A Review of $B^0$ Mixing and Lifetimes
Intl. Workshop on Weak Interaction and Neutrinos

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January 25, 2002
Outline

• Mixing of neutral flavor states
• Measuring the mixing
• Mixing in the Standard Model
• Measuring the lifetimes
• What lies ahead

Many summary plots and combined fits can be found at:
B-Oscillations Working Group: http://lepbosc.web.cern.ch/LEPBOSC/
Last updated for 2001 summer conferences
Where the data come from

- **\( Y_{4s} \) Symmetric e^+e^- Colliders:**
  - Argus, CLEO
    - \( B_d \) only
    - produced \textasciitilde \text{at rest}
    - \( 9 \times 10^6 \) events

- **\( Z^0 \) Pole e^+e^- Colliders:**
  - LEP expts., SLD
    - \( B_d \) and \( B_s \)
    - boost
    - \( \sim 1 \times 10^6 \) pairs/LEP expt.
    - \( \sim 1 \times 10^5 \) SLD + pol. beam

- **Hadron Colliders:**
  - CDF, D0
    - \( B_d \) and \( B_s \)
    - boost
    - several million events
    - trigger issues
    - Run II in progress

- **\( Y_{4s} \) Asymmetric e^+e^- Colliders:**
  - BABAR, BELLE
    - \( B_d \) only
    - boost
    - \( 3 \times 10^7 \) pairs and counting
Mixing of Neutral Flavor States

- Neutral flavor states (K, D, B mesons) are produced in strong interaction flavor eigenstates $B^0, \bar{B}^0$.
- They decay in eigenstates of definite mass and lifetime

$$|B^0_\pm(t)\rangle = e^{-\Gamma t/2} e^{-i m t} |B^0_\pm\rangle$$

If CP is conserved: $\text{CP} |B^0_\pm\rangle = \pm |B^0_\pm\rangle$

$$|B^0_\pm\rangle = \frac{1}{\sqrt{2}} \left( |B^0\rangle \mp |\bar{B}^0\rangle \right)$$

$$|B^0\rangle = \frac{1}{\sqrt{2}} \left( |B^+_0\rangle + |B^-_0\rangle \right)$$

$$\left| \langle B^0(t) | B^0 \rangle \right|^2 = \frac{1}{4} \left( e^{-\Gamma^+_t} + e^{-\Gamma^- t} - 2 \cos(\Delta m t) \right);$$

$$= e^{-\tilde{\Gamma} t} \left( \cosh\left( \frac{\Delta \Gamma}{2} t \right) - \cos(\Delta m t) \right)$$
Time-integrated B Mixing Measurements

\[ \chi^{B^0 \rightarrow B^0} = \chi^{ \bar{B}^0 \rightarrow B^0} \]

\[ \chi = \frac{1}{\Delta m \tau} \left( \frac{\Delta \Gamma}{2} \right) \left( \frac{\Delta m \tau}{2} \right)^2 \]

Most recent measurement of this type:

Uses 9.6 x 10^6 events (9.1 fb^{-1})

One B is flavor tagged by partially reconstructing:
The other B is tagged by high-p_T lepton in:

\[ \bar{B}^0 \rightarrow D^{*+} \pi^- \text{ or } D^{*+} \rho^- \]

\[ \chi_d = \frac{B^0 B^0 + \bar{B}^0 \bar{B}^0}{B^0 B^0 + B^0 B^0 + B^0 \bar{B}^0 + B^0 \bar{B}^0} = 0.198 \pm 0.013 \pm 0.014 \]
Measuring the B Mixing Oscillations

\[ \frac{\Delta \Gamma}{\Delta m} \approx O \left( \frac{m_b^2}{m_t^2} \right) \sim 5 \times 10^{-3}; \text{ for } B_d, \frac{\Delta \Gamma}{\Gamma} < 1\% \]

\[ e^{-\Gamma t} \left( 1 \pm \cos(\Delta m t) \right) \]

- Need to flavor tag both initial and final state
- Need to reconstruct both primary and decay vertices to measure decay length: \( t = mL/p = L/\beta \gamma \)
- beam spot, vertexing from tracks, intrinsic detector resolution

\[ \sigma_t = \left( \frac{\sigma_t m}{p} \right) \pm t \left( \frac{\sigma_p}{p} \right) \]

decaying B momentum or good estimator needed (boost)

\( \text{BaBar: } \delta(\beta \gamma)/\beta \gamma = 0.1\% \)
Tagging Methods

• **Initial State:**
  - can tag same side or opposite side
  - same side: jet charge and topology
  - opposite side: techniques similar to final state tags

• **Final State:**
  - $b \rightarrow c \rightarrow s$ decays
    - charge of primary lepton in sl decay
    - charge of charm meson (fully or partially reconstructed)
    - charge of kaon
  - momentum-weighted jet charge
  - dipole charge between D and B
  - jet angles (polarized beams)
  - multivariate constraints (perhaps realized in a neural net)
  - full reconstruction

DELPHI 9 Discr. variables:
5 opposite side; 3 same side uncorrelated with final state tag; 1 both sides: polar angle

SLD sign(QD-QB)*separation
Improvement in Detection

Statistical Significance

\[ S \approx \sqrt{\frac{N}{2}} f(1-2w) e^{-\frac{1}{2}(\Delta m \sigma_t)^2} \]

\( \sigma_t \) more important for large \( \Delta m \)

Likelihood fit to:

\[ \Gamma e^{-\Gamma t} \left( 1 \pm (1-2w) \cos(\Delta m t) \right) \otimes \text{res} \]

w is mistag fraction; (1-2w) = (1-2w_i)(1-2w_f)


ALEPH

\[ C_3 = \frac{N^{+/-} - N^+}{N^{+/-} + N^+} \]
What’s different about the Asymmetric B-factories

Coherent Production with boost:
finding flavor of one B fixes the flavor of the second B at the same time

Either the tagging B or the reconstructed B can decay first, $\Delta z$ (or $\Delta t$, proper time) can be $< 0$

No sharp edge due to finite resolution

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Update to July 2001 WG Summary

New Results:

1. BABAR (submitted for publication):
   - Inclusive Dilepton Events (20.7 fb⁻¹)
   - Fully reconstructed hadronic events (29.7 fb⁻¹)
   Both update preliminary results in WG summary

2. BELLE (3 preliminary results) in Nov based on 29.1 fb⁻¹:
   - Fully reconstructed hadronic events
   - Partial D*π reconstruction
   - Partial D*lv reconstruction
Asymmetric B-factories Analyses

Full Reconstruction Analyses: Hadronic Decay Modes

\[ D^{(*)-}\pi^+; D^{*-}\rho^+ \quad D^{(*)-}\pi^+; D^{(*)-}\rho^+; D^{(*)-}a_1^+; J/\psi K^{*0} \]

Simultaneous Maximum Likelihood Fit to mixed and unmixed events

\[ \Gamma e^{-\Gamma \Delta t} \left( 1 \pm (1 - 2w) \cos(\Delta m \Delta t) \right) \otimes res \]

Output is \( \Delta m \); \( w \) (mistag rates), and resolution parameters for signal and background (44 total for BABAR); \( \tau_{Bd} \) fixed at PDG value (1.548 ps)

Other analyses: dilepton events

Belle: \( D^*\ell \nu \)

\( D^{*+}\pi \) partial reconstruction (high and low \( p \) \( \pi \)'s)

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**Δm_d New Results**

**Hadronic**

![Graph showing Hadronic results with asymmetry values](image)

- Asymmetry / 0.24 ps
- Δt (ps)
- 0.516 ± 0.016 ± 0.010

**Dileptons**

![Graph showing Dileptons results with asymmetry values](image)

- Asymmetry / 0.24 ps
- Δt (ps)
- 0.493 ± 0.012 ± 0.009

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In the Standard Model, the transformation takes place via box diagrams and gives us information on $V_{td}$ ($B_d$ mesons) and $V_{ts}$ ($B_s$ mesons).

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_{B_d} m_W^2 S \left( \frac{m_t^2}{m_W^2} \right) \times |V_{tb}^* V_{td}|^2 \ f_{B_d}^2 \ B_{B_d} \ \eta_B$$

**Bag parameter**

**QCD correction**

**Decay constant**
Standard Model and Mixing

• The flavor eigenstates of the quarks are not the weak interaction eigenstates--there are transitions between the families.

\[
\begin{pmatrix}
  u \\
  d \\
  c \\
  s \\
  t \\
  b
\end{pmatrix}
\]

• This mixing is described by the CKM Matrix. Where the three diagonal elements \( V_{ud}, V_{cs}, \) and \( V_{tb} \approx 1 \), family is almost a good quantum number.

\[
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

• The CKM matrix is unitarity which can be expressed as:

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0
\]
Constraining the Unitarity Triangle

- Lattice QCD calculations: uncertainty in $B_{Bd} \otimes f_{Bd} \sim 20\%$
  $\Rightarrow$ limits extraction of $V_{td}$ from $\Delta m_d$

- Ratio $\Delta m_s/\Delta m_d$ gives us a better limit

\[
\Delta m_d \propto |V_{td}|^2 = A^2 \lambda^6 \left( (1 - \rho)^2 + \eta^2 \right); \quad \Delta m_s \propto |V_{ts}|^2 = A^2 \lambda^4
\]

Lattice QCD: “known” to $\sim 5\%$
**B^0_s Mixing**

- Oscillations not yet observed
- Express results as Amplitude $\mathcal{A}$ of oscillations as a function of $\Delta m_s$ by a likelihood fit for fixed value of $\Delta m_s$ to

$$\Gamma e^{-\Gamma t} (1 \pm \mathcal{A} \cos \Delta m_s t)$$

Bands show 95% C.L. Limits

**LEP/SLD and CDF provide data**

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New Results from SLD

Same initial state tag: Pol. $A_{FB}$ or NN multivariate using opposite side var.

Three different final state tags – hierarchical to remove overlaps:
Lepton + topological $D$  \[ B_s \rightarrow D_sX; D_s \text{ reconstructed} \]  Dipole Charge

Dipole $D_s \rightarrow \phi \pi$ or $K^*K$

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New $\Delta m_s$ Limits

$B^0_s \rightarrow D_s^- \pi^+, D_s^- a_1^+, D^0 K^- \pi^+, D^0 K^- a_1^+$

Complete $D$ reconstruction
also exclusive $D_s$ and high-$p_T$ hadron

Current Status of $B_s$ Mixing

Graph showing the world average (preliminary) $\Delta m_s (ps^{-1})$ vs Amplitude. The graph includes data points, 95% CL limit, sensitivity, and predicted regions in SM.

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Lifetime Measurements

\[ \left| \langle \bar{B}^0(t) | B^0 \rangle \right|^2 = e^{-\bar{\Gamma}t} \frac{1}{2} \left( 1 - \cos(\Delta m t) \right) \]

\[ \left| \langle B^0(t) | \bar{B}^0 \rangle \right|^2 = e^{-\Gamma t} \frac{1}{2} \left( 1 + \cos(\Delta m t) \right) \]

- Tag event as B
- Measure B momentum
- Measure Decay length

Exclusive Event Tagging
- \( B \rightarrow D^* l \nu (X) \); reconstruct D*
- \( B \rightarrow D X \)
- \( B \rightarrow J/\psi K \)

Inclusive

Topology:
- Displaced vertex; event shape

Full reconstruction (new)
Working Group Summary

Belle (new) 1.554 ± 0.030 ± 0.019 ps

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New Results on Lifetime

110 pb⁻¹ J/ψK

2.1 M Z → hadrons topology tag

1.497 ± 0.073 ± 0.033 ps
1.546 ± 0.018 ± 0.035 ps
1.546 ± 0.032 ± 0.022 ps
1.554 ± 0.030 ± 0.019 ps

20.0 fb⁻¹

FULLY RECONSTRUCTED HADRONS

1.636 ± 0.058 ± 0.025 ps
1.631 ± 0.012 ± 0.021 ps
1.673 ± 0.032 ± 0.023 ps
1.695 ± 0.026 ± 0.015 ps

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**B_s Lifetime**

**DELPHI: D_s \rightarrow \phi/h**

![Graph](image)

**ALEPH D_s 1**
91-95
1.54 ± 0.14 ± 0.04 ps

**ALEPH D_s h**
91-95
1.47 ± 0.14 ± 0.08 ps

**CDF D_s 1**
92-96
1.36 ± 0.09 ± 0.05 ps

**DELPHI D_s 1**
91-95 Prel.
1.42 ± 0.14 ± 0.03 ps

**DELPHI D_s h**
91-95 Prel.
1.49 ± 0.16 ± 0.07 ± 0.15 ± 0.08 ps

**DELPHI inclusive D_s**
91-94
1.60 ± 0.26 ± 0.12 ± 0.15 ps

**OPAL inclusive D_s**
90-95
1.72 ± 0.20 ± 0.18 ± 0.19 ± 0.17 ps

**OPAL D_s 1**
90-95
1.50 ± 0.16 ± 0.04 ps

**Average of above 8**
1.471 ± 0.059 ps

**CDF \psi / \phi**
92-95
1.34 ± 0.23 ± 0.05 ps

**World average**
1.464 ± 0.057 ps
Lifetime Ratios

Spectator process

\[ \Gamma \approx \frac{G_F^2 m^5}{129 \pi^3} \left( A_1 + \frac{A_2}{m^2} + \frac{A_3}{m^3} + O \left( \frac{1}{m^4} \right) \right) \]

baryon/ meson

meson/meson

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What lies ahead?

- **BABAR and Belle:**
  - 30 fb\(^{-1}\) → 300 fb\(^{-1}\) 2004
  - Go beyond Δm: ΔΓ
  - CP effects: ε (|q/p|)

- **CDF/D0 100 pb\(^{-1}\) → 2 fb\(^{-1}\) 2004
  - probe B\(_s\) mixing to x\(_s\)≈30 with semileptonic decays (x\(_s\)=Δm\(_s\)/Γ\(_s\))
  - probe B\(_s\) mixing to x\(_s\)≈60 in hadronic mode
  - Precision lifetime ratios (B\(_s\)/B\(^0\))
CDF Expectations

Figure 1: Left: Required luminosity to measure at 5σ $x_s$ as function of $x_s$. Right: Error on $x_s$ as function of $x_s$.

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And much later ……

LHC start in 2006?

<table>
<thead>
<tr>
<th>Channels used:</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
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<tr>
<td>$B_0^0$ decay channels</td>
<td>$D_s^0 \pi^+$</td>
<td>$D_s^- \pi^+$</td>
<td>$D_s^- \pi^+$</td>
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<td>$D_s^-$ decay channels</td>
<td>$\phi \pi^-$</td>
<td>$\phi \pi^-$</td>
<td>(see text)</td>
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<tr>
<td>$\phi$ decay channel</td>
<td>$K^+ K^-$</td>
<td>$K^+ K^-$</td>
<td>$K^+ K^-$</td>
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<tr>
<td>$\psi_1^0$ decay channel</td>
<td>$\rho^0 \pi^+$</td>
<td>$\rho^0 \pi^+$</td>
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<tr>
<td>$K^{*0}$ decay channel</td>
<td>$K^+ \pi^-$</td>
<td>$K^+ \pi^-$</td>
<td>$K^+ \pi^-$</td>
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<table>
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<tr>
<th>Assumptions:</th>
<th></th>
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<tr>
<td>$B(b \rightarrow B_0^0)$</td>
<td>0.105</td>
</tr>
<tr>
<td>$B(B_0^0 \rightarrow D_s^- \pi^+)$</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$B(B_0^0 \rightarrow D_s^- a_1^0)$</td>
<td>$6.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$B(D_s^- \rightarrow \phi \pi^-(K^+ K^-))$</td>
<td>0.036</td>
</tr>
<tr>
<td>$B(D_s^- \rightarrow \pi^-(K^+ K^-))$</td>
<td>–</td>
</tr>
<tr>
<td>$B(B_s^0$ lifetime</td>
<td>1.54 ps</td>
</tr>
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</table>

Analysis performance:

- Reconstructed signal events per year: 3457 | 4500 | 86000
- Rec. and tagged signal events per year: 3457 | 4500 | 34500
- $B_s^0$ purity of tagged sample: 0.38 | 0.5 | 0.95
- Wrong tag probability: 0.22 | 0.22 | 0.30
- Proper time resolution (Gaussian function(s)): 50 fs (60.5%) | 65 fs | 43 fs

$\Delta M_s$ reach after one year of running:

- Measurable values of $\Delta M_s$ up to 30 ps$^{-1}$ | 26 ps$^{-1}$ | 48 ps$^{-1}$
- $95\%$ CL excl. of $\Delta M_s$ values up to – | 29 ps$^{-1}$ | 58 ps$^{-1}$
- $\sigma(\Delta m_s)$ for $\Delta m_s = 20$ ps$^{-1}$ 0.11 | – | 0.011

$x_s$ reach after one year of running:

- Measurable values of $x_s$ up to 46 | 42 | 75
- $95\%$ CL excl. of $x_s$ values up to – | 47 | 91

Table 22: Summary of the analyses and results for $B_0^0$ oscillation frequency measurements by the LHC experiment.

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