Study of $e^+e^-$ collisions with a hard initial state photon at BaBar

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Initial state radiation (ISR) method

\[
\frac{d\sigma(s,x)}{dx d(cos\theta)} = W(s,x,\theta) \cdot \sigma_0(s(1-x)),
\]

\[
W(s,x,\theta) = \frac{\alpha}{\pi x} \left( \frac{2 - 2x + x^2}{\sin^2 \theta} - \frac{x^2}{2} \right), \quad x = \frac{E_\gamma}{\sqrt{s}}
\]

- High PEP-II luminosity at \(\sqrt{s} = 10.58\) GeV → precise measurement of the e+e- cross section \(\sigma_0\) at low c.m. energies with BaBar.
- Improved hadron spectroscopy
- Input to \((g_\mu - 2)\) and \(\alpha_{em}\) calculations.
- Few previous data in the 1.4-3.0 GeV range.
- Comprehensive program at BaBar.
- Today: preliminary results for \(\pi^+\pi^-\pi^0\), \(2\pi^+2\pi^-\), \(K^+K^-\pi^+\pi^-\), \(2K^+2K^-\) from 89.3 fb⁻¹
$e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma$

**Event selection:**
- Isolated ISR photon with $E_{CM} > 3$ GeV
- At least 2 good photons with $E_\gamma > 0.1$ GeV
- Two good, non-K tracks from IP

**Kinematic fit:**
- Energy and momentum balance enforced
- Mass of two soft photons constrained to $\pi^0$
- $\chi^2 < 40$ for fit in $\pi^+\pi^-\pi^0\gamma$ hypothesis selects signal events
- Reject events with extra photons if $\chi^2 < 40$ for $\pi^+\pi^-\pi^0\pi^0\gamma$ hypothesis
Background for $3\pi\gamma$

Most dangerous backgrounds:
\[ e^+ e^- \rightarrow K^+ K^- \pi^0 \gamma, \ e^+ e^- \rightarrow q\bar{q} \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \]

Other backgrounds:
\[ e^+ e^- \rightarrow 2\pi \gamma, \ 4\pi \gamma, \ 5\pi \gamma, \ ..., \ \tau^+ \tau^- , \ \tau^\pm \rightarrow \pi^\pm \pi^0 \nu \]

Two methods of background subtraction:
1. background mass distribution measured in data, subtracted bin-by-bin from signal mass distribution
2. taken from simulation, corrected to real experimental distribution

Total background level:
- (0.5 - 1.5)% in $\omega, \ \phi \ \text{regions}$
- (15 - 50)% at higher masses
- accuracy in background level
  ~25% up to 2 GeV
Detection efficiency for $3\pi\gamma$

The detection efficiency $\varepsilon(m)$:

- determined from a Monte Carlo simulation that includes additional corrections extracted from special control event-samples
- quite uniform in $0.5 < m < 3$ GeV/c$^2$
- systematic error currently $\sim 4\%$ - will be improved with more data
Fit of the $\pi^+\pi^-\pi^0$ mass spectrum

\[
\left( \frac{dN}{dm} \right)_{th} = \sigma_{3\pi} (m) \frac{dL}{dm} \cdot \varepsilon (m) \cdot R(m),
\]

\[
\left( \frac{dN}{dm} \right)_{exp} = \left( \frac{dN}{dm'} \right)_{th} \otimes f(m,m')
\]

- $\sigma_{3\pi} (m)$ - Born cross section of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ is a coherent sum of 4 resonances: $\omega, \phi, \omega', \omega''$
- $R(m)$ - radiative correction function from calculation
- $dL/dm$ - ISR luminosity taken from total integrated luminosity and photon radiator function $W(s,x)$; (checked with $\mu\mu\gamma$ events)
- $f(m,m')$ - detector resolution taken from simulation (with floating extra Gaussian smearing)
- Fix $\omega, \phi$ widths to PDG values
- Fix $\omega-\phi$ relative phase to experimental value $(163\pm7)^\circ$
- Fix $\omega-\omega'$ relative phase to $180^\circ$, $\omega - \omega''$ to $0^\circ$
Fit results: $\omega - \phi$ region

$\chi^2$/d.f. = 146/148

consistent with known properties of these resonances ($\omega, \phi$ widths fixed to PDG values)

The resolution is about 6, 7, 9 MeV/c$^2$ at $\omega, \phi, J/\psi$ masses.

Fitted resolution smearing is $\sim1$ MeV/c$^2$

<table>
<thead>
<tr>
<th>BaBar Preliminary</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(\omega\to e\bar{e})B(\omega\to 3\pi) = (6.70 \pm 0.06 \pm 0.27) \times 10^{-5}$</td>
<td>$(6.35 \pm 0.11) \times 10^{-5}$</td>
</tr>
<tr>
<td>$B(\phi\to e\bar{e})B(\phi\to 3\pi) = (4.30 \pm 0.08 \pm 0.21) \times 10^{-5}$</td>
<td>$(4.59 \pm 0.14) \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Fit results: higher mass region

- Good fit obtained for the range up to 1.8 GeV/c^2.
- Extending the fit to masses above 1.8 GeV/c^2 may require a more complicated fitting function taking into account non-resonant $3\pi$ production.
- Mass and width parameters are dependent upon our assumed phases - interference effect is strong.

<table>
<thead>
<tr>
<th>Process</th>
<th>$B(\omega'\rightarrow ee)B(\omega'\rightarrow 3\pi)$</th>
<th>$M(\omega')=1350\pm20\pm20$ MeV/c^2</th>
<th>$\Gamma(\omega')=450\pm70\pm70$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(0.82\pm0.05\pm0.06)\cdot10^{-6}$</td>
<td>1400 - 1450</td>
<td>180 - 250</td>
</tr>
<tr>
<td>Process</td>
<td>$B(\omega''\rightarrow ee)B(\omega''\rightarrow 3\pi)$</td>
<td>$M(\omega'')=1660\pm10\pm2$ MeV/c^2</td>
<td>$\Gamma(\omega'')=230\pm30\pm20$ MeV</td>
</tr>
<tr>
<td></td>
<td>$(1.3\pm0.1\pm0.1)\cdot10^{-6}$</td>
<td>1670 ± 30</td>
<td>315 ± 35</td>
</tr>
</tbody>
</table>
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ cross section

$\sigma_{3\pi}(m) = \frac{(dN/dm)}{\epsilon(m) \cdot R(m) \cdot dL/dm}$

- coverage of wide region in this experiment - no point-to-point normalization problems
- consistent with SND data $E_{c.m.} < 1.4$ GeV
- inconsistent with DM2 results
- overall normalization error $\sim 5\%$ up to 2.5 GeV
**J/ψ → 3π decay**

- The $J/\psi$ meson is narrow - clean signal
- After sideband subtraction -
  \[ N_{J/\psi} = 920 \pm 34 \]
- Detection efficiency - \( \varepsilon = (9.2 \pm 0.6)\% \)
- The result:
  \[ \Gamma(J/\psi \rightarrow ee) \cdot B(J/\psi \rightarrow 3\pi) = 0.122 \pm 0.005 \pm 0.08 \text{ keV} \]
- We previously measured ($\mu\mu\gamma$)
  \[ \Gamma(J/\psi \rightarrow ee) = 5.61 \pm 0.20 \text{ keV} \]

**Radiative decay**

- $B(J/\psi \rightarrow 3\pi) = (2.18 \pm 0.19)\%$
  - BaBar Preliminary
  - (1.50 \pm 0.20)\%  
    - PDG
  - (2.10 \pm 0.12)\%  
    - BES 2003

**References**

$e^+ e^- \rightarrow 2\pi^+ 2\pi^- \gamma, \; K^+ K^- \pi^+ \pi^- \gamma, \; 2K^+ 2K^- \gamma$

Event selection:
- Isolated ISR photon with $E_{cm} > 3$ GeV
- At least four good tracks from IP

Kinematic fit:
- Energy and momentum balance enforced
- Energy and angles of hard ISR photon are not used - 1C fit
- Fit in 3 hypotheses:
  - $4\pi$ for all events
  - $2K2\pi$ if 1 or 2 identified kaons
  - $4K$ if 2, 3 or 4 identified kaons

Background subtraction:
- Other ISR processes ($5\pi\gamma, \ldots$) – using difference in $\chi^2$ distributions
- $e^+ e^- \rightarrow qq$ – using JETSET simulation
e^+e^- → 2\pi^+ 2\pi^- cross section

Systematic errors:
- 12% for m_{4\pi} < 1 GeV,
- 5% for 1 < m_{4\pi} < 3 GeV,
- 16% for higher masses
- best measurement above 1.4 GeV

Coverage of wide region in one experiment
No point-to-point normalization problems

Intermediate states:
- a_1(1260)\pi - dominant, structure which may be f_0(1370)\rho final state is seen.
- For detailed study, a simultaneous analysis of 2\pi^+ 2\pi^- and \pi^+\pi^- 2\pi^0 final states is required.
$e^+ e^- \rightarrow 2\pi^+ 2\pi^-$ cross section

Good agreement with direct $e^+ e^-$ measurements
Most precise result above 1.4 GeV
\[ \pi^+\pi^-\pi^+\pi^- \text{ substructures} \]

BaBar preliminary

MC generator:
- Includes \( a_1(1260) \pi \) and \( f_0(1370)\rho \)
- Does not include \( J/\psi \)
$e^+ e^- \rightarrow K^+ K^- \pi^+ \pi^-$

Substantial resonance sub-structures observed:
- $K^*(890)K\pi$ dominant
- $\phi\pi\pi$, $\rho KK$ contribute strongly
- $K^*_2(1430)K\pi$ seen.

Systematic error – 15% (model dependence, kaon identification)

Much more precise than previous measurement
$K^+K^-\pi^+\pi^-$ substructures

BaBar preliminary

$K^*(890)K\pi$ dominated

No studies in previous $e^+e^-$ experiments!

$K^*\bar{K}\pi\gamma$ MC generators are not available yet

$K^*$ regions excluded

No signal from $\phi f_0(980)$ yet

Connection to $\phi \rightarrow f_0(980)\gamma$?

Events from $\phi$ band
$e^+ e^- \rightarrow 2K^+ 2K^-$

No clear mass structure in the two- or three-body subsystems

No $\phi$'s!

BaBar preliminary

First measurement

Overall normalization systematic error – 25% (model dependence, kaon identification)
J/ψ and ψ(2S) decays

BaBar preliminary
B(J/ψ → π⁺π⁻π⁺π⁻) = (3.61 ± 0.26 ± 0.26) · 10⁻³
B(J/ψ → K⁺K⁻π⁺π⁻) = (6.09 ± 0.50 ± 0.53) · 10⁻³
B(J/ψ → K⁺K⁻K⁺K⁻) = (6.7 ± 1.1 ± 1.0) · 10⁻⁴
B(ψ(2S) → J/ψ π⁺π⁻) = 0.361 ± 0.015 ± 0.028

PDG
(4.0 ± 1.0) · 10⁻³
(7.2 ± 2.3) · 10⁻³
(9.2 ± 3.3) · 10⁻⁴
0.317 ± 0.011
Summary

- Using ISR method the cross sections of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, $K^+K^-\pi^+\pi^-$, $2K^+2K^-$ reactions have been measured from threshold to 4.5 GeV.

- These are the most precise measurements to date for c.m. energies greater than 1.4 GeV.

- Example: contribution to $a_{\mu}^{\text{had}} (\times 10^{-10})$ from $2\pi^+ 2\pi^-$ (0.56 – 1.8 GeV)
  - from all $e^+ e^-$ exp. $14.21 \pm 0.87_{\text{exp}} ^{\pm 0.23}_{\text{rad}}$
  - from all $\tau$ data $12.35 \pm 0.96_{\text{exp}} ^{\pm 0.40}_{\text{SU(2)}}$
  - from BaBar $12.95 \pm 0.64_{\text{exp}} ^{\pm 0.13}_{\text{rad}}$

  Davier-Eidelman-Hoecker-Zhang 2003

- Several $B(J/\psi \rightarrow X)$ measurements better than current world average

- Detailed papers to be submitted to PRD

- More modes to come; aim for systematic errors $\leq 1\%$ (in $\pi^+\pi^-$)