Ion Effects in the PEP-II High Energy Ring¹

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Abstract

We summarize the expected ion effects in the PEP-II High Energy Ring, considering both a uniform vacuum pressure and a local pressure bump in the IR quadrupoles.

In this note we describe the ion effects predicted for the PEP-II High Energy Ring (HER). We consider two sets of beam and machine parameters. The first (case I) corresponds to a uniform pressure of 10 nTorr and design emittances. Parameters for this case are listed in Table 1. The second (case II) represents a local pressure bump of 2 μ Torr over a total length of 10 m near IR quadrupole QD4, and, in addition, a 20% emittance coupling. Table 2 lists the parameters for case II.

variable	symbol	value
circumference	C	2200 m
current/bunch	I_b	1 mA (0.5 mA)
no. particles per bunch	N_b	4.6×10^{10} (2.3×10^{10})
average pressure	p	10 nTorr
emittance coupling	κ	5% (50%)
bunch spacing	L_{sep}	variable (1.26 m minimum)
av. hor. beta function	β_x	25 m
av. vert. beta function	β_y	20 m
av. dispersion function	η_x	1 m
rms energy spread	δ_{rms}	6.1×10^{-4}
horizontal rms beam size	σ_x	1.25 mm
vertical rms beam size	σ_y	220 μ m (693 μ m)
no. of bunches	n_b	variable

Table 1: PEP-II HER parameters corresponding to a uniform vacuum pressure (case I).

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Ions are trapped by the beam if they are heavy enough, *i.e.*, if their mass A (in units of the proton mass) is larger than a critical mass [1]

$$A_c \equiv \frac{Q_i N_b r_p L_{sep}}{2\sigma_y(\sigma_x + \sigma_y)} \quad (1)$$

where Q_i is the charge of the ion (in units of the electron charge, in most cases $Q_i = 1$), r_p is the classical proton radius (1.5×10^{-18} m), and other quantities were defined in Table 1. Figures 1 (a)–(d) show the critical mass as a function of the number of bunches, assuming that the bunches are uniformly distributed around the ring. The four pictures correspond to different currents per bunch (1 mA and 0.5 mA), to two different emittance coupling factors ($\kappa \approx 5\%$ and 50%), and to two different beam sizes (case I and case II), respectively. The figure demonstrates that ions are easily trapped. Depending on current and vertical emittance, the trapping threshold for CO ions lies in the range 1–8 bunches, the threshold for H₂ between 10 and 120 bunches.

The ions are produced by collisional ionization as the beam circulates around the ring. The ionization cross section for CO molecules is about 2 Mbarn [2, 3]. This translates into an ion production rate of

$$\dot{\lambda}_{ion} [\text{m}^{-1} \text{ s}^{-1}] \approx 6N_b n_b f_{rev} p [\text{Torr}] \quad (2)$$

where λ_{ion} is the ion line density and the dot denotes the derivative with respect to time. This amounts to 2700 ions per meter produced by an 1-mA bunch on each revolution. Even if the ions are trapped from turn to turn, the ion accumulation does not continue forever. There are at least two mechanisms which limit the ion build-up and give rise to an equilibrium density:

- The ion density cannot exceed the average beam density (neutralization effect) [1]. Hence, $\lambda_{ion} \leq N_b n_b / C$. If this limit applies, the ion density increases in proportion to the beam current.
- The ion density is smaller than the residual-gas density [1]. Hence, $\lambda_{ion} \leq 2\pi \sigma_x \sigma_y p / (kT)$. This follows from the fact that the cross sections for first and multiple ionization are of similar size. The above formula also assumes that the multiply ionized molecules are not trapped.

For PEP-II the predicted equilibrium ion line density is plotted in Fig. 2, as a function of the number of bunches. Implicitly assumed is that the single-charge ions, and only those, are trapped by the beam. Note that the equilibrium density is about 3 orders of magnitude higher than the ion density reached after a single passage of 1000 bunches with 1 mA each.

If the ions are trapped in the beam, they give rise to additional focusing. The ion induced coherent tune shift is given by

$$\Delta Q_{x,y;\text{coh}} = \frac{\beta_{x,y} r_e Q_i \lambda_{ion} C}{\gamma 4\pi \sigma_{x,y} (\sigma_x + \sigma_y)} \quad (3)$$

The horizontal and vertical tune shifts are depicted in Fig. 3, again as a function of the total number of bunches. Tune shifts up to an integer are predicted. The incoherent tune shift would

Figure 1: Critical ions mass in units of the proton mass for a uniform bunch pattern, as a function of the number of bunches: (a) for 1 mA/bunch and $\epsilon_y/\epsilon_x = 5\%$, (b) for 0.5 mA/bunch and $\epsilon_y/\epsilon_x = 5\%$, (c) for 1 mA/bunch and $\epsilon_y/\epsilon_x = 50\%$, (d) for 1 mA/bunch, $\epsilon_y/\epsilon_x = 20\%$, and beam sizes in quadrupole QD4. Picture (d) corresponds to case II. The parameters for (a)–(c) are those of case I, except for those being varied.

variable	symbol	value
current/bunch	I_b	1 mA
no. particles per bunch	N_b	4.6×10^{10}
maximum pressure	p_{max}	2 μ Torr
effective length of pressure bump	l_{eff}	10 m
emittance coupling	κ	20%
bunch spacing	L_{sep}	variable (1.26 m minimum)
hor. beta function	β_x	100 m
vert. beta function	β_y	400 m
av. dispersion function	η_x	0 m
horizontal rms beam size	σ_x	2.2 mm
vertical rms beam size	σ_y	2.0 mm
no. of bunches	n_b	variable

Table 2: Parameters representing a localized pressure bump in quadrupole QD4 and 20% emittance coupling (case II).

Figure 2: Line density of trapped ions for a uniform bunch pattern, as a function of the number of bunches, for 1 mA/bunch, and 10 nTorr average pressure. Solid line: case I; dashed line: case II.

Figure 3: Horizontal and vertical coherent tune shifts for a uniform bunch pattern, as a function of the number of bunches, for 1 mA/bunch, and 10 nTorr average pressure. Solid lines: case I; dashed lines: case II.

Figure 4: Horizontal and vertical oscillation frequencies of two different ion species (H_2 and CO) for a uniform bunch pattern and average beam sizes (case I), as a function of the number of bunches. Other parameters are those listed in Table 1.

be two times larger than the coherent tune shift, whereas the transient tune shift between head and tail of a short 100-bunch train would be 2×10^{-4} only.

When the ions are trapped by the beam, they oscillate in the potential well. The ion oscillation frequency is [1]

$$f_{i; x, y} = \frac{c}{2\pi} \left(\frac{2Q_i N_b r_p}{AL_{sep} \sigma_{x, y} (\sigma_x + \sigma_y)} \right)^{1/2} \quad (4)$$

The horizontal and vertical ion frequencies for CO and H_2 ions are shown in Fig. 4. They vary between a few hundred kHz and several MHz.

Trapped ions can cause a coherent trapped ion instability, where the beam and ion cloud are oscillating with respect to each other. The growth rate for this instability has been estimated in the literature as [4, 5]

$$\frac{1}{\tau} \approx 2\pi^2 f_{rev} \frac{\Delta Q_{y; coh}}{(\Delta f_i / f_i)_{FWHM}} \approx 2\pi^2 f_{rev} \Delta Q_{y; coh} \quad (5)$$

where $(\Delta f_i / f_i)_{FWHM} \approx 1$ denotes the full-width half-maximum ion frequency spread. The predicted growth rates are exceedingly fast, with growth times of about $1 \mu s$. This is illustrated in Fig. 5.

Figure 5: Growth rate of conventional trapped ion instability for a uniform bunch pattern, as a function of the number of bunches. Solid line: case I; dashed line: case II.

Figure 6 shows the growth rates predicted for the transient fast beam-ion instability (FBII), where we now consider successive bunches in a train, separated by 1.26 m, and only include the effect of single-turn ions (no trapping from turn to turn). There are two solid curves, one corresponding to the quasi-exponential growth $y \sim \exp(\sqrt{t/\tau_c})$ predicted by a simple linear treatment [6], the other corresponding to an exponential evolution $y \sim \exp(t/\tau_e)$, which is expected when the effect of ion decoherence is taken into account [7, 8]. The analytical formula for the quasi-exponential time constant τ_c is [6]

$$\frac{1}{\tau_c} \equiv \frac{4d_{gas}\sigma_{ion}\beta_y N_b^{3/2} n_b^2 r_p^{1/2} L_{sep}^{1/2} c}{\sqrt{3} 3\gamma\sigma_y^{3/2}(\sigma_x + \sigma_y)^{3/2} A^{1/2}} \quad (6)$$

with $d_{gas} = p/(k_B T) \approx 1.9 \times 10^{13} \text{ m}^{-3}$ the residual gas density, $\sigma_{ion} \approx 2 \text{ Mbarn}$ the ionization cross section, and $A \approx 28$ the ion mass (in units of the proton mass). The exponential time constant τ_e is given by [8]

$$\frac{1}{\tau_e} \approx \frac{1}{\tau_c} \frac{c}{2\sqrt{2}l_{train}(\Delta\tilde{\omega}_i)_{\text{rms}}} \quad (7)$$

where l_{train} is the length of the train, and $(\Delta\tilde{\omega}_i)_{\text{rms}}$ denotes the rms variation of the coherent (angular) ion frequency,

$$\tilde{\omega}_i = c \sqrt{\frac{4N_b r_p}{3AL_{sep}\sigma_y(\sigma_x + \sigma_y)}} \quad (8)$$

as a function of the azimuthal position. We assume that $\Delta\tilde{\omega}_i/\tilde{\omega}_i$ is about 10%. After 200 bunches, the FBII growth times are about 25 ms^{-1} (τ_c) and 3 ms^{-1} (τ_e), respectively.

Chao and Stupakov [9] investigated the FBII equilibrium state in the presence of feedback and noise. They found that the rms orbit motion along the train can be characterized by the parameter $\bar{\eta}$,

$$\bar{\eta}(z) = z\sqrt{\tau_d/\tau_c}/(n_b L_{sep}), \quad (9)$$

where τ_d is the feedback damping time. In the following, we assume $\tau_d \approx 1 \text{ ms}$. The parameter $\bar{\eta}$ is plotted in the left picture of Fig. 7, as a function of the bunch number along the train. The rms vertical orbit motion in the two limits of small and large $\bar{\eta}$ is [9]:

$$\langle y^2(z) \rangle \approx \sqrt{\tau_d/\tau_c} \langle y^2(0) \rangle \frac{\bar{\eta}^3}{24n_b} \quad \text{for } \bar{\eta} \ll 1 \quad (10)$$

$$\langle y^2(z) \rangle \approx \sqrt{\tau_d/\tau_c} \langle y^2(0) \rangle e^{\bar{\eta}^2/2} \frac{1}{4\bar{\eta}n_b} \quad \text{for } \bar{\eta} \gg 1 \quad (11)$$

where $\langle y^2(0) \rangle^{1/2}$ denotes the rms oscillation amplitude of the first bunch in the train, driven by noise. The rms orbit motion amplification $(\langle y^2(z) \rangle / \langle y^2(0) \rangle)^{1/2}$ is shown in Fig. 7 right. After about 200 bunches, the calculated orbit motion increases steeply.

In the July-98 PEP-II, a strong vertical instability was observed under unusually poor vacuum conditions in the IR region. How would the above results change if the increased vacuum pressure is only in the interaction region? the parameters for case II were chosen to answer this question. They give rise to the largest conceivable beam sizes, for which the

Figure 6: FBII growth rates as a function of bunch number, for the design bunch spacing of 1.26 nm, an average CO pressure of 10 nTorr, and 1 mA/bunch current. Solid lines: case I; dashed lines: case II.

Figure 7: FBII equilibrium in the presence of feedback and noise, for a feedback damping time of 1 ms, an average CO pressure of 10 nTorr, 1.26 m bunch spacing, and 1 mA bunch current. Left: parameter $\bar{\eta}$ defined by Chao and Stupakov [9], right: rms orbit motion along the train, normalized to the motion of the first bunch. Solid lines: case I; dashed lines: case II.

vertical size is still smaller than the horizontal size. For even larger emittance coupling, the horizontal ion instability would be stronger than the vertical, contrary to observations. Case-II parameters were summarized earlier in Table 2. The average pressure of the two cases is the same (10 nTorr).

Figure 1 (d) shows that in this case almost all ionized molecules, except for hydrogen, would be trapped from turn to turn, even if there is only a single bunch or a short train. We may thus speculate that residual ions, left over from the previous turn, might re-excite the leading bunches in a short train on its next passage, similar to what has been observed [10]. Different from case I, in case II the average ion line density is limited only by neutralization. It thus always grows with the total number of bunches. For the maximum number of bunches, the ion line density averaged over the circumference is about three times smaller than for case I; see dashed line in Fig. 2. Also the coherent tune shifts are reduced by a factor 4–5 (compare solid and dashed lines in Fig. 3), and so is the growth rate of the conventional trapped ion instability, depicted in Fig. 5. While the characteristic rise time of the fast beam-ion instability, τ_c , is a factor 6 smaller than in case I, the FBII e-folding time τ_e remains almost the same. The FBII growth rates are illustrated by the dashed lines in Fig. 6. In Fig. 7 (a) we see a reduction of the Chao-Stupakov parameter $\bar{\eta}$ (dashed line) by a factor of 2, implying negligible orbit motion for trains up to 300–400 bunches. Finally, Fig. 8 displays the single-ion oscillation frequency inside quadrupole QD4. For CO ions and the design bunch spacing, it is about 1 MHz. The coherent oscillation frequency is about 20% lower. If the emittance coupling is 50% instead of

Figure 8: Horizontal and vertical oscillation frequencies of two different ion species (H_2 and CO) for a pressure bump in QD4 (case II), as a function of the number of bunches. In all cases a uniform bunch pattern is assumed. This figure should be compared with Fig. 4 for case I.

20%, and if we consider molecules with 30% higher mass, and a beta beat of about 10–20%, the ion frequency in QD4 could be smaller than 400 kHz, which is not too far from the measured instability frequency.

In conclusion, assuming average beam and machine parameters (case I), we find that ions in PEP-II are easily trapped, unless there is a significant gap in the train. CO ions are trapped for a uniform fill of only 8 bunches. The predicted instability growth rates are very large, up to $1 \mu\text{s}^{-1}$ for trapped-ion instabilities, and about 3–25 ms^{-1} in case of the fast beam-ion instability. For the predicted equilibrium density of trapped ions the tune shift can be as large as 1 unit. The ion frequencies are in the range 100 kHz to a few MHz, depending on ion species, plane of oscillation (horizontal or vertical), the bunch current, and the vertical emittance. The rms orbit motion along a train is expected to increase strongly after about 200 bunches, due to the fast beam-ion instability. In the IR region, ions are trapped even more easily. Inside quadrupole QD4 (case II), already a single bunch can trap CO ions from turn to turn. Such multi-turn ions could excite the leading bunches of a train on the next turn. Expected tune shifts and conventional growth rates are roughly a factor 5 smaller than for case I, while the exponential FBII rise time is almost the same. The Chao-Stupakov theory would predict that under these conditions a train of 300 consecutive bunches is still stable, but this theory does not

fully include the effect of residual multi-turn trapping. The estimated coherent ion oscillation frequencies in QD4 are within a factor 3–4 equal to the instability frequency observed in July 1998 (around 200 kHz). The remaining discrepancy might be explained by a lower current or by larger emittances than assumed here, or it could reflect the nonlinear character of the beam-ion force.

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