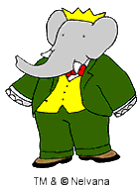


BaBar RooFit Workshop

Integration and Convolution

Jake Anderson
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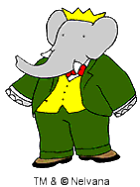


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Outline

- **Integration**
 - When does RooFit integrate?
 - What methods does RooFit use?
 - Numeric integration
 - Analytic integration
 - Projection with data
- **Convolution**
 - How does the analytic convolution work?
 - Numeric convolution



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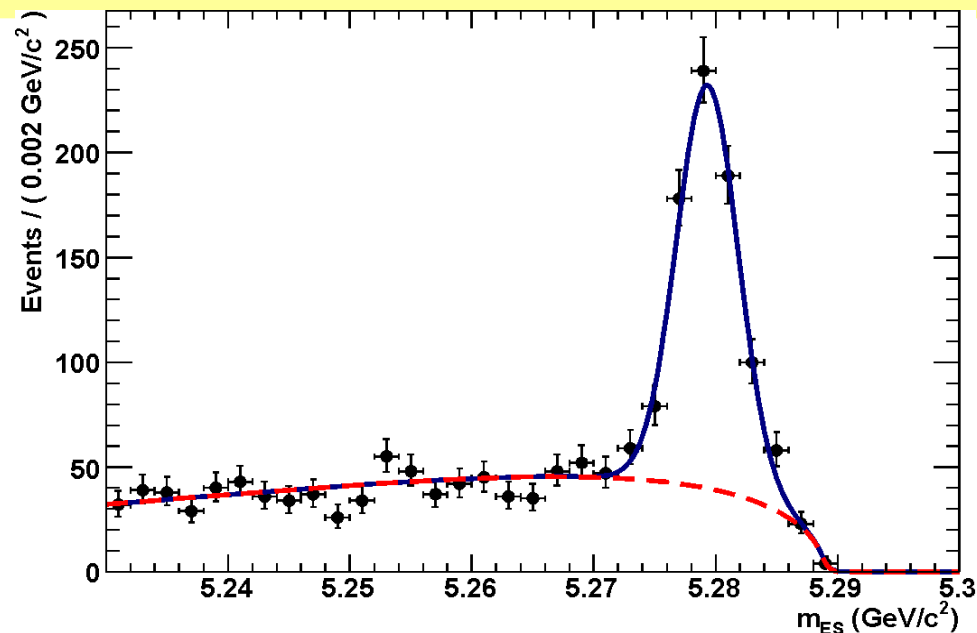
When RooFit integrates

- Plotting

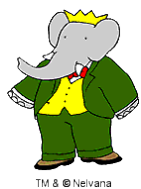
- Automatic normalization of PDF to data
- Projecting out unplotted observables.

- Fitting

- Generating

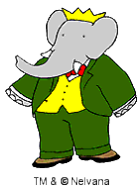


- RooFit will integrate many times.
- How can it be done most effectively?



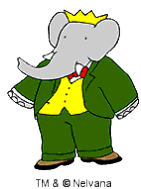
RooFit Integration Methods

- **Analytic integration**
 - Most efficient
 - Requires implementation in code.
- **Numeric integration**
 - Generally works in 1D and low dimensions.
 - Can be CPU intensive.
 - May not converge.
- **Projection with data**
 - When plotting you can avoid integration by using data.
 - Can be expensive depending on size of data.



Implementing Analytic Integrals

- Your PDF may have a well defined definite integral or you may know an efficient approximation.
- You can tell RooFit how to use that method.
- This can really speed up the time it takes to do most non-trivial RooFit operations.
- It requires you to write a little code, but it isn't hard.



Advertise to RooFit Your Integral

- In the class interface for the PDF you need to tell RooFit you know integrals.

```
class RooDstarDstarAngPdf : public RooAbsPdf {  
public:
```

```
...
```

```
Int_t getAnalyticalIntegral(RooArgSet& allVars,  
    RooArgSet& analVars,  
    const char* rangeName=0) const;
```

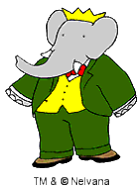
```
Double_t analyticalIntegral(Int_t code,  
    const char* rangeName = 0) const;
```

```
...
```

```
}
```

What variables you can integrate

Calculations for the integral



Specify the Analytic Variables

- In the implementation of your PDF you specify the variables you can integrate using the `MatchArgs()` method.

```
Int_t RooDstarDstarAngPdf::getAnalyticalIntegral(RooArgSet& allVars,  
                                                  RooArgSet& analVars,  
                                                  const char* /*rNm*/) const {  
    if (matchArgs(allVars, analVars, x,z,phi_tr)) return 7;  
    if (matchArgs(allVars, analVars, x,z)) return 6;  
    if (matchArgs(allVars, analVars, x,phi_tr)) return 5;  
    if (matchArgs(allVars, analVars, z,phi_tr)) return 4;  
    if (matchArgs(allVars, analVars, x)) return 3;  
    if (matchArgs(allVars, analVars, z)) return 2;  
    if (matchArgs(allVars, analVars, phi_tr)) return 1;  
    return 0;  
}
```

Start with the most variables you can do.

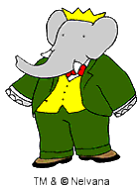


Implementing the Integral Calculations

- Add the code that does the actual integration for each advertised integral.

```
Double_t RooDstarDstarAngPdf::analyticalIntegral(Int_t code,
  const char* rangeName) const {
  If (code == 1) {
    //calculation of integral for variables from code 1
    ...
    return valueOfIntegral;
  } else ...
}
```

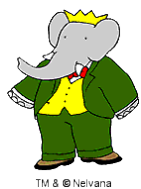
- This is where you show that you are smarter than the computer.



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Numeric Integration

- “Integrals of functions of several variables, over regions with dimension greater than one, are *not easy*”[1]
- RooFit has a couple techniques for numeric integration
 - Midpoint and trapeziod quadrature (1D,2D)
 - Gauss-Kronrod quadrature (and adaptive) (1D)
 - Segmented integrators (1D, 2D)
 - MC integration (ND)



Numeric Integration

- Parameters to control the numeric integration methods are in the class `RooNumIntConfig`.

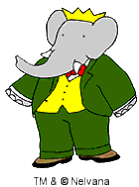
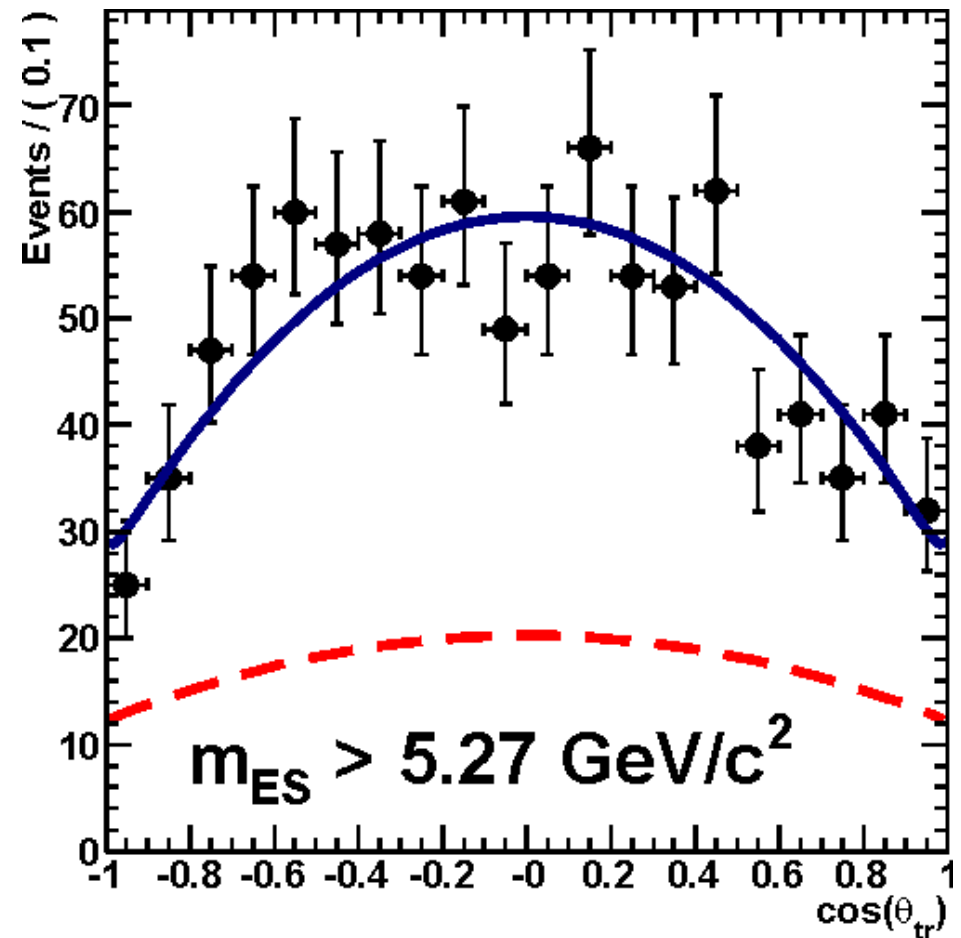
```
RooNumIntConfig& RooNumIntConfig::defaultConfig();
```

- From here you can change the precision of convergence and tweak integration parameters to hopefully better suit your PDF.
- “The fact that integrals of elementary functions could not, in general, be computed analytically, while derivatives *could* be, served to give the field a certain panache, and to set it a cut above the arithmetic drudgery of numerical analysis...” [1]



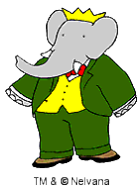
Projecting with Data

- When plotting you can skirt the whole integration of unplotted observables by projecting using a data sample.
 - For plotting only a signal box
 - Conditional observables
 - Discreet observables



Projecting with Data

- Projecting with data can use either a binned or an unbinned dataset.
 - The binned datasets can be much faster if the number of bins that have to be projected over is fewer than the number of data points.
 - You can also create toy data for the purpose of projecting a PDF.
 - This is similar to MC integration but uses only the data you provide.



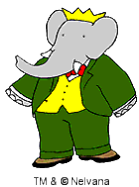
Convolution

- In an ideal experiment quantities are measured precisely.
- In a real experiment there is finite measurement resolution, meaning that a distribution is a convolution of the physical distribution and this resolution.
- RooFit provides methods to convolve physics PDF's with resolution models to accurately model measured distributions.

– Analytic

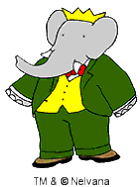
$$f(x') \otimes R(x-x') \equiv \int f(x') R(x-x') dx'$$

– Numeric



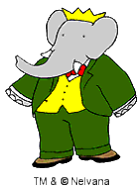
Analytic Convolution

- In RooFit analytic convolution is accomplished through physics PDF's that inherit from RooAbsAnaConvPdf and resolution models that inherit from RooResolutionModel.
- The physics PDF is expressed as a sum of basis functions that the resolution model knows how to do the convolution integral for.
- An important resolution model is the RooTruthModel, which is $\delta(x-x')$.



Convolutions

- Convolution can be computationally expensive even when implemented analytically
 - Generation takes longer.
 - Fitting takes longer.
- Numeric Convolution is also computationally expensive.
 - RooNumConvPdf can convolve any 2 PDF's.
 - “This class should not be used blindly as numeric convolution is computing intensive and prone to stability fitting problems.” — RooNumConvPdf.cc



Conclusion

- Integration is a hard problem, but RooFit makes it easy to add analytic integrals to a PDF to speed up common tasks associated with an analysis.
- The numeric integration techniques of RooFit can be customized to suit your problem.
- Convolution is a powerful tool for unfolding the physics of a distribution from the measurement resolution.
- RooFit brings these techniques together in an easily extended toolkit.

