SEMILEPTONIC B DECAY METHODS

~~LATTICE~~ → next talk QCD

HQET

OPE

SCET

A little on methods how they are used and what some of the issues are. Set the table for talks that start tomorrow.

A little about Rare Decays as well. They overlap with semileptonic in some cases. $B \to X_s \gamma$ inclusive.

EMPHASIS NOT ON NEW physics much more likely in $B-\bar{B}$ mixing, $B \to X_s \gamma$, $B \to X_s e^+e^-$ where standard model contribution starts at 1-loop. But beyond SM physics might be small correction. Need precision B physis, both theory + expt

I A Exclusive Semileptonic Decay to Charmed Final STATES

Decays

$\rightarrow (D^0, D^*)\, e\bar{\nu}_e$

$\rightarrow (D_1(2420), D_2^*(2460))\, e\bar{\nu}_e$

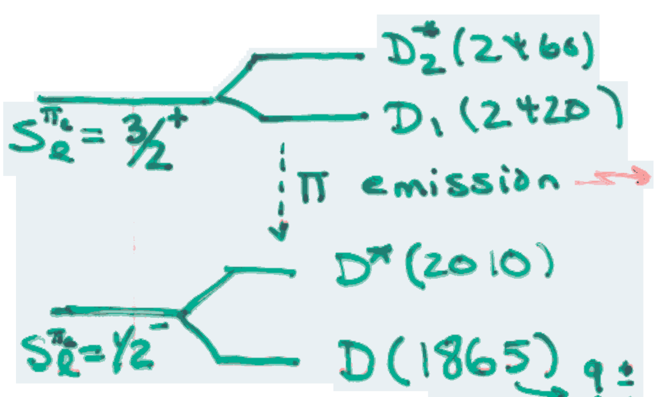
Theoretical Framework HQET

$$\Psi(x) = e^{i m_Q v\cdot x}(1 + \ldots)\, h_v(x)$$

$$\mathcal{L}_{QCD} = \bar{h}_v(x)\, i v\cdot D\, h_v(x) + \ldots$$

Heavy quarks velocity not changed by soft interactions at leading order $\Delta v \sim \Delta p / m_Q$.

Quarks of same v have same interactions + interactions independent of quark spin so new quantum # to label states at rest S_ℓ spin of light degrees of freedom



Leading order HQET must be L=2 partial wave suppressed width. $S_\ell^{\pi_\ell} = 1/2^+$ can decay L=0 so broad.

Decay Matrix Elements expressed in terms of Lorentz invariant form factors. Eg

$$\langle D(p') | \bar{c}\gamma^\mu b | B(p) \rangle$$
$$= f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$

$q = p - p'$, $q^2 = m_B^2 + m_D^2 - 2p\cdot p'$

But at leading order in Λ_{QCD}/m_Q expansion quarks of same four velocity have same strong interactions

Crazy to use q^2 as kinematic variable and p to label states

$p' = m_D v'$, $p = m_B v$

$q^2 = m_B^2 + m_D^2 - 2m_B m_D v\cdot v'$

$w = v\cdot v'$
$1 \le w \le 1.5$

zero recoil

Leading order in Λ_{QCD}/m_Q

$\bar{c}\gamma^\mu b \to \bar{c}_{v'}\gamma^\mu b_v$ + perturbative α_s corrections

Spin-Flavor Symmetry Find at leading order

$\langle D(v')| \bar{c}_{v'} \gamma_\mu b_v |B(v)\rangle - \xi(w)\,(v+v')_\mu$

$\langle D(v'\varepsilon)\, \bar{c}_v \gamma_\mu \gamma_5 b_v\, B(v)\rangle \quad i\xi(w)[(1+w)\epsilon^*_\mu - (\epsilon^* v)\, v'_\mu]$

$\langle D^*(v\,\varepsilon)\, \bar{c}_v \gamma_\mu b_v\, B(v)\rangle = \xi(w)\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu} v'^{\alpha} v^{\beta}$

v v changes

Perturbative QCD corrections calculable

Λ_{QCD}/m_Q corrections new functions of $w = v\, v'$ Some involve same parameter as occurs in Λ_{QCD} m_Q expansion of hadron masses

$$m_B \quad m_b + \Lambda \quad \frac{\lambda}{2m_b} \quad \frac{3\lambda_2}{2m_b} +$$

$$m_{B^*} \quad m_b + \Lambda \quad \frac{\lambda_1}{2m_b} + \frac{\lambda_2}{2m_b} +$$

$$m_D = m_c + \Lambda \quad \frac{\lambda}{2m_c} \quad \frac{3\lambda_2}{2m_c} +$$

$$m_{D^*} = m_c + \Lambda \quad \frac{\lambda}{2m_c} + \frac{\lambda_2}{2m_b} + \cdots$$

$\lambda_2 \sim 0$ GeV^2 from B^* B mass splitting

At zero recoil no Λ_{QCD}/m_Q corrections

$$\langle D(v)|\bar{c}_v \gamma_\mu b_v|\bar{B}(v)\rangle = (v+v')_\mu + \mathcal{O}\left(\left(\frac{\Lambda_{QCD}}{m_Q}\right)^2\right)$$

$$\langle D^*(v)|\bar{c}_v \gamma_\mu \gamma_5 b_v|\bar{B}(v)\rangle = -2i\,\varepsilon^*_\mu + \mathcal{O}\left(\left(\frac{\Lambda_{QCD}}{m_Q}\right)^2\right)$$

$\Rightarrow$ Way to get precise $|V_{cb}|$ from $\bar{B}\to D^* e\bar{\nu}_e$ (Lattice QCD for order Λ^2_{QCD}/m_Q^2 piece)

Similar story for $\bar{B}\to D_1 e\bar{\nu}_e$, $\bar{B}\to D_2^* e\bar{\nu}_e$ except at zero recoil $w=1$ matrix elements vanish. At leading order all form factors expressed in terms of single function $\tau(w)$. At order Λ_{QCD}/m_Q predict zero recoil matrix element

$$\langle D_1(v)|\bar{c}_v \gamma_\mu b_v|\bar{B}(v)\rangle = \frac{i8}{\sqrt{6}}\left(\frac{1}{2m_c}\right)\left(\bar{\Lambda}_{s_\ell^\pi=\frac{3}{2}^+} - \bar{\Lambda}_{s_\ell=\frac{1}{2}^+}\right) \cdot \tau(1)\,\varepsilon^*_\mu$$

$1 \le w \le 1.32$

$\bar{\Lambda}_{s_\ell^\pi=\frac{3}{2}^+} - \bar{\Lambda}_{s_\ell=\frac{1}{2}^+} \sim 0.4$ GeV

from known mass splittings

Bauer, Fleming Luke PRD 63, 014006 (2001)
Bauer, Stewart, Phys. Lett B 516, 134 (2001)
Bauer, Pirjol, Stewart PRD 65, 054022 (2002), PRD 66, 054005 (2002)

Ib Semileptonic B decay to Light Hadrons

$\bar{B} \to \pi e \bar{\nu}_e$, $\bar{B} \to \rho e \bar{\nu}_e$ low q^2 region where some components of light hadrons four mom large but its invariant mass $\sim \Lambda_{QCD}$.

Bauer, Fleming Pirjol, Stewart, PRD 66, 054005 (2002)

Effective theory used SCET and above

Say have quark with large component of momentum along $\hat{z}$ direction. Introduce lightlike four vectors

$$n^\mu = (1, 0, 0, 1) = (1, \vec{0}_\perp, 1)$$

$$\bar{n}^\mu = (1\ 0\ 0\ -1) = (1, \vec{0}_\perp, -1)$$

$$n \cdot n = \bar{n} \cdot \bar{n} = 0 \qquad n \cdot \bar{n} = 2$$

$$p = \frac{\bar{n} \cdot p}{2} n^\mu + \frac{n \cdot p}{2} \bar{n}^\mu + p_\perp^\mu$$

$$p^2 = 2p^+p^- + p_\perp^\mu p_{\perp\mu} \Rightarrow \begin{array}{l} p^+ \sim \lambda^2 E, \; p_\perp \sim \lambda E \\ p^- \sim E \end{array}$$

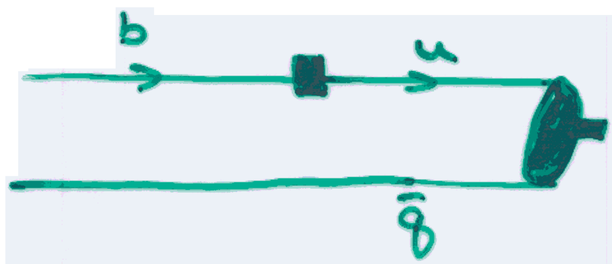
big, small

$$p^+ = n \cdot p, \quad p^- = \bar{n} \cdot p, \quad p_\perp = (0, \vec{p}_\perp, 0)$$

Imagine "offshellness"

$$p^2 \sim \Lambda_{QCD} E^2$$

For example



$p^2 \sim \Lambda_{QCD}\, m_B$ (m_B '' E)

$\lambda \sim \sqrt{\frac{\Lambda_{QCD}}{m_B}}$

Might think much like HQET

$$\Psi = e^{-i\tilde{p}\cdot x}\, \xi_n(x) \quad , \quad \not{n}\, \xi_n(x) = 0$$

$$\tilde{p}^\mu = \frac{\bar{n}\cdot p}{2} n^\mu + p_\perp^\mu \quad , \quad \tilde{p}_+ = 0$$

and turn crank. But $\tilde{p}$ not fixed

Interactions with collinear gluons can change collinear momenta and still leave you in same effective theory

Need collinear gluons as well a quarks (and antiquarks)
Large components of mom p can change (but still labels)
Also must add ultrasoft degrees of freedom with four momentum

$$k^{\mu} \sim \lambda^2 E$$

since

$$(\tilde{p}+k)^2 \sim \lambda^2 E^2$$

using field redefinition

Doesn't seem like gained much
But can decouple ultrasoft degrees of freedom from collinear at leading order in λ. Gives rise to factorization theorems.
Bauer, Fleming, Pirjol, Stewart, Rothstein, PRD 66, 014017 (2002)
Also constraint $\not{n}\,\xi_n$ powerful

Match $\bar{q}\Gamma b \to \bar{\xi}_n \Gamma h_v + \dots$ 0

perturbative corrections — Wilson lines omitted.

Spinor Constraints Powerful

$$\bar{\xi}_n \Gamma h_v = \bar{\xi}_n \Gamma_1 h_v$$ — using $\not{n}\,\xi_n = 0$, $\not{v}\, h_v = h_v$

$$\Gamma_1 = \frac{\not{\bar{n}}}{2}\,\mathrm{tr}\left[\frac{\not{n}}{2}\Gamma\left(\frac{1+\not{v}}{2}\right)\right] + \frac{\not{\bar{n}}\gamma_5}{2}\,\mathrm{tr}\left[\frac{\not{n}}{2}\gamma_5\Gamma\left(\frac{1+\not{v}}{2}\right)\right] + \gamma_\perp^\mu\,\mathrm{tr}\left[\gamma_{\perp\mu}\frac{\not{\bar{n}}\not{n}}{4}\Gamma\left(\frac{1+\not{v}}{2}\right)\right]$$

4 terms since 2 comp spinors

How many form factors for $\bar{B} \to \rho e \bar{\nu}_e$?

$$\langle V(\varepsilon, E, n)|\,\bar{\xi}_n \not{\bar{n}}\, b_v\,|B(v)\rangle \sim \varepsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\mu n_\nu \bar{n}_\lambda v_\sigma = 0 \qquad v_\sigma = \tfrac{1}{2}(n+\bar{n})_\sigma$$

$$\langle V(\varepsilon, E, n)|\,\bar{\xi}_n \not{\bar{n}}\gamma_5 b_v\,|B(v)\rangle = -i\,\zeta_{\parallel}(E)\,\bar{n}\cdot\varepsilon^* \quad (\bar{n}\cdot\varepsilon^* = 2v\cdot\varepsilon^*)$$

$$\langle V(\varepsilon, E, n)|\,\bar{\xi}_n \gamma_\perp^\mu b_v\,|B(v)\rangle$$
= [redacted] $\zeta_{\parallel}(E)\,\varepsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu n_\lambda v_\sigma$

The four possible form factors expressed in terms of 2.

But $m_\rho^2 \sim \Lambda_{QCD}^2$ not $\Lambda_{QCD} m_b$. So should introduce further matching onto effective theory for this regime. ρ made up of colinear quark + colinear antiquark (interpolating field). So need suppressed interactions to change "soft" spectator antiquark to colinear antiquark.

Matrix Element   Bauer, Pirjol, Stewart hep-ph/0211069

$$\alpha_s(\sqrt{\Lambda_{QCD} m_b}) \cdot \left[\begin{array}{l}\text{Part that is convolution of} \\ \text{l.c. wave functions for } \bar{B} \text{ meson} \\ \text{and rho with other computable factors} \\ \text{Violates above form factor relations}\end{array}\right]$$

?

$$+ \left[\begin{array}{l}\text{Part that cannot be written in that} \\ \text{way but obeys form factor relations}\end{array}\right]$$

II Inclus ve Semileptonic B decay

Basic too Operator Product Expansi
OPE (+ transition to HQET) f na
resu t expresso for some nclus ve
sem eptonic decay (d fferential , tegrated
rates as expans in Λ_{QCD}/m_Q

Also for some quantities where thi
expansion breaks down can get needed
nonperturbative quantity from weak radiative
B decay

For Inclus ve decay need Hadronic tensor

$$W^{\alpha\beta} \sum_X (2\pi)^3 \delta^4(p_B\ q - p_X)$$

$$\frac{1}{2m_B} \langle \bar{B}(p_B) | J_L^{+\alpha}\ X(p_X) \rangle \langle X(p_X) | J_\beta^\beta | \bar{B}(p_B) \rangle$$

Expand in terms of Lorentz Scalars

$$W_{\alpha\beta} \quad g_{\alpha\beta} W_1\ q^2\ v\,q) \qquad p_B = m_B v$$

$+$

other W's

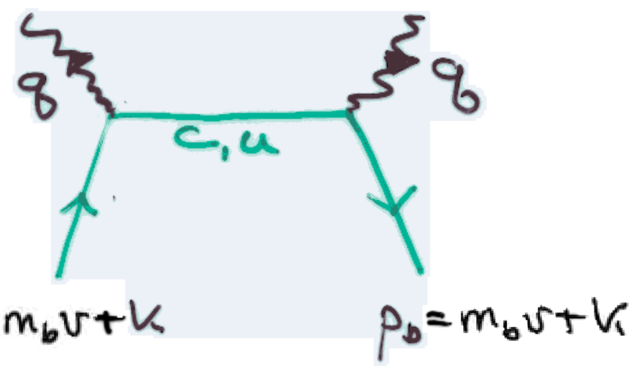
For time orded product

$$T^{\alpha\beta} \quad i \int d^4x\, e^{iq\cdot x} \frac{\langle \bar{B} | T\{J_L^{\alpha\dagger}(x)\, J_L^{\beta}(0)\} | B\rangle}{2m_B}$$

$$\frac{}{\pi} \mathrm{Im}\, T_j - W_j$$

Large energy ava'lable to hadron c states

for nclusive decay perfor OPE + transition to HQET



Expand n esidual momentu and get operators

Leading o de

$$\langle B(v) | \bar{b}_v \gamma_\lambda b_v | B(v)\rangle = 2v_\lambda$$

know g ves
free quark deca reso t

Next order get operator with one covariant derivative D_λ and its matr x element vanishes by HQET equations of motion

Λ_{QCD}/m_b corrections Next order operators get are same as give correction meson masses. Need matrix elements $\lambda_{1,2}$ mentioned already know λ_2

Wow thats great. Lets pack up its Miller Time!

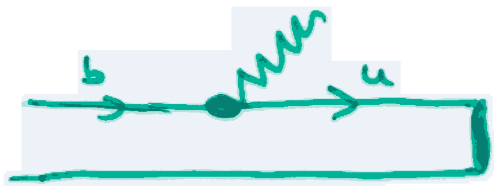
Not quite some issues Consider $d\Gamma/dy$ $\quad y = 2E_e/m_b$ Has singular terms in its expansion near maximum "parton" value of y. For $b \to u$ decay

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_b^5}{192\pi^3}\Big\{ 2y^2(3-2y)\,\theta(1-y) - \frac{2\lambda_2}{m_b^2}\Big[\frac{11}{2}\delta(1-y) - y^2(6+5y)\,\theta(1-y)\Big] - \frac{2\lambda_1}{m_b^2}\Big[\frac{1}{6}\delta'(1-y) + \frac{1}{6}\delta(1-y) - \frac{5}{3}y^3\theta(1-y)\Big] + \dots\Big\}$$

For given Final state $E_e^{max} = \dfrac{m_B^2 - m_X^2}{2m_B}$

so not surprising it breaks down very close to y_{max} where can only make low mass states

Actually breaks down earlier when states have $m_X^2 \sim \Lambda_{QCD} m_B$



$\bar{q}$ → change in $\bar{q}$ momentum of order Λ_{QCD} effects invariant mass of final state of order $m_X^2 \sim \Lambda_{QCD} m_B$

This called shape function region $\Delta E_e \sim \Lambda_{QCD}$ from endpoint. Must sum most singular terms at each order in expansion in this region go into shape function.

Related story for $B \to X_s \gamma$ and can get shape function from endpoint photon spectrum. {Including PQCD effects, Leibovich, Low, Rothstein PLB 486, 86 (2000) Neubert PLB 513, 88 (2001)

(ii) $|V_{cb}|$ from Integrated Semileptonic Decay rate

Prediction similar to Z $\to$ hadronic decay width depends on local parton hadron duality since cannot change m_B experimentally

Bigi + Mannel, hep-ph/0212021 recent discussion

For Z decays available hadronic energy $m_Z \sim 100$ GeV. For $B \to X_c e \bar{\nu}_e$ about $m_b - m_c \sim 3$ GeV. Get confidence threshold effects that might cause OPE to not work are small by measuring many things, $\langle m_x^2 \rangle$ $\langle m_x^4 \rangle$, R_1, R_2 etc. Eliminate m_c, m_b in favor of masses of hadrons, $\bar{\Lambda}$, λ_1 λ_2, ... Measure these things and check consistency. Right now things don't seem to fit together well.

(Renormalon ambiguity)

e.g. Leave out preliminary BABAR results on $\langle m_x^2 \rangle$ as a function of lepton energy cut (too large) and D, D^* branching ratio (too small) and things would fit together.

Assuming no duality violations $|V_{cb}|$ at 2% level of theoretical uncertainty. Consistent with Exclusive extraction.

Bauer, Ligeti, Luke, Manohar, hep-ph/0210027

Battaglia, Calvi, Gambino, Oyanguren, Roudeau, Salmi, Salt, Stocchi, Uraltsev hep-ph/0202175.

(1) |Vub| From Inclusive Semileptonic B decay

Remove large charmed final state background by going to large enough electron energies. Puts us in $\Delta E_e \sim \Lambda_{QCD}$ endpoint region but get shape function from $B \to X_s \gamma$ "Subleading shape functions" down

CLEO Collab hep-ex/0202019

only $\Lambda_{QCD}/m_b \sim 10\%$ But one is enhanced by numerical factors $1\frac{1}{2} \sim 5$

Ligeti, Leibovich, W, PLB 539, 242, 2002.
Bauer, Luke, Mannel, PLB 543, 261, 2002.

However, know integral of this shape function which helps some if can make ΔE_e large enough

Neubert PLB 543 269 2002.

Four quark operators a serious problem. Of order Λ_{QCD}^3/m_b^3 suppressed but enhanced in some ways

Voloshin, Mod. Phys. Lett. A17, 245, 2002

$$\frac{d\Gamma^{(4\text{-quark})}}{dy} = -\frac{G_F^2 m_b^2}{12\pi} f_B^2 m_B (B_1 - B_2)\,\delta(1-y)$$

$(\bar{b}_v u)_{V-A} \otimes (\bar{u} b_v)_{V-A}$

$$\frac{1}{2}\langle B(v)|\, O_{V-A}\,|B(v)\rangle = \frac{f_B^2 m_B}{8} B_1$$

$$\frac{1}{2}\langle B(v)|\, O_{S-P}\,|B(v)\rangle = \frac{f_B^2 m_B}{8} B_2$$

Large N_c
charged B meson
$B_1 = B_2 = 1$
neutral
$B_1 = B_2 = 0$

$$\frac{\delta\Gamma}{\Gamma_{SL}} \quad 0.02\left(\frac{f_B}{0.2\,GeV}\right)^2\left(\frac{B_1-B_2}{0.1}\right)$$

$\frac{1}{N^2}$

$B_1\ B_2 \sim 0$ then 2% of semileptonic rate. If focus on region near endpoint that contains 0% of rate then 20% effect. And we just made $B_1 - B_2 = 0$ up maybe its 0.3 (or 0.03)

Other ways to emove $b \to c$ contamination

At fixed m_X

$$(m_B\ m_X)^2 \leq q^2 \leq m_B^2$$

c $q^2 > (m_B\ m_D)^2$ remove char background and no shape function since states not 'rapidly recoiling" st I have to worry about 4-quark operator but by combined $m_X^2\ q^2$ cut get more of spectrum without shape function Bauer, Ligeti, Luke, PRD64, 113004 (2001)

Just say no!