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We are interested in obtaining the long-range, dipole wakefield of a linac structure with internal misalignments. The NLC linac structure is composed of a collection of cups that are brazed together, and such a calculation, for example, is important in setting the straightness tolerance for the composite structure. Our derivation, presented here, is technically applicable only to structures for which all modes are trapped. The modes will be trapped at least at the ends of the structure, if the connecting beam tubes have sufficiently small radii and the dipole modes do not couple to the fundamental mode couplers in the end cells. For detuned structures (DS), like those in the injector linacs of the JLC/NLC[1], most modes are trapped internally within a structure, and those that do extend to the ends couple only weakly to the beam; for such structures the results here can also be applied, even if the conditions on the beam tube radii and the fundamental mode coupler do not hold. We believe that even for the damped, detuned structures (DDS) of the main linac of the JLC/NLC[2], which are similar, though they have manifolds to add weak damping to the wakefield, a result very similar to that presented here applies.

We assume a structure is composed of many cups that are misaligned transversely by amounts that are very small compared to the cell dimensions. For such a case we assume that the mode frequencies are the same as in the ideal structure, and only the mode kick factors are affected. To first order we assume that for each mode, the kick factor for the beam on-axis in the imperfect structure is the same as for the case with the beam following the negative of the misalignment path in the error-free structure. In Fig. 1 we sketch a portion of such a misaligned structure (top) and the model used for the kick factor calculation (bottom). Note that the relative size of the misalignments is exaggerated from what is expected, in order to more clearly show the principle. Given this model, the method of calculation of the kick factors can be derived using the so-called “Condon Method”[3],[4] (see also [5]). Note that this application to cell-to-cell misalignments in an accelerator

structure is presented in Ref. [6]. The results of this perturbation method have been shown to be consistent with those using a 3-dimensional scattering matrix analysis[7]. We will only sketch the derivation below.

Figure 1: Sketches of part of a misaligned structure (top) and the model used for the kick factor calculation (bottom). Note that the relative size of the misalignments here is much exaggerated.

Consider a closed cavity with perfectly conducting walls. For such a cavity the Condon method expands the vector and scalar potentials, in the Coulomb gauge, as a sum over the empty cavity modes. As function of position $\mathbf{x}$ (x, y, z) and time t the vector potential in the cavity is given as

$$\mathbf{A}(\mathbf{x}, t) = \sum_{\lambda} q_{\lambda}(t) \mathbf{a}_{\lambda}(\mathbf{x}) \quad , \quad (1)$$

where

$$\nabla^2 \mathbf{a}_\lambda + \frac{\omega_\lambda^2}{c^2} \mathbf{a}_\lambda = 0 \quad , \quad (2)$$

with ω_λ the frequency of mode λ , and $\mathbf{a}_\lambda \times \hat{\mathbf{n}} = 0$ on the metallic surface ($\hat{\mathbf{n}}$ is a unit vector normal to the surface). Using the Coulomb gauge implies that $\nabla \cdot \mathbf{a}_\lambda = 0$. The q_λ are given by

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{1}{2U_\lambda} \int_{\mathcal{V}} d\mathcal{V} \mathbf{j} \cdot \mathbf{a}_\lambda \quad , \quad (3)$$

with the normalization

$$\frac{\epsilon_0}{2} \int_{\mathcal{V}} d\mathcal{V} \mathbf{a}_{\lambda'} \cdot \mathbf{a}_\lambda = \mathcal{U}_\lambda \delta_{\lambda\lambda'} \quad , \quad (4)$$

with $\mathbf{j}$ the current density. Note that the integrations are performed over the volume of the cavity $\mathcal{V}$.

The scalar potential is given as

$$\Phi(\mathbf{x}, t) = \sum_{\lambda} r_\lambda(t) \phi_\lambda(\mathbf{x}) \quad , \quad (5)$$

where

$$\nabla^2 \phi_\lambda + \frac{\Omega_\lambda^2}{c^2} \phi_\lambda = 0 \quad , \quad (6)$$

with Ω_λ the frequencies associated with ϕ_λ , and with $\phi_\lambda = 0$ on the metallic surface. The r_λ are given by

$$r_\lambda = \frac{1}{2T_\lambda} \int_{\mathcal{V}} d\mathcal{V} \rho \phi_\lambda \quad , \quad (7)$$

with ρ the charge distribution in the cavity. Note that one fundamental difference between the behavior of $A(\mathbf{x}, t)$ and $\Phi(\mathbf{x}, t)$ is that when there are no charges in the cavity the vector potential can still oscillate whereas the scalar potential must be identically equal to 0.

Let us consider an ultra-relativistic driving charge Q that passes through the cavity parallel to the z axis, and (for simplicity) a test charge following at a distance s behind on the same path. Both enter the cavity at position $z = 0$ and leave at position $z = L$. The transverse wakefield at the test charge is then

$$\mathbf{W}(s) = \frac{1}{QLx_0} \int_0^L dz [c\nabla_\perp A_z - \nabla_\perp \Phi]_{t=(z+s)/c}$$

$$= \frac{1}{QLx_0} \sum_{\lambda} \int_0^L dz \left[cq_{\lambda} \left(\frac{z+s}{c} \right) \nabla_{\perp} a_{\lambda z}(z) - r_{\lambda} \left(\frac{z+s}{c} \right) \nabla_{\perp} \phi_{\lambda}(z) \right], \quad (8)$$

where the integrals are along the path of the particle trajectory. The parameter x_0 is a parameter for transverse offset (the transverse wake is usually given in units of V/C per longitudinal meter per transverse meter); for a cylindrically-symmetric structure it is usually taken to be the offset, from the axis, of the driving bunch trajectory. For $s > L$ we can drop the scalar potential term (it must be zero when there is no charge in the cavity), and the result can be written in the form[5]

$$\mathbf{W}(s) = \sum_{\lambda} \frac{c}{2U_{\lambda}\omega_{\lambda}Lx_0} \Im m \left[V_{\lambda}^* \nabla_{\perp} V_{\lambda} e^{i\omega_{\lambda}s/c} \right] \quad [s > L], \quad (9)$$

with

$$V_{\lambda} = \int_0^L dz a_{\lambda z}(z) e^{i\omega_{\lambda}z/c}. \quad (10)$$

Note that the arbitrary constants associated with the parameter $\mathbf{a}_{\lambda}$ in the numerator and the denominator of Eq. 9 cancel. Note also that—to the same arbitrary constant— $|V_{\lambda}|^2$ is the square of the voltage lost by the driving particle to mode λ and U_{λ} is the energy stored in mode λ .

Consider now the case of a cylindrically-symmetric, multi-cell accelerating cavity, and let us limit our concern to the effect of the dipole modes of such a structure. We will allow the charges to move on an arbitrary, zig-zag path in the $x - z$ plane that is close to the axis, and for which the slope is everywhere small (so that $\nabla_{\perp} \sim \partial/\partial x$). For dipole modes in a cylindrically-symmetric, multi-cell accelerator structure, it can shown that the synchronous component of $a_{\lambda z}$ (the only component that, on average, is important) can be written in the form $a_{\lambda z} = x f_{\lambda}(z)$ (see *e.g.* Ref. [8]). Then Eq. 9 becomes

$$W_x(s) = \sum_{\lambda} \frac{c}{2U_{\lambda}\omega_{\lambda}Lx_0} \times \Im m \left[e^{i\omega_{\lambda}s/c} \int_0^L dz' x(z') f_{\lambda}(z') e^{-i\omega_{\lambda}z'/c} \int_0^L dz f_{\lambda}(z) e^{i\omega_{\lambda}z/c} \right] \quad [s > L]. \quad (11)$$

Note that this equation can be written in the form:

$$W_x(s) = \sum_{\lambda} 2k'_{\lambda} \sin \left(\frac{\omega_{\lambda}s}{c} + \theta_{\lambda} \right) \quad [s > L], \quad (12)$$

with k'_λ a kind of kick factor and θ_λ the phase of excitation of mode λ . Note that in the special case where the particles move parallel to the axis, at offset a , $k'_\lambda = k_\lambda = c|V_\lambda|^2/(4U_\lambda\omega_\lambda a^2 L)$, the normal kick factors for the structure, and $\theta_\lambda = 0$. For this case it can be shown that Eq. 12 is valid for all $s > 0$ [5]. Finally, note that, for the general case, Eq. 12 can obviously not be extrapolated down to $s = 0$, since it implies that $W_x(0) \neq 0$, which is nonphysical, since a point particle cannot kick itself transversely. To obtain the proper equation valid down to $s = 0$ we would need to include the scalar potential term that was dropped in going from Eq. 8 to Eq. 9.

To estimate the wakefield associated with very small, random cell-to-cell misalignments in accelerator structures we assume that we can use the mode eigenfrequencies and eigenvectors of the error-free structure. We obtain these from the circuit program. Then to find the kick factors we replace $x(z)$ in the first integral in Eq. 11 by the zig-zag path representing the negative of the cell misalignments, a path we generate using a random number generator. The normalization factor x_0 is set to the rms of the misalignments.

In Ref. [1] this method is used to estimate the wake at the bunch spacings in the S-band injector linacs of the JLC/NLC. How can we justify this? For example, for the $3\pi/4$ S-band structure, one possible bunch spacing is only 42 cm whereas the whole structure length $L = 4.46$ m. Therefore, in principle, Eq. 11 is not valid until the 11th bunch spacing. We believe, however, that the scalar potential fields will not extend more than one or two cells behind the driving charge (the cell length is 4.375 cm), and therefore this method will be a good approximation at all bunch positions behind the driving charge. This belief should be tested in the future by repeating the calculation, but now also including the contribution from scalar potential terms.

In Fig. 2 we give a numerical example. Shown, for the optimized $3\pi/4$ S-band structure for the injector linacs of the NLC (see Ref. [1]), are the kick factors and the phases of the modes as calculated by the method described here. Note that θ_λ is not necessarily small.

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Figure 2: The kick factors and phases of the modes for a cell-to-cell misalignment example. The structure is the optimized $3\pi/4$ S-band structure for the injector linacs of the NLC (see Ref. [1]).

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