Heavy Quark Parameters from Inclusive Semileptonic B Decays

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Total width $\Gamma(B \to X\,\ell\nu)$ is well measured

Strong interactions are controlled by

QCD theorem (1992)

Bigi, Shifman, Uraltsev, Vainshtein

No $\Lambda_{\rm QCD}/m_b$ corrections to inclusive widths of heavy flavor hadrons

Applies to all types: semileptonic, nonleptonic, $b \to s+\gamma$, $b \to s\,\ell^+\ell^-$, ...

$\Gamma(B)$ is expressed in terms of quark masses and local heavy quark expectation values

expansion runs in $E_r = m_b - m_c \simeq 3.5\,\mathrm{GeV}$

$$\mu_G^2 = \tfrac{1}{2M_B}\langle B|\,\bar b\,\tfrac{i}{2}\,g_s\,\sigma_{\mu\nu}G^{\mu\nu}\,b\,|B\rangle \;\longleftrightarrow\; \langle B|-g_s\,\vec\sigma_b\vec B_{\rm chr}(0)\,|B\rangle_{\rm QM}$$

$$\mu_\pi^2 = \tfrac{1}{2M_B}\langle B|\,\bar b\,(i\vec D\,)^2\,b\,|B\rangle \;\longleftrightarrow\; \langle B|\,\vec p_b^{\,2}\,|B\rangle_{\rm QM}$$

$$\vec p_b \to \vec\pi_b = -i\vec D = -i\vec\partial - g_s\vec A$$

$$\rho_D^3 = -\langle B|\,\bar b\,\frac{1}{2}(\vec D\vec E)\,b\,|B\rangle, \qquad \rho_{LS}^3 = \langle B|\,\bar b\,(\vec\sigma\cdot\vec E\times\vec\pi)\,b\,|B\rangle$$

Theoretical status

Can go down to a $\%$ level in $|V_{cb}|$ if relevant parameters are determined:

- $m_{b,c}(\mu)$, $\mu_\pi^2(\mu)$, $\mu_G^2(\mu)$, ... are completely defined and can (in principle) be determined from experiment with an unlimited accuracy
- Duality violation is very small in $\Gamma_{\rm sl}(B)$ BU 2001
- α_s corrections to Wilson coefficients are feasible
- We know how to analyze higher power corrections

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s+\gamma)$ decay distributions themselves BSUV, 1993-1994

Nowadays is being implemented in a number of experiments

Comprehensive approach: measure many observables to extract the 'theoretical' input parameters

Extreme opposite – B experiment provides only $\Gamma_{\rm sl}$ the rest must theory

Until recently relied on charm mass expansion

$$m_b - m_c = \overline{M}_B - \overline{M}_D + \frac{\mu_\pi^2}{2}\left(\frac{1}{m_c} - \frac{1}{m_b}\right) + \frac{\rho_D^3 - (\rho_{\pi\pi}^3 + \rho_S^3)}{4}\left(\frac{1}{m_c^2} - \frac{1}{m_b^2}\right) + \ldots$$

Expansion in $1/m_c$ is questionable: $\frac{1}{m_c^2} > 14\,\frac{1}{m_b^2},\quad 8\,\frac{1}{(m_b - m_c)^2}$

Non-local correlators $(\rho_{\pi\pi}^3,\ \rho_S^3)$ are probably quite large

	N.U.	2001
't Hooft model	Lebed, N.U.	2000
Lattice	Kronfeld, Simone	2000

Non-local correlators are not measured in B decays

Expansion for $M_B - M_D$ enjoys much smaller power corrections compared to $\overline{M}_B - \overline{M}_D$ 'BPS' limit

We can do without relying on $1/m_c$ expansion at all

rather check it

DELPHI

$$\begin{aligned}\langle E_\ell\rangle &= 1.383 \pm 0.015\,\mathrm{GeV}\\ \langle (E_\ell - \langle E_\ell\rangle)^2\rangle &= 0.192 \pm 0.009\,\mathrm{GeV}^2\\ \langle (E_\ell - \langle E_\ell\rangle)^3\rangle &= -0.029 \pm 0.008\,\mathrm{GeV}^3\end{aligned}$$

CLEO $R_1,\ R_2,\ R_0$

$$\begin{aligned}\langle E_\ell\rangle_{E_\ell>1.5\,\mathrm{GeV}} &= 1.5 \;\, + 0.2295 \pm 0.0007 \pm 0.0007\,\mathrm{GeV}\\ \langle E_\ell^2\rangle_{E_\ell>1.5\,\mathrm{GeV}} &= 1.5^2 + 0.9482 \pm 0.0025 \pm 0.0025\,\mathrm{GeV}^2\\ \frac{\Gamma(E_\ell>1.7\,\mathrm{GeV})}{\Gamma(E_\ell>1.5\,\mathrm{GeV})} &= 0.6173 \pm 0.0016 \pm 0.0014\end{aligned}$$

$b \to s + \gamma$ CLEO: $\langle E_\gamma\rangle = 2.346 \pm .034\,\mathrm{GeV}$

at $E_\gamma > 2\,\mathrm{GeV}$

Hadronic moments: if m_c were large enough first would yield $\overline{\Lambda}$, second μ_π^2, third ρ_D^3 more or less directly BSUV 1993-94

CLEO 2001

$$\begin{aligned}\langle M_X^2\rangle_{E_\ell>1.5\,\mathrm{GeV}} - M_{\bar D}^2 &= 0.251 \pm 0.066\,\mathrm{GeV}^2\\ \langle (M_X^2 - M_{\bar D}^2)^2\rangle_{E_\ell>1.5\,\mathrm{GeV}} &= 0.576 \pm 0.170\,\mathrm{GeV}^4\end{aligned}$$

DELPHI

$$\begin{aligned}\langle M_X^2\rangle - M_{\bar D}^2 &= 0.534 \pm 0.085\,\mathrm{GeV}^2\\ \langle (M_X^2 - \langle M_X^2\rangle)^2\rangle &= 1.226 \pm 0.22\;\;\mathrm{GeV}^4\\ \langle (M_X^2 - \langle M_X^2\rangle)^3\rangle &= 2.97\;\; \pm 0.83\;\;\mathrm{GeV}^6\end{aligned}$$

BaBar

$$\langle M_X^2\rangle_{E_\ell>1\,\mathrm{GeV}} - M_{\bar D}^2 = 0.62\;\; \pm 0.10\;\;\mathrm{GeV}^2$$

With the comprehensive studies can do a robust analysis

- width is affected only by ρ_D^3 to order $1/m_b^3$
  moments also depend (weakly) on ρ_{LS}^3
- No non-local correlators ever enter
- Deviations from the HQ limit are driven by $1/m_b$
  actually, $\propto \mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$ in B BPS limit
- Exact sum rules and inequalities for properly defined parameters
  e.g., $\mu_\pi^2 > \mu_G^2 \simeq 0.35\,\mathrm{GeV}^2$

$$\langle E_\ell \rangle = 1.38\,\mathrm{GeV} + 0.38[(m_b - 4.6\,\mathrm{GeV}) - 0.71\,(m_c - 1.15\,\mathrm{GeV}) + 0.09\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.22\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)]$$

$$\mathcal{L}_2 = 0.18\,\mathrm{GeV}^2 + 0.1[(m_b - 4.6\,\mathrm{GeV}) - \ldots] + 0.01(m_c - 1.15\,\mathrm{GeV}) + 0.04\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.04\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)$$

$$\mathcal{L}_3 = -0.033\,\mathrm{GeV}^3 - 0.03[(m_b - 4.6\,\mathrm{GeV}) - \ldots] + 0(m_c - 1.15\,\mathrm{GeV}) + 0.03\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.04\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)$$

Practically the same combination $m_b - 0.7 m_c$

weak dependence on μ_π^2, ρ_D^3

$$\frac{|V_{cb}|}{0.042} = 1 - 0.65\,[(m_b - 4.6\,\mathrm{GeV}) - \ldots] - 0.06\,(m_c - 1.15\,\mathrm{GeV}) - 0.07\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.05\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)$$

Precise value of m_c is irrelevant!

Need to know with some accuracy μ_π^2 and ρ_D^3

no hidden assumptions

Hadronic moments

$$\langle M_X^2\rangle \simeq 4.54\,\mathrm{GeV}^2 - 5\,[(m_b-4.6\,\mathrm{GeV}) - 0.62\,(m_c-1.15\,\mathrm{GeV}) + 0.13\,(\mu_\pi^2-0.4\,\mathrm{GeV}^2)-0.2\,(\tilde\rho_D^3-0.12\,)]$$

Nearly the same combination $m_b-0.7m_c-0.1\mu_\pi^2-0.2\tilde\rho_D^3$ as in $\langle E_\ell\rangle$

Not very constraining ... – instead checks how HQ expansion works: Theory predicts $\langle E_\ell\rangle = 1.377\,\mathrm{GeV}$

Experiment: $\langle E_\ell\rangle = 1.383 \pm 0.015\,\mathrm{GeV}$

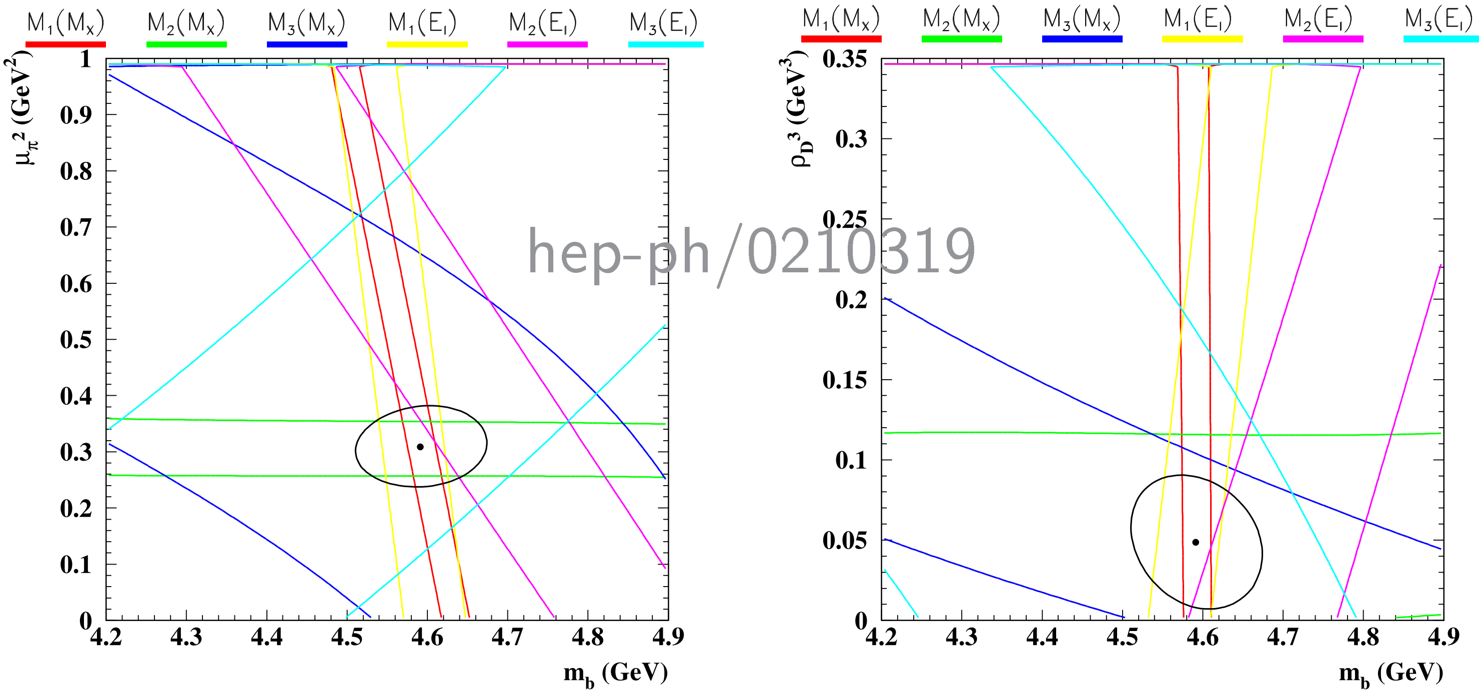
A highly nontrivial nonperturbative check of the OPE: the sum rule for $M_B-m_b \simeq 650\,\mathrm{MeV}$ verified at a $40\,\mathrm{MeV}$ level!

$$\langle (M_X^2-\langle M_X^2\rangle)^2\rangle \simeq 1.2\,\mathrm{GeV}^4 + 0.02(m_b-4.6\,\mathrm{GeV}) - 0.7\,(m_c-1.15) + 4.5\,(\mu_\pi^2-0.4\,\mathrm{GeV}^2) - 5.3\,(\tilde\rho_D^3-0.12\,\mathrm{GeV}^3)$$

$$\langle (M_X^2-\langle M_X^2\rangle)^3\rangle \simeq 4\,\mathrm{GeV}^6 - (m_b-4.6\,\mathrm{GeV}) - 3\,(m_c-1.15\,\mathrm{GeV}) + 5\,(\mu_\pi^2-0.4\,\mathrm{GeV}^2) + 12\,(\tilde\rho_D^3-0.12\,\mathrm{GeV}^3)$$

Ideally, these moments measure kinetic and Darwin expectation values. In practice, for ρ_D^3 only approximate evaluation and an informative upper bound

Current sensitivity to μ_π^2 is about $0.1\,\mathrm{GeV}^2$, $0.1\,\mathrm{GeV}^3$ to ρ_D^3



$$\frac{|V_{cb}|}{0.042} = 1 \;-\; 0.13\,[\langle M_X^2\rangle - 4.54\,\mathrm{GeV}^2] \;-\; 0.005\,(m_c - 1.15\,\mathrm{GeV})$$
$$+\; 0.10\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) \;-\; 0.03\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

Measuring $\langle M_X^4\rangle$ and $\langle M_X^6\rangle$ is the real step in implementing the comprehensive program of extracting $|V_{cb}|$

more work is required

Example:

$$|V_{cb}| = 0.0421 \cdot (1 \pm 0.015_{\mathrm{SL\,width}} \pm 0.02_{\mathrm{HQ\ param}})$$

solely from DELPHI hadronic moments

Does not rely on expansion in $1/m_c$!

small uncertainties are not dominated by theory

Moreover,

assuming $\bar{m}_c(m_c) = 1.23\,\mathrm{GeV}$ $\quad m_b(1\,\mathrm{GeV}) \simeq 4.58\,\mathrm{GeV}$

(too) good agreement with the theoretical expectations ?

$$\frac{R_1}{1.78} = 0.986 + 0.15[(m_b - 4.6\,\mathrm{GeV}) - 0.63\,(m_c - 1.15\,\mathrm{GeV}) + 0.27\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.36\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)]$$

$$\frac{R_2}{3.20} = 0.975 + 0.31[(m_b - 4.6\,\mathrm{GeV}) - ...] + 0.004\,(m_c - 1.15\,\mathrm{GeV}) + 0.007\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.013\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

$$\frac{R_0}{.617} = 0.953 + 0.76[(m_b - 4.6\,\mathrm{GeV}) - ...] - 0.03\,(m_c - 1.15\,\mathrm{GeV}) - 0.17\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) + 0.18\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

Practically the same combination $m_b - 0.6 m_c + 0.3\mu_\pi^2$ in R_1 and R_2; slightly different dependence on μ_π^2, ρ_D^3 in R_0

Yield close values, but do not literally coincide

Problem? Need more checks

Convergence of power expansion:

A separate and nontrivial issue. Strictly speaking, cannot be convergent $\Longleftrightarrow$ local duality violation

In practice, a different concern: accuracy provided by the first few terms

Upshot: depends on what is studied, and `how`

Quite safe for $\Gamma_{\rm tot}$, including duality violation Sub-% level

Not necessarily for higher moments, and/or with cuts duality violation would first emerge at much higher level extracting the HQ parameters itself used to compute $\Gamma_{\rm sl}(B)$

How to study: differential distributions $\left.\frac{{\rm d}\Gamma}{{\rm d}M_X^2}\right|_{M_X>3\,{\rm GeV}}$

BU 2001

BaBar 2002 (!)

Saturation of sum rules and onset of perturbative regime

Similarly, in moments of distributions (lepton spectra, M_X^2, ...) – in general, of `structure functions`

Even if found significant in distributions, does not imply a noticeable effect for $\Gamma_{\rm sl}$

My opinion: a `potential` concern only if the differential widths/moments would be `totally off`

The only relevant fact is saturation of sum rules (primarily SV)

't Hooft model	Lebed, N.U. 1999
SV regime	Le Yaouanc et al. 2000-01

A cartoon illustration (asymptotics): $\delta_{\rm DV} \propto 1/\mathcal{Q}^7$

$$
\begin{array}{llllll}
\Gamma_{\rm sl} = & M_0 & \longleftrightarrow & \mathcal{Q} = E_r & \delta_{\rm DV} & \\
& M_1 & \longleftrightarrow & \mathcal{Q} = E_r/2 & \delta_{\rm DV}\cdot 2^7 & \\
& M_2 & \longleftrightarrow & \mathcal{Q} = E_r/3 & \delta_{\rm DV}\cdot 3^7 & \text{etc.}
\end{array}
$$

Bottle neck: 'Hardness' too low with the cut on E_ℓ

CLEO's extraordinary accuracy cannot be even nearly used

For total width $\mathcal{Q} \simeq m_b - m_c$ with the cut ?

Generally $\mathcal{Q} \lesssim \omega_{\rm max}$ $\omega_{\rm max}$ is the threshold energy at which the process disappears if $m_b \to m_b - \omega$

In semileptonic decays

$$\mathcal{Q} \simeq m_b - E_{\rm min} - \sqrt{E_{\rm min}{}^2 + m_c^2}$$

This is only about $1.25\,\mathrm{GeV}$ for cut at $E_\ell = 1.5\,\mathrm{GeV}$

and below $1\,\mathrm{GeV}$ for $E_\ell > 1.7\,\mathrm{GeV}$

marginal $\mathcal{Q} \simeq 2\,\mathrm{GeV}$ for $E_\ell > 1\,\mathrm{GeV}$

In $b \to s + \gamma$ $\mathcal{Q} \simeq M_B - 2E_{\rm min} \simeq 1.2\,\mathrm{GeV}$

if the cut is at $E_\gamma = 2\,\mathrm{GeV}$

Reliability of theory is questionable ...

Hardness deteriorates for higher moments

Experiment must strive to lower the cuts ! Bigi, N.U. 2002

$\langle (M_X^2)^k \rangle$ are important to scrutinize HQ parameters

Even without a cut on E_ℓ convergence for $k \geq 2$ is not great...

$\langle (M_X^2)^3 \rangle$ seems a bit too low

Peculiarity of M_X^2 :

$$M_X^2 \equiv (P_B - q)^2 = p_c^2 + 2(M_B - m_b)\,(m_b - q_0) + (M_B - m_b)^2$$

$\mathrm{OPE} \longrightarrow$ parton + small $1/m_b^2$ corrections

Large corrections are traced to $2(M_B - m_b)\,E_c$ rather than to $p_c^2 - m_c^2$

$\mathrm{Cure:}$ use the combinations of M_X^2 and E_X moments, viz.

Trade M_X^2 for

$$\mathcal{N}_X^2 = M_X^2 - 2\tilde{\Lambda} E_x \qquad \text{with} \qquad \tilde{\Lambda} \approx 650\,\mathrm{MeV}$$

Say, $\langle \mathcal{N}_X^4 \rangle - \langle \mathcal{N}_X^2 \rangle^2$:

$$[\langle M_X^4 \rangle - \langle M_X^2 \rangle^2] - 4\tilde{\Lambda}[\langle M_X^2 E_X \rangle - \langle M_X^2 \rangle \langle E_X \rangle] + 4\tilde{\Lambda}^2[\langle E_X^2 \rangle - \langle E_X \rangle^2]$$

`Can you do this?`

Distribution over $\mathcal{N}_X^2$ is a counterpart of E_γ-distribution in $b \to s + \gamma$