma^+ \gamma_5 \psi | p \rangle , \quad (15)$$

$$X^{(4)}(\alpha) = \frac{\alpha^3}{(p^+)^3} g \langle p | \bar{\psi} Q^2 g^2(\alpha) F^{+B} g(\alpha) F^{+B} g^2(\alpha) \gamma^+ \psi | p \rangle , \quad (16)$$

$$X^{(5)}(\alpha) = \frac{\alpha^2}{(p^+)^2} g \langle p | \bar{\psi} Q^2 g^2(\alpha) (\bar{\psi} \gamma^+ \tau_a \psi) g^2(\alpha) \gamma^+ \tau_a \psi | p \rangle . \quad (17)$$

In $A^+ = 0$ gauge,

$$*F^{+B} \gamma_B = 2\epsilon^{jk} [(\partial^j A^k) \gamma^+ + (\partial^+ A^j) \gamma^k - g/2 f_{abc} \tau_a A_b^j A_c^k \gamma^+] , \quad (18)$$

$$F^{+B} F^{+B} = -(\partial^+ A^j)(\partial^+ A^j) , \quad (19)$$

$$*F^{+B} F^{+B} = 2\epsilon^{jk} (\partial^+ A^j)(\partial^+ A^k) . \quad (20)$$

The ∂^+ derivatives in $g(\alpha)$ in $X_{T=q}^{\mu\nu}$ act on all fields to the right. It is not surprising that $\langle p | X_{T=q}^{\mu\nu} | p \rangle$ has a relatively simple expression in terms of a sum of products of three $g(\alpha)$'s because the corresponding coefficient functions arose through the expansion of a sum of products of three propagators (see fig. 2). There is a direct correspondence between each propagator and factor of $g(\alpha)$. The structure of the factors of $g(\alpha)$ reflects the flow of momentum in the coefficient function calculation. It is not transparent that $\langle p | X^{\mu\nu} | p \rangle$ should be so simply expressed in terms of $g(\alpha)$.

On the basis of the coefficient functions alone in eq. (12), it is possible to make the following general statement regarding $\langle p | X_{T=q}^{\mu\nu} | p \rangle$: as $x \rightarrow 1$, its contribution to F_2 will have the same

dependence on $(1-x)$, or will be enhanced by at most one power of $(1/(1-x))$ relative to its contribution to F'_L , barring unusual cancellations among terms.

The n -dependence of the twist-4 matrix elements can be investigated in a well-organized fashion using Eqs. (7)-(20). In evaluating the generating functions graphically it is useful to associate a hatched line with each factor of $g(\alpha)$ or $g^2(\alpha)$. The operator itself is still considered to be local, but the momentum flow within each graph related to a generating function is made more transparent by this procedure.

For the purposes of illustration $Y_{RR}^{(1)}(\alpha)$ will be examined in more detail. The result which emerges is identical to that from a direct calculation of the related contribution to the structure function. Consider the graph of fig. 4. Its integrand is proportional to $\{[1-\alpha z][1-\alpha(z-y')][1-\alpha(1-y')]\}^{-1}$. In this simple case, $\psi(y'p^+, \vec{x}'_\perp)$ is assumed to be strongly peaked near $y' = 0.5$, so y' is near 0.5 in this expression. Consequently, the dominant term in this contribution to $Y_{n,RR}^{(1)}$ involves z^{n-2} . The significant integration region in z is again concentrated near $z = 1$. Counting powers in $(1-z)$, the dominant contribution is proportional to

$$\int_{z_0}^1 dz (1-z) z^{n-2} \quad .$$

The analogous contribution in a direct calculation of the structure function is illustrated in fig. 5. $|\vec{k}_\perp|^2 \ll Q^2$ in the integration. In other regions of momentum space, the graph corresponds to a one-loop correction to a coefficient function. The cut in fig. 5 results in a delta function: $\delta(x-z)$, $\delta[x-(z-y')]$, or $\delta[x-(1-y')]$, depending on the

position of the cut. As $x \rightarrow 1$, only the first contributes significantly and the graph leads to a result proportional to $(1-x)$, as expected. In terms of moments, this twist-4 effect is enhanced over the twist-2 result by a factor of n .

The dominant twist-4 effects in the pion structure functions arise from terms in $X^{(1)}(\alpha)$, $X^{(2)}(\alpha)$, and $X^{(3)}(\alpha)$ such as shown in fig. 6(a-d). A contribution to $X^{(2)}(\alpha)$ will be examined in a little bit of detail. One of two important contributions to $X^{(2)}(\alpha)$ is shown in fig. 6(a). Only transverse gluons appear in leading order in n . Using eq. (A.10) and the discussion thereafter, the contribution can be written as

$$\begin{aligned}
X_{6(a)}^{(2)}(\alpha) = & \left\{ -\alpha^3 \frac{g^3}{2(2\pi)^3} \text{Tr} [t^a t^a t^d t^d] e_q^2 \int d^2 r_\perp d^2 r'_\perp \right. \\
& \int_0^1 dy dy' \psi_p^{(-)}(y p^+, \vec{r}_\perp) \psi_p^{(-)*}(y' p^+, \vec{r}'_\perp) \int_{x_0}^1 dx \\
& \int d^2 k_\perp \theta(x-y) \theta(x-y') \frac{1}{1-\alpha y} \frac{1}{(1-\alpha x)^2} \frac{1}{1-\alpha y'} \\
& \frac{\bar{v}_\downarrow(1-x, -\vec{k}_\perp)}{\sqrt{1-x}} \gamma_j (\gamma^- \gamma_5) \gamma^+ (\gamma^- \gamma_5) \gamma_j \frac{v_\downarrow(1-x, -\vec{k}_\perp)}{\sqrt{1-x}} \\
& \left. \frac{1}{\frac{M^2}{2} - \frac{\vec{k}_\perp^2}{1-x} - \frac{(\vec{k}_\perp - \vec{r}'_\perp)^2}{x-y'} + i\epsilon} - \frac{1}{\frac{M^2}{2} - \frac{\vec{k}_\perp^2}{1-x} - \frac{(\vec{k}_\perp - \vec{r}_\perp)^2}{x-y} + i\epsilon} \right\} \\
& + (\downarrow \rightarrow \uparrow)
\end{aligned} \tag{21}$$

where e_q^2 is the square of the quark's charge. After introducing the ad hoc cutoffs described in sect 2, the term is negative definite. Again, ignoring the dependence of the energy denominators on $\vec{r}_\perp$ and $\vec{r}'_\perp$, it is convenient to define

$$\Phi_p^{(-)}(yp^+) = \int d^2r_\perp \psi_p^{(-)}(yp^+, \vec{r}_\perp) \quad . \quad (22)$$

Note that the integral over $\vec{r}_\perp$ is logarithmically divergent. The operator matrix element is to be renormalized at a scale μ^2 , though. Consequently, the result is proportional to $\ln \mu^2/\kappa^2$. There is nothing peculiar in the dependence of the matrix element on $\ln \mu^2$, as the renormalization-group-improved coefficient functions depend on $\ln \mu^2$ as well. Presumably, the dependence on the arbitrary scale, μ^2 , cancels. For consistency, however, μ^2 must be greater than κ^2 , say, $\mu^2 = 1.5 \text{ GeV}^2$, or perhaps $n(1.5) \text{ GeV}^2$. In order to compare with other workers, the result will be evaluated in the weak binding approximation, where $\Phi_p^{(-)}(yp^+) = \Phi_6(y-1/2)$. To leading order in n , the contribution of the graphs of figs. 6(a) and 6(b) is

$$X_n^{(2)} = -2^7 \cdot \frac{g^3 \langle e_p^2 \rangle}{2(2\pi)^3} \cdot 3c_3^2 \cdot |\Phi|^2 \cdot 2\pi \ln \frac{\mu^2}{\kappa^2} \quad . \quad (23)$$

Here $\langle e_p^2 \rangle$ is the sum of the squares of the constituents' charges. The evaluation of $X^{(3)}(\alpha)$ follows along nearly identical lines. The result is

$$X_n^{(3)} = -X_n^{(2)} \quad . \quad (24)$$

Two of the important contributions to $X^{(1)}(\alpha)$ are shown in Figs. 6(c) and 6(d). The others are obtained just by interchanging the quark and anti-quark lines. The diagrams are not symmetric; correspondingly, the sign of the contribution is model dependent. In the loose-binding approximation,

$$X_n^{(1)} = -\frac{1}{n} \cdot 2^9 \cdot \frac{g^3 \langle e_p^2 \rangle}{2(2\pi)^3} \cdot 3c_3^2 \cdot |\Phi|^2 \cdot 2\pi \ln \frac{\mu^2}{\kappa^2} \quad . \quad (25)$$

In a convention where $W^{\mu\nu} = (1/2\pi) \text{Im } T^{\mu\nu}$,

$$W^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W'_L + \frac{1}{M^2} \left[p^\mu p^\nu - \frac{p \cdot q}{q^2} (p^\mu q^\nu + p^\nu q^\mu) + \frac{(p \cdot q)^2}{q^2} g^{\mu\nu} \right] W_2 ,$$

and $F'_L = W'_L$, $F_2 = (p \cdot q)/M^2 W_2$, the usual moment analysis [11] applied to eqs. (1), (7) and (12) in the Bjorken limit gives, for large n ,

$$\int_0^1 dx x^n F_2(x, Q^2) = \frac{1}{2} X_{n, T=2} + \frac{g}{Q^2} \left[\frac{5}{2n} (X_n^{(3)} - X_n^{(2)}) + \frac{1}{4} X_n^{(1)} \right] ,$$

$$\int_0^1 dx x^n F'_L(x, Q^2) = \frac{g}{Q^2} \frac{1}{n} (X_n^{(3)} - X_n^{(2)}) .$$

(26)

Note that the leading term in F'_L is positive independent of the model for the longitudinal momentum distribution, as is suggested by the positivity condition on F'_L based on general principles.

In terms of the structure functions themselves, the twist-4 contributions are of the form

$$F_2^{T=4}(x, Q^2) = \frac{K_2}{Q^2} ,$$

$$F'_L, T=4(x, Q^2) = \frac{K'_L}{Q^2} ,$$

$$\text{as } x \rightarrow 1 . \quad (27)$$

In [12] it was claimed implicitly that $F_2^{T=4}$ is suppressed by some powers of $(1-x)$ relative to $F'_L, T=4$ as $x \rightarrow 1$. More recently, other workers [13] have found the form of eq. (27) as well. According to eqs. (23)-(26), $K_2/K'_L = 2$ in the weak binding limit.

The analysis of the matrix elements for a nucleon target is quite similar. In OFPT $_{\infty}$ the leading behavior in n of A_n , defined by eq. (5), is determined by the graphs of fig. 7 [14]. By power counting, the result is roughly proportional to

$$\int_{x_0}^1 dx_1 x_1^n \int_0^{1-x_1} dx_2 \left(\frac{1}{x_2} \right)^2 [(1-x_1-x_2) \text{ or } x_2]^4 \sim \int_0^1 dx x^n (1-x)^3 .$$

[The smaller of the two quantities in brackets should be taken.]

Equivalently, $F_2(x, Q^2) \rightarrow A(1-x)^3$ as $x \rightarrow 1$, as observed experimentally.

In lowest order, the same sort of power counting arguments apply in the analysis of the twist-4 operator matrix elements. Somewhat

surprisingly, $X_n^{(2)}$ and $X_n^{(3)}$ do not contribute to the leading dependence of the transverse piece on n , only $X_n^{(1)}$ and $X_n^{(4)}$ do so.

Figures 8(a,b) contain important diagrams for $X^{(1)}(\alpha)$, while fig. 9 contains a typical one for $X^{(4)}(\alpha)$. Consider for example the graph of fig. 8(a). It is proportional to

$$\int_{x_0}^1 dx_1 \frac{1}{1-\alpha x_1} \int_0^{1-x_1} dx_2 \left(\frac{1}{x_2} \right)^2 [(1-x_1-x_2) \text{ or } x_2]^3 .$$

Equivalently, $X_{n,[8a]}^{(1)} \sim \int_0^1 dx x^n (1-x)^2$. The same holds for $X_{n,[8b]}^{(1)}$, while $X_{n,[9]}^{(4)} \sim n \int_0^1 dx x^n (1-x)^2$. Looking back at eq. (12) and applying the standard moment analysis, these results translate into the statements that as $x \rightarrow 1$

$$F_2^{T=4}(x, Q^2) \sim \frac{A(1-x)^2}{Q^2} ,$$

$$F_L^{T=4}(x, Q^2) \sim \frac{B(1-x)^3}{Q^2} . \quad (28)$$

No attempt has been made to evaluate the coefficients in a model. One obvious observation is that the signs of terms are model dependent, because the graphs of figs. 8 and 9 are not symmetric. The result for $F_2^{T=4}$ is in agreement with the old result of [15], but conflicts with [16]. It is interesting to examine the source of the discrepancy. The

workers in [16] calculated directly the $x \rightarrow 1$ behavior of the structure functions. Strictly in lowest order, when α_s was held fixed, they found an apparently coincidental cancellation between transverse momentum and off-shellness effects. The remainder had the behavior given here. Once α_s was allowed to run with the loop momenta, the effects no longer canceled. However, from the work presented here and in [5], it is apparent that the initial cancellation must be due to the two exact relations used in the calculation of the coefficient functions: the single-particle forward matrix elements of operators which either vanish by the equations of motion, or are traceless and have no totally symmetric part are zero. The second feature is maintained in perturbation theory only after all loop integrations have been performed. One should be very careful to preserve these properties exactly when estimating higher order effects. In particular, when working in a non-covariant gauge, gauge invariance must be strictly satisfied or else another 4-vector, the gauge fixing vector, is introduced. Probably the most convenient way to preserve both features is to work within the framework of the longitudinal basis, where all the model independent information has been used initially. However, the controversy may not be settled until the one-loop anomalous dimensions of the twist-4 longitudinal operators have been calculated.

4. CONCLUSION

Twist-4 effects may be important in kinematical regions where twist-2 effects are suppressed. Physically, the twist-2 structure function falls in the limit $x \rightarrow 1$ because it is difficult to transfer virtually all the hadron's longitudinal momentum to one constituent. It is natural, then, to describe twist-4 effects in terms of multi-constituent longitudinal momentum distributions. From this perspective added constituents enter the hard-interaction region and can inject into it the longitudinal momentum carried from the spectators. At some time in the hard-interaction process one constituent may carry virtually all the available longitudinal momentum. Twist-4 effects in structure functions can be enhanced as $x \rightarrow 1$, then, because the transfer of longitudinal momentum in the hard-interaction region does not lead to additional suppression.

Based on the perturbative scheme described here, the actual importance of twist-4 effects appears to be quite process-dependent. Both the longitudinal and transverse piece of the pion structure function receive significant twist-4 contributions of nearly equal magnitude, constant as $x \rightarrow 1$. On the other hand, the twist-4 transverse nucleon structure function is enhanced by only one factor of $1/(1-x)$, while the longitudinal piece is not enhanced at all as $x \rightarrow 1$, relative to the twist-2 transverse part.

From the remarks in sect. 2 and the appendix, though, it should be clear that strong assumptions were made in this calculation. Furthermore, it is possible, unlike the case of twist-2, that higher order corrections could modify the leading behavior. It would be quite

interesting if it were necessary phenomenologically for twist-4 effects in the nucleon structure function to be strongly dependent on n (or x), as has been suggested recently. [17]

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APPENDIX

The material in this appendix is included to establish notation and for the sake of completeness; it is not original.² The description of bound states formulated on the light-cone has been used extensively in the literature [9].

Reference [18] discusses the expression for a bound-state matrix element of an operator in the Bethe-Salpeter formalism for a non-gauge theory.³ The derivation given there is appropriate for a system quantized at equal t . The same proof would naturally follow through if the system is quantized on the light-cone [10] at equal τ provided that $t \rightarrow \tau = x^+ = t + x^3$ and τ -orderings replace T -orderings. For the case of a meson,

$$\begin{aligned} \langle p' | O(z) | p \rangle = & \int d^4x_1 \cdots d^4x_4 \bar{\chi}_{p', \alpha_1 \alpha_2}(x_1, x_2) \\ & K_O(z), \alpha_1 \alpha_2 \alpha_3 \alpha_4(x_1, x_2, x_3, x_4) \chi_{p, \alpha_3 \alpha_4}(x_3, x_4) \end{aligned} \quad (A.1)$$

where

$$\chi_{p, \alpha_3 \alpha_4}(x_3, x_4) = \langle 0 | \tau [\bar{\psi}_{\alpha_3}(x_3) \psi_{\alpha_4}(x_4)] | p \rangle, \quad (A.2)$$

$$\bar{\chi}_{p', \alpha_1 \alpha_2}(x_1, x_2) = \langle p' | \tau [\psi_{\alpha_1}(x_1) \bar{\psi}_{\alpha_2}(x_2)] | 0 \rangle, \quad (A.3)$$

and

² I thank S. Brodsky for discussions concerning the issues addressed here. He may not agree with all the statements made in this appendix, though.

³ In a gauge theory, one must add a phase factor to $\bar{\psi}(x_3)\psi(x_4)$ in order to form a gauge-invariant wave function. This can be decomposed into gauge non-invariant components with specific numbers of gluons. The various components satisfy a tower of coupled integral equations. Only the component with no gluons will be considered here, because others with more than the minimal number of constituents give rise to sub-leading effects as $x \rightarrow 1$.

$$\begin{aligned}
& K_0(z), \alpha_1 \alpha_2 \alpha_3 \alpha_4 (x_1, x_2, x_3, x_4) \\
& = \langle 0 | \tau [\bar{\psi}_{\alpha_1}(x_1) \psi_{\alpha_2}(x_2) O(z) \psi_{\alpha_3}(x_3) \bar{\psi}_{\alpha_4}(x_4)] | 0 \rangle_{\text{B.S.I.}}
\end{aligned} \tag{A.4}$$

where the subscript, B.S.I., indicates that only the Bethe-Salpeter irreducible piece should be kept. Define

$$\begin{aligned}
g_{p, \alpha_3 \alpha_4} \left(\frac{k_3 - k_4}{2} \right) &= \int \frac{d^4 z}{(2\pi)^4} \exp \left[i \left(\frac{k_3 - k_4}{2} \cdot z \right) \right] \\
&\quad \langle 0 | \tau \left[\bar{\psi}_{\alpha_3} \left(\frac{z}{2} \right) \psi_{\alpha_4} \left(-\frac{z}{2} \right) \right] | p \rangle,
\end{aligned} \tag{A.5}$$

and

$$\bar{g}_{p, \alpha_1 \alpha_2} \left(\frac{k_1 - k_2}{2} \right) = \int \frac{d^4 z}{(2\pi)^4} \exp \left[-i \left(\frac{k_1 - k_2}{2} \cdot z \right) \right] \langle p | \tau \left[\psi_{\alpha_1} \left(\frac{z}{2} \right) \bar{\psi}_{\alpha_2} \left(-\frac{z}{2} \right) \right] | 0 \rangle. \tag{A.6}$$

Equation (A.1) can be rewritten in terms of g and $\bar{g}$ as

$$\begin{aligned}
\langle p' | O(z) | p \rangle &= \int d^4 k_1 \cdots d^4 k_4 \delta^4(k_1 + k_2 - p') \delta^4(k_3 + k_4 - p) \bar{g}_{p', \alpha_1 \alpha_2} \left(\frac{k_1 - k_2}{2} \right) \\
&\quad [\int d^4 x_1 \cdots d^4 x_4 \exp[i(k_1 \cdot x_1 + k_2 \cdot x_2 - k_3 \cdot x_3 - k_4 \cdot x_4)] \\
&\quad K_0(z), \alpha_1 \alpha_2 \alpha_3 \alpha_4 (x_1, x_2, x_3, x_4)] g_{p, \alpha_3 \alpha_4} \left(\frac{k_3 - k_4}{2} \right).
\end{aligned} \tag{A.7}$$

Hereafter, the Dirac indices will be dropped except where needed.

As a first remark, K_0 is unchanged if the τ -ordering is replaced by a T -ordering. This is shown most simply by recalling that the free propagators themselves have this property [10]. For the purposes of calculations it is easiest to write down a contribution to K_0 in coordinate space according to the rules of Feynmann perturbation theory, and then write the propagators in a form suitable for the light-cone, following [10].

As an example relevant to the $x \rightarrow 1$ behavior of the pion, consider the operator $O = \bar{\psi} A^k A j \gamma^+ \psi$. A simple contribution to K_0 , $K_0^{(1)}$, is shown in fig. 10a.

$$\begin{aligned}
 K_0(z), \alpha_1 \alpha_2 \alpha_3 \alpha_4^{(1)}(x_1, x_2, x_3, x_4) &= (-ig)^2 \delta^4(x_4 - z) \delta^4(x_2 - z) \\
 &\quad (\gamma^+)_{\alpha_2 \alpha_4} \{ \gamma_\mu [iS_F(x_3 - x_1)] \gamma_\nu \}_{\alpha_3 \alpha_1} \\
 &\quad [iD_F^{\mu j}(z - x_3)] [iD_F^{\nu k}(x_1 - z)]
 \end{aligned} \tag{A.8}$$

Making the substitutions

$$\begin{aligned}
 iS_F(x_3 - x_1) &= - \frac{1}{2(2\pi)^3} \int d^2 \ell_3^\perp \int_0^\infty \frac{d\ell_3^+}{\ell_3^+} \theta(\tau_1 - \tau_3) v(\ell_3) \bar{v}(\ell_3) \\
 &\quad \exp[i\ell_3 \cdot (x_3 - x_1)] + \dots, \\
 iD_F^{\mu j}(z - x_3) &= \frac{1}{2(2\pi)^3} \int d^2 \ell_1^\perp \int_0^\infty \frac{d\ell_1^+}{\ell_1^+} \theta(z^+ - \tau_3) d^{\mu j}(\ell_1) \\
 &\quad \exp[-i\ell_1 \cdot (z - x_3)] + \dots, \\
 iD_F^{\nu k}(x_1 - z) &= \frac{1}{2(2\pi)^3} \int d^2 \ell_2^\perp \int_0^\infty \frac{d\ell_2^+}{\ell_2^+} \theta(\tau_1 - z^+) d^{\nu k}(\ell_2) \\
 &\quad \exp[-i\ell_2 \cdot (x_1 - z)] + \dots,
 \end{aligned} \tag{A.9}$$

with

$$\ell_i^- = \frac{(\ell_i^\perp)^2 + m^2}{\ell_i^+}$$

and

$$d^{\mu\nu}(\ell) = -g^{\mu\nu} + \frac{\eta^\mu \ell^\nu + \eta^\nu \ell^\mu}{\eta \cdot \ell}$$

projects out the contribution of the graph in OFPT $_\infty$ shown in fig. 10b.

Then,

$$\begin{aligned}
& \int d^4z \int d^4x_1 \cdots d^4x_4 \exp[i(k_1 \cdot x_1 + k_2 \cdot x_2 - k_3 \cdot x_3 - k_4 \cdot x_4)] \\
& K_0(z), \alpha_1 \alpha_2 \alpha_3 \alpha_4^{(1)}(x_1, x_2, x_3, x_4) \\
& = \frac{-(-ig)^2}{2(2\pi)^3} \cdot (2\pi)^4 \delta^4(k_3 + k_4 - k_1 - k_2) \int d^2\ell_1^\perp \cdots d^2\ell_3^\perp \int_0^\infty \frac{d\ell_1^+}{\ell_1^+} \cdots \frac{d\ell_3^+}{\ell_3^+} \\
& \delta^2(k_1^\perp - \ell_3^\perp - \ell_2^\perp) \delta(k_1^+ - \ell_3^+ - \ell_2^+) \delta^2(k_3^\perp - \ell_3^\perp - \ell_1^\perp) \delta(k_3^+ - \ell_3^+ - \ell_1^+) \\
& d^{\mu j}(\ell_1) d^{\nu k}(\ell_2) \frac{i}{(k_1 - \ell_3 - \ell_2)^-} \frac{i}{(k_3 - \ell_3 - \ell_1)^-} \{\gamma^+ \}_{\alpha_2 \alpha_4} \\
& \{\gamma_{\mu\nu}(\ell_3) \bar{\psi}(\ell_3) \gamma_\nu\}_{\alpha_3 \alpha_1} \quad . \quad (A.10)
\end{aligned}$$

Integrating (A.7) over z , and substituting in (A.10) would give one contribution to the matrix element.

To make additional progress it is necessary to assume that

$$\bar{g}_P \left[\frac{k_1 - k_2}{2} \right] , \quad \left[g_P \left[\frac{k_3 - k_4}{2} \right] \right] ,$$

are strongly peaked about $(k_1 - k_2)^- = 0$, $[(k_3 - k_4)^- = 0]$, respectively.

Then, one can substitute $k_i^- = M^2/2p^+$ throughout K_0 . Afterwards the only dependence on $(k_1 - k_2)^-$, $[(k_3 - k_4)^-]$, is in $\bar{g}, [g]$, and the integrals over these variables can be done. For example, after parameterizing k_3 and k_4 as

$$k_3 = \left[(1-y)p^+, -r^- + \frac{M^2}{2p^+}, -\vec{r}^\perp \right] \text{ and } k_4 = \left[yp^+, r^- + \frac{M^2}{2p^+}, \vec{r}^\perp \right] ,$$

$$\psi_{P, \alpha_3 \alpha_4}(yp^+, \vec{r}^\perp) = \int dr^- g_P[-(y - 1/2)p^+, -r^-, -\vec{r}^\perp]$$

$$= \int \frac{d^2z^\perp}{(2\pi)^2} \frac{dz^-}{2\pi}$$

$$\exp[i(yp^+ z^-/2 - \vec{r}^\perp \cdot \vec{z}^\perp)] \langle 0 | \tau [\bar{\psi}_{\alpha_3}(0) \psi_{\alpha_4}(z)] | p \rangle \Big|_{z^+=0} .$$

(A.11)

A similar equation defines $\bar{\psi}_P$.

In general, $\psi_{p,\alpha_3\alpha_4}$ and $\bar{\psi}_{p,\alpha_1\alpha_2}$ can be expanded in terms of the 16 Dirac matrices, restricted by discrete symmetries. In fact, the dominant terms in the $x \rightarrow 1$ limit of the pion structure function all arise from the terms

$$\begin{aligned}\psi_{p,\alpha_3\alpha_4}(yp^+, \vec{r}_\perp) &= (\gamma^- \gamma_5)_{\alpha_4\alpha_3} \psi_p^{(-)}(yp^+, \vec{r}_\perp) + \dots, \\ \bar{\psi}_{p,\alpha_1\alpha_2}(y'p^+, \vec{r}'_\perp) &= (\gamma^- \gamma_5)_{\alpha_1\alpha_2} \bar{\psi}_p^{(-)}(yp^+, \vec{r}_\perp) + \dots\end{aligned}\quad (A.12)$$

In light-cone quantization, the Fermi field which satisfies canonical commutation relations is $1/4 (\gamma^- \gamma^\mu \psi)_\alpha$. [10] Fortunately, $\psi_p^{(-)}$ and $\bar{\psi}_p^{(-)}$ receive contributions from fields of this type only. Using the commutation relations, one can quickly show that

$$\bar{\psi}_p^{(-)}(yp^+, \vec{r}_\perp) = -\psi_p^{(-)*}(yp^+, \vec{r}_\perp) \quad (A.13)$$

The support of $\psi_p(yp^+, \vec{r}_\perp)$ in the variable y is critical. It will be taken to be the interval $[0,1]$. One can argue that free fields carry only positive longitudinal momentum [10]. The QCD interactions conserve longitudinal momentum, so the positivity property must hold for the interacting fields as well. The difficulty with this argument is that the constituents of a hadron cannot be thought of as having evolved adiabatically from free particles. If nothing else, the success of twist-2 phenomenology supports the claim.

The final assumption is that $\psi_p(yp^+, \vec{r}_\perp)$ falls off rapidly for $\vec{r}_\perp^2 > \rho^2$ as explained in sect. 2.

Figure Captions

- Fig. 1. Tree graphs required for the calculation of $X^{\mu\nu}$.
- Fig. 2. Tree graphs required for the calculation of $Y_{T=1}^{\mu\nu}$.
- Fig. 3. One graph contributing to $X_{T=2}(\alpha)$.
- Fig. 4. A simple contribution to $Y^{(1)}(\alpha)$.
- Fig. 5. The diagram in a direct calculation of the structure function corresponding to Fig. 4.
- Fig. 6. (a) and (b): The two graphs of order g^2 giving the leading behavior of $X^{(2)}(\alpha)$ and $X^{(3)}(\alpha)$. (c) and (d): Two graphs giving the leading behavior of $X^{(1)}(\alpha)$. Those with quark and anti-quark interchanged are not shown.
- Fig. 7. (a) and (b): Typical graphs in OFPT $_{\infty}$ which give the leading behavior of $F_2^{T=2}(x, Q^2)$ as $x \rightarrow 1$.
- Fig. 8. (a) and (b): Some leading contributions to $X^{(1)}(\alpha)$.
- Fig. 9. A leading contribution to $X^{(4)}(\alpha)$.
- Fig. 10. (a) A Feynmann graph discussed in the text of the appendix.
(b) One of several graphs in OFPT $_{\infty}$ corresponding to Fig. 10a.

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Fig. 1

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Fig. 2

Fig. 3

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Fig. 4

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Fig. 5

Fig. 6

(a)

(b)

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Fig. 7

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Fig. 8

Fig. 9

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Fig. 10