Decay constants, $\xi$ and $B - \bar{B}$ mixing

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HPQCD and UKQCD collaborations
Decay constants $f_X$

- Related to purely leptonic decay rate

- Formula contains $V_{xy}$

$$\Gamma(X \rightarrow l\nu) = \frac{G_F^2 f_X^2 m_l^2 M_X (1 - \frac{m_l^2}{M_X^2})}{8\pi} |V_{xy}|^2$$

2 possibilities:

EITHER: measure decay rate, and use a given $V_{xy}$ to get $f_X$ and compare to lattice QCD

OR: measure decay rate and use lattice QCD value for $f_X$ to get $V_{xy}$.
D decay constants

Use as test of lattice QCD, taking $V_{cs/d}$ from elsewhere

$V_{cs} = 0.9737(?)$

Unitarity, PDG2004

$V_{cd} = V_{us} = 0.2238(29)$

$\sim 10\%$ errors

theory and expt

FNAL/MILC/HPQCD, hep-lat/0506030; CLEO, hep-ex/0508057, 0607074; BaBar, hep-ex/0607074
Using lattice + expt, get \( V_{cs} = 1.09(7)(5) \)

Ratios of decay constants

Lattice errors much reduced
- few percent is possible

Accurate \( \frac{V_{cs}}{V_{cd}} \)
possible if expt error also reduced

Simone, FNAL/MILC, LAT06 Update
Lattice calc for D/D_s done with following ingredients:

- use MILC unquenched gluons, inc. effect of u, d, s sea quarks with many values of m_u/d down to 0.1 m_s and 2 values of lattice spacing, 0.12 fm and 0.09 fm.

- c valence quarks use ‘FNAL formalism’. Main systematic error in results comes from discretisation errors \( O((m_c a)^2) \), \( O(\alpha_s m_c a) \) with m_c a \( \sim 0.4-0.7 \)

- Most of these errors cancel in ratio. To improve individual results need a better action.

- NEW Highly Improved Staggered Quark action for c quarks. Errors now \( O(\alpha_s (m_c a)^2) \) \( O((m_c a)^4) \)

Few % errors now possible for c physics!
Tests of HISQ formalism

Can tune $m_c$ very accurately from $m_{\eta_c}$ and get right answer for $m_{D_s}/D$

Preliminary results for $D_s/D$ decay constants

Follana et al, HPQCD/UKQCD, LAT06
B decay constants - how well can we do from lattice QCD?

Very accurate NRQCD for b quarks on MILC configs - tested on Upsilon physics

ICHEP06, BELLE: $Br(B \rightarrow \tau\nu)$ using HFAG $V_{ub}$

$\frac{f_B}{f_B} = 216(22)\text{MeV}$

$\frac{f_{B_s}}{f_B} = 1.20(3)$

Exptl error limiting factor on usefulness of this. $B_s$ has no leptonic decay, of course!

Ratio already 3% accurate - can we do better?

log dependence on light quark mass

$\frac{\phi(B)}{\phi(B)}$
Try double ratio to $f_K / f_\pi$

MILC results - $f_K / f_\pi = 1.208(2) \left( ^{+7}_{-14} \right)$

yield $V_{us} = 0.2223(26)$

Competitive with PDG from SL decay

Sugar, MILC, LAT06

$\frac{f_{B_s}}{f_B} \times \frac{f_\pi}{f_K} = 1.019(11)$

$\frac{f_{B_s}}{f_B}$ Total error 2%

Becirevic et al, hep-ph/0211271
Why were we doing this?

B/B_s oscillation rate determined by box diagram. Calculate in lattice QCD as 4-q operator.

\[
\begin{align*}
B^0 & \quad B^0 = \quad H_W \\
V_{td}V_{tb} & \\
\end{align*}
\]

Parameterise with \( f_B^2 B_B \) where \( f_B \) is decay constant.

\[
\Delta M_x = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tx}^*V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_x} f_{B_x}^2 \hat{B}_{B_x}
\]

Ratio really required is \( \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_B \sqrt{B_B}} \)
\[ \left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} \]

From HPQCD \( f_{B_s}/f_B \) and JLQCD 2-flavor \( B_{B_s}/B_B \)
\[ \xi = 1.21(5) \quad \text{OKAMOTO, LAT05} \]

+CDF measurement of \( \Delta M_s \)
\[ \left| \frac{V_{td}}{V_{ts}} \right| = 0.208(8) \]

New \( f_{B_s}/f_B \) gives \( \xi = 1.25(5) \)

Prospects for \( \xi \): Calcs underway on MILC configs

Expect errors of 2-3% in 2007
2006: first full lattice QCD result for

\[ f_{B_s} \sqrt{\hat{B}_{B_s}} = 281(21)\text{MeV} \quad \text{Shigemitsu, HPQCD, LAT06} \]

gives \[ \Delta M_{B_s} = 20(3)\text{ps}^{-1} \quad \text{using unitarity} \quad |V_{ts}V_{tb}| \]

cf CDF \[ \Delta M_{B_s} = 17.3(3)\text{ps}^{-1} \]

or, use expt to extract CKM elements:

\[ |V_{ts}^*V_{tb}| = 3.8(3)(1) \times 10^{-2} \]
CONCLUSIONS

• Decay constants have 10% systematic errors from lattice QCD. This will improve with better actions/matching.

• Ratios of decay constants are much more accurate. 2% errors possible.

• Expect 2-3% error in $\xi$ next year

• To EXPT: be clear about what $V_{xy}$ you are using if extracting decay constants from leptonic rates