A LOWER BOUND FOR THE TRIPLE POMERON VERTEX*

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ABSTRACT

We establish a lower bound for the triple-Pomeron contribution to inclusive distributions at small values of t , and observe that this minimum contribution is larger than that suggested by data on inclusive reactions, unless the Pomeron slope is extremely small.

*Work supported in part by the Air Force Office of Scientific Research, Office of Aerospace Research, U.S. Air Force, under AFOSR Contract No. F44620-71-C-0044 and in part by the U.S. Atomic Energy Commission.

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There has been considerable interest in the possibility of a coupling between three Pomerons, which, if it existed, would dominate inclusive reactions of the type $a+b \rightarrow a+X$ in the limit $M^2 \rightarrow \infty$, $S/M^2 \rightarrow \infty$ (the kinematics are indicated in Fig. 1). It is known that, if the Pomeron intercept is unity, this coupling must vanish as the momentum transfer t approaches zero.¹ In this note we derive a non-zero lower bound for the derivative of the triple-Pomeron coupling at $t = 0$. The existence of this bound establishes that this coupling is not identically zero, and that the zero at $t = 0$ is in fact linear as has been suggested by dual-model calculations.² Furthermore, for those finite values of t sufficiently small so that one may make a linear approximation to the triple-Pomeron coupling, this bound provides a lower limit on the triple-Pomeron contribution to inclusion cross sections. We observe below that it is difficult, although not impossible, to reconcile this bound with presently available data, unless the value of the Pomeron slope α' is less than about 0.25 GeV^{-2} .

Consider the reaction $pp \rightarrow pX$. In the triple-Regge region, this distribution is given by

$$\frac{d\sigma}{dt dM^2}(s, t, M^2) = \frac{1}{s^2} \sum_{i,j,k} \Gamma_{ijk}(t) \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} (M^2)^{\alpha_k(0)} \quad (1)$$

where Γ_{ijk} is proportional to the triple-Regge coupling between Reggeons i , j , and k (see Fig. 2). We shall show, for $i = j = k = \text{Pomeron (P)}$, that

$$\Gamma_{PPP}(t) \geq (-2\alpha' t) \frac{d\sigma}{dt} + O(t^2) \quad (2)$$

Here $d\sigma/dt$ is the forward elastic pp differential cross section at high energy, which has a value estimated from the optical theorem of about 80 mb/GeV^2 .

We shall assume that the Pomeron is a simple, moving Regge pole with intercept one; if it is a more complicated singularity, then it is quite possible that none of the triple-Regge formalism applies to it. We shall also use a result of Abarbanel and Green³ on fixed poles in Pomeron-proton amplitudes which, although not rigorously established, has been made sufficiently plausible to motivate us to explore its consequences.

As shown in Ref. 4, one may write finite-energy sum rules for inclusive distributions, which in our case read

$$\int_0^L dM^2 (M^2)^n \frac{d\sigma(s, t, M^2)}{dt dM^2} = \sum_{i,j,k} \Gamma_{ijk}(t) s^{\alpha_i(t)+\alpha_j(t)-2} \frac{(L)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)+n+1}}{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)+n+1} \quad (3)$$

where n is odd. One may also write Schwarz⁵ sum rules, obtained by evaluating (3) for n even, if one remembers to include fixed-pole residues on the RHS of (3). Taking then $n = 0$ in (3), and equating coefficients of the leading power of s (which is $s^{2\alpha_P(t)-2}$), we obtain

$$\int_0^L dM^2 \lim_{s \rightarrow \infty} \left[s^{2-2\alpha_P(t)} \frac{d\sigma(s, t, M^2)}{dt dM^2} \right] = \sum_k \Gamma_{PPk}(t) \frac{(L)^{\alpha_k(0)-2\alpha_P(t)+1}}{\alpha_k(0)-2\alpha_P(t)+1} + R(t) \quad (4)$$

where $R(t)$ is proportional to the residue of a nonsense wrong-signature fixed pole in the Pomeron-proton amplitude. At $t = 0$, we may take $L \rightarrow \infty$, in which case the only Regge term that survives is the one in which $k = P$; this gives

$$\int_0^\infty dM^2 \lim_{s \rightarrow \infty} \frac{d\sigma(s, t=0, M^2)}{dt dM^2} = \lim_{t \rightarrow 0} \left[\frac{\Gamma_{PPP}(t)}{2-2\alpha_P(t)} \right] + R(t=0) \quad (5)$$

We now use a result of Abarbanel and Green³ that $R(t=0)$ is zero. Since the integrand in (5) is non-negative, we can obtain a lower estimate for this integral by including only the contribution of the single-proton state in the M^2 integration. This gives

$$\lim_{t \rightarrow 0} \frac{\Gamma_{PPP}(t)}{2 - 2\alpha_P(t)} > \lim_{s \rightarrow \infty} \frac{d\sigma(s, t=0)}{dt} \quad (6)$$

Equation (2) then follows from (6) and analyticity in t .

We shall try to use the bound in (2) and available data on $p+p \rightarrow p+X$ to set an upper bound on α' . The data of Allaby et al⁶ obtained at $p_{\text{lab}} = 19.2 \text{ GeV}/c$ for $p+p \rightarrow p+X$ is remarkably flat, for small t values, as a function of M^2 in the region of M^2 where one may hope the triple-Regge formalism applies ($M^2 \sim 4-6 \text{ GeV}^2$). This behavior of $\frac{d\sigma}{dt dM^2}$ has been taken⁷ as an indication that the triple-Pomeron contribution is small. This may not be necessarily the case since there could be cancellations between various terms in Eq. (1) and it could be that the contribution of Γ_{PPP} to $\frac{d\sigma}{dt dM^2}$ is indeed larger than $\frac{d\sigma}{dt dM^2}$. We regard this possibility, however, as rather unlikely, especially in view of the remarkable cancellations needed to produce a distribution which is M^2 independent. Let us then take (very conservatively) the actual value of $\frac{d\sigma}{dt dM^2}$ as an estimate for the triple-Pomeron contribution and for small t ignore the $O(t^2)$ term in Eq. (2). For $t = -.13 \text{ GeV}^2$ and $M^2 = 4.1 \text{ GeV}^2$ we have that $\frac{d\sigma}{dt dM^2} = 1.38 \text{ mb/GeV}^4$. Equation (2) then provides the bound $\alpha' \leq 0.25 \text{ GeV}^{-2}$. Since we suspect that the single Pomeron contribution is in fact much less than our estimate, the value of α' should probably be significantly smaller. At higher energies one should be able to isolate the Pomeron contributions better and hence sharpen the bound on α' .

Finally, we should emphasize that the assumption of simple analytic properties of the Pomeron singularity as well as the assumptions that led to the Abarbanel-Green result are certainly open to very serious questions. It may well be that their result is incorrect: in fact, if the inequality (2) turns out to be violated, this may be taken as evidence against the result of Ref. 3. If the fixed-pole residue $R(t) \rightarrow \infty$ as $t \rightarrow 0$, one could still obtain a result similar to (2); however, if $R(t) \rightarrow \text{constant}$ as $t \rightarrow 0$, no such bound is obtained. We have shown that simple analyticity assumptions, including those of Ref. 3, lead to a lower bound in the triple-Pomeron contribution to inclusive reactions, and seem to require a quite small value of α' .

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FIGURE CAPTIONS

Fig. 1. Kinematics for the reaction $a+b \rightarrow a+X$.

Fig. 2. Contribution of Reggeons R_i , R_j , and R_k to the inclusive reaction $a+b \rightarrow a+X$.

Fig. 1

Fig. 2