Resistive Wall Contribution to the Impedance of a Collimator ¹

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Abstract

Effect of the wall resistivity is rigorously taken into account in the calculations of the impedance of a collimator.

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The first case is relevant for calculation of the longitudinal impedance (azimuthal harmonics $m = 0$), and the second for the calculations of the transverse impedance ($m = 1$). The only difference in the monopole and dipole cases are the coefficients c_m , $c_0 = 1$, $c_1 = 2$.

The BC at $r = b$ are:

$$\begin{aligned} E_t &= -E_t^P - (1 - i)\frac{k\delta}{2}c_m E_r^P, \\ E_\phi &= -E_{phi}^P. \end{aligned} \quad (7)$$

For a taper with the angle α with the z-axes, $E_t^P = E_r^P \sin(\alpha)$. Eq. (7) gives E_t at the boundary

$$E_t = -[\sin(\alpha) + (1 - i)k\delta]E_r^P. \quad (8)$$

3 Calculation of the Impedance

The impedance calculations may be based on the standard technique[1, 2]. BC are taken into account modifying Maxwell equations by adding magnetic current and charge:

$$\begin{aligned} \text{curl } E &= ikH - Z_0 j, \quad \text{div } E = 0 \\ \text{curl } H &= -ikE, \quad \text{div } H = 4\pi\rho. \end{aligned} \quad (9)$$

The current j is the surface current $j = \delta(r - b(z))I(z)/r$ where $I(z)$ is the 2D tangential vector defined by the boundary condition:

$$I = \frac{1}{Z_0}(n \times E)_{r=b}. \quad (10)$$

Induced fields are given as superposition of the eigen modes, E_ν , H_ν , which are solution of Maxwell equations with ideal BC,

$$\begin{aligned} \text{curl } E &= ikH, \quad \text{div } E = 0 \\ \text{curl } H &= -ikE, \quad \text{div } H = 0. \end{aligned} \quad (11)$$

The combination of these equations, gives (Lorentz) after integration over the volume V ,

$$\int dS(E \times H_\nu^* - H \times E_\nu^*) = -Z_0 \int dV H_\nu^* j. \quad (12)$$

Here the surface integral is taken only over the cross-sections of the pipe perpendicular to the z-axes.

5 Conclusion

Resistivity of the wall can be taken into account in calculations of the impedance of a taper as a multiplicative factor to the geometric impedance. In the usual case of a shallow collimator, it can be written as the sum of the geometric impedance and the impedance of a straight pipe with small additional term. This justifies the standard recipe of taking RW into account.

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References

- [1] Veinstein
- [2] Jackson. "Physics of Collective Beam Instabilities in High Energy Accelerators", Wiley, 1993.