Precision Physics with inclusive B decays: A Global Fit

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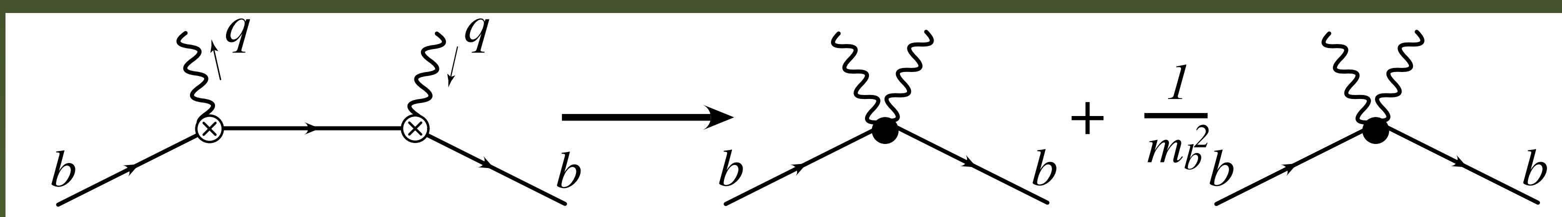
Operator Product Expansion

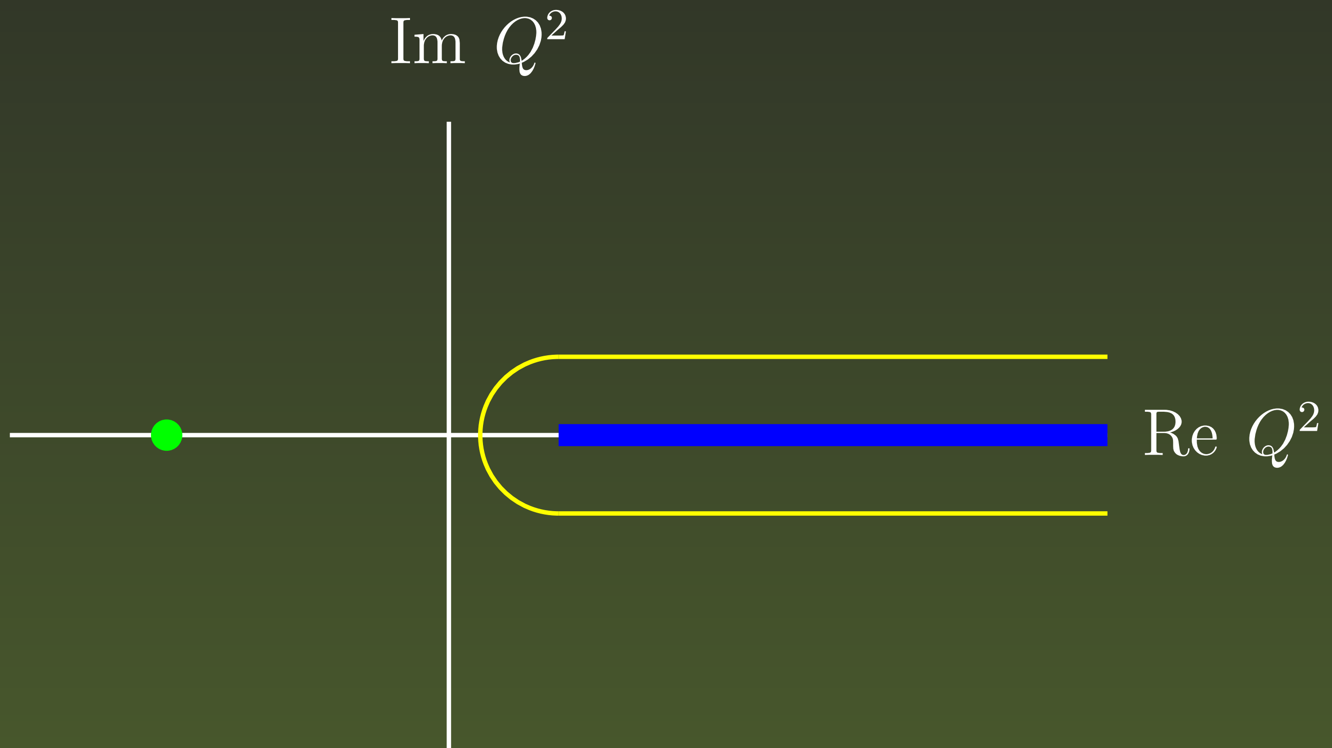
Describe the decay $B \to X\ell\bar{\nu}$ using optical theorem

$$\Gamma \sim \sum_X |\langle B|J^\mu|X\rangle|^2 \sim \int d^4q\, e^{-iq\cdot x}\mathrm{Im}\langle B|T\{J^{\mu\dagger}(x)J^\nu(0)\}|B\rangle$$

If the intermediate state is far off-shell, one can expand in terms of local operators (OPE)

Similar to Deep inelastic scattering or $e^+e^- \to \mathrm{hadrons}$



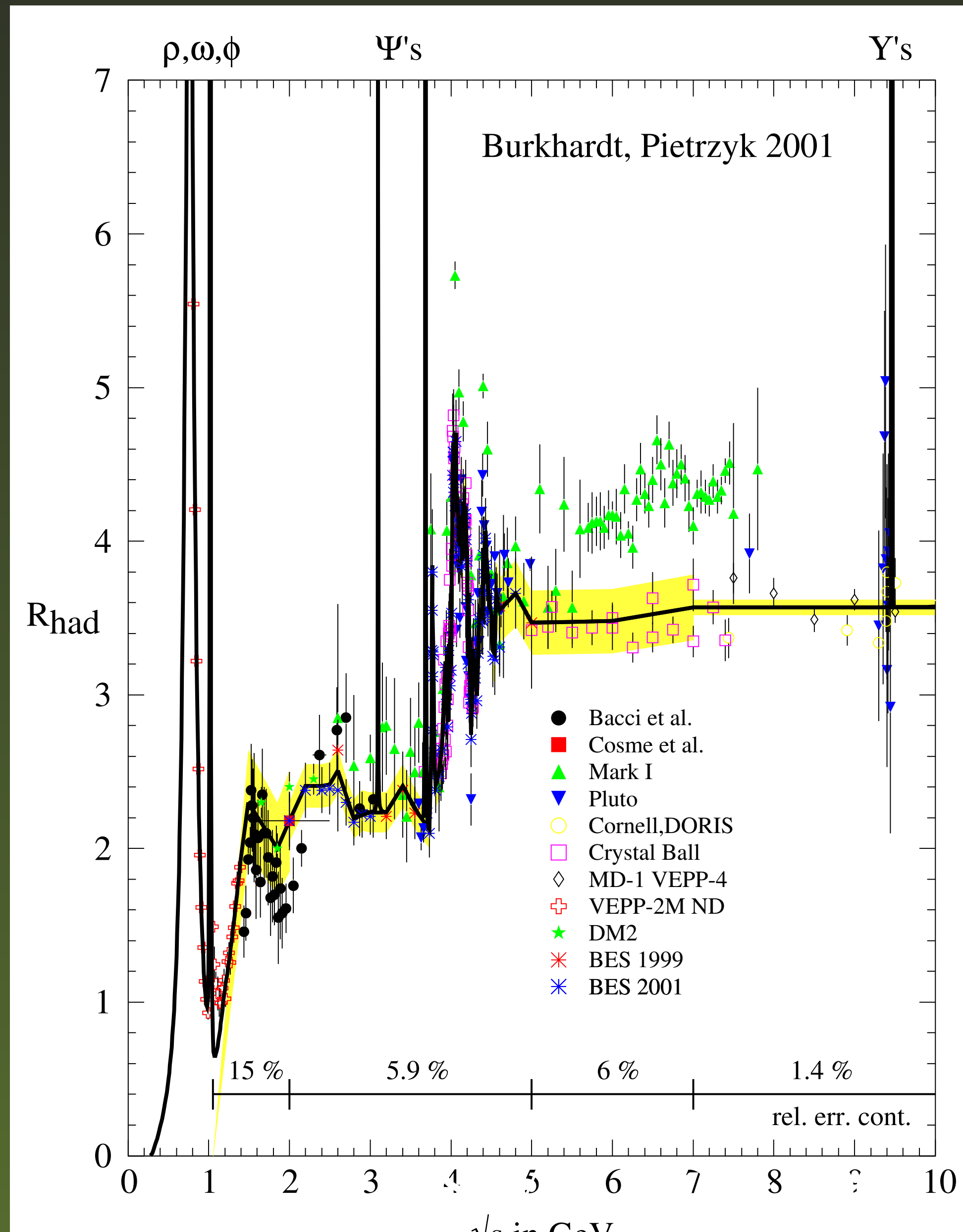


Need the integral over the *entire* physical cut

Assumption of local duality

Local Duality

- To compare OPE results with data, have to smear result
- Smearing has to be over "many resonances"



Inclusive B decays

Typical OPE result for differential spectrum looks like

$$\frac{d\Gamma}{dX} = \frac{d\Gamma_{\rm part}}{dX} + 0\frac{\bar\Lambda}{m_b} f_\Lambda(X) + \frac{\lambda_i}{m_b^2} f_{\lambda_i}(X) + \frac{\rho_i}{m_b^3} f_{\rho_i}(X) + \ldots$$

Typical OPE result for moments of spectra looks like

$$\langle X \rangle = \langle X \rangle_{\rm part} + 0\frac{\bar\Lambda}{m_b} F_\Lambda + \frac{\lambda_i}{m_b^2} F_{\lambda_i} + \frac{\rho_i}{m_b^3} F_{\rho_i} + \ldots$$

Each coefficient function has an expansion in $\alpha_s(m_b)$ and depends on m_c/m_b

Order

Compute to order

$$1, \quad \frac{\Lambda_{\rm QCD}^2}{m_b^2}, \quad \frac{\Lambda_{\rm QCD}^3}{m_b^3}$$

$$\alpha_s, \quad \alpha_{s,BLM}^2$$

For hadronic moments, $\alpha_s \Lambda_{\rm QCD}/m_b$ terms known with no lepton energy cut.

Parameters

All inclusive results given in terms of parameters:

$$V_{cb}, \qquad m_b,\ m_c, \qquad \lambda_1,\ \lambda_2$$

$$\mathcal{T}_1,\ \mathcal{T}_2,\ \mathcal{T}_3,\ \mathcal{T}_4,\ \rho_1,\ \rho_2$$

B and D masses can be used: $m_b,\ m_c \to \bar{\Lambda}$

B^* and D^* masses can be used to fix λ_2 and $\rho_2 - \mathcal{T}_2 - \mathcal{T}_4$

Inclusive results depend on $\mathcal{T}_1 + 3\mathcal{T}_2,\ \mathcal{T}_2 + 3\mathcal{T}_4$

masses depend on $\mathcal{T}_1 + \mathcal{T}_3$ and $\mathcal{T}_2 + \mathcal{T}_4$

$$V_{cb}, \qquad \bar{\Lambda}, \qquad \lambda_1$$

$$\mathcal{T}_1 - 3\mathcal{T}_4,\ \ \mathcal{T}_2 + \mathcal{T}_4,\ \ \mathcal{T}_3 + 3\mathcal{T}_4,\ \ \rho_1$$

Global Fit

- Use more data $\Rightarrow$ reduce uncertainties
- See if there are inconsistencies between different measurements. Allows one to test local duality *experimentally*
- Investigate the effect of theoretical uncertainties
- Include theoretical correlations between different observables
- All quantities are fit using a consistent scheme

Data Used

- Lepton energy moments from CLEO

CLEO ('02)

$$R_0(1.5\,\mathrm{GeV}) = 0.6187 \pm 0.0021$$
$$R_1(1.5\,\mathrm{GeV}) = (1.7810 \pm 0.0011)\,\mathrm{GeV}$$
$$R_2(1.5\,\mathrm{GeV}) = (3.1968 \pm 0.0026)\,\mathrm{GeV}^2$$

R_0 and R_1: e/μ averaged value including correlation matrix, R_2: weighted average of e, μ

- Lepton energy moments from DELPHI

DELPHI ('02)

$$R_1(0) = (1.383 \pm 0.015)\,\mathrm{GeV}$$
$$R_2(0) - (R_1(0))^2 = (0.192 \pm 0.009)\,\mathrm{GeV}^2$$

$$S_1 = m_X^2 - \bar{m}_D^2, \qquad S_2 = \left\langle (m_X^2 - \left\langle m_X^2 \right\rangle)^2 \right\rangle$$

- Hadron invariant mass moments from CLEO

CLEO ('01)

$$\begin{aligned} S_1(1.5\,\mathrm{GeV}) &= (0.251 \pm 0.066)\,\mathrm{GeV}^2 \\ S_2(1.5\,\mathrm{GeV}) &= (0.576 \pm 0.170)\,\mathrm{GeV}^4 \end{aligned}$$

- Hadron invariant mass moments from DELPHI

DELPHI ('02)

$$\begin{aligned} S_1(0) &= (0.553 \pm 0.088)\,\mathrm{GeV}^2 \\ S_2(0) &= (1.26 \pm 0.23)\,\mathrm{GeV}^4 \end{aligned}$$

- Hadron invariant mass moments from BABAR

BABAR ('02)

$$S_1(1.5\,\mathrm{GeV}) = (0.354 \pm 0.080)\,\mathrm{GeV}^2$$
$$S_1(0.9\,\mathrm{GeV}) = (0.694 \pm 0.114)\,\mathrm{GeV}^2$$

- Photon energy moments from CLEO

CLEO ('01)

$$T_1(2\,\mathrm{GeV}) = (2.346 \pm 0.034)\,\mathrm{GeV}$$
$$T_2(2\,\mathrm{GeV}) = (0.0226 \pm 0.0069)\,\mathrm{GeV}^2$$

- Average of semileptonic decay width for B^+ and B^0

PDG ('02)

$$\Gamma(B \to X\ell\bar{\nu}) = (42.7 \pm 1.4) \times 10^{-12}\,\mathrm{MeV}$$

Cannot use data that includes B_s, Λ_b

Mass Schemes

- Pole Mass:
  - Has a renormalon ambiguity of order $\Lambda_{\rm QCD}$
  - Perturbation series poorly behaved
  - The two problems are related, asymptotic nature of perturbation series (i.e. divergent) related to non-perturbative corrections
- $\overline{\rm MS}$ Mass:
- 1S Mass using the upsilon expansion
- Other Schemes: PS Mass, . . . PS mass requires introducing a factorization scale μ_f that enters linearly in the mass: $m_{\rm pole} = m_{\rm PS} + \ldots \mu_f$

Higher Hadron Moments

- Second hadron moment seems to give orthogonal information to most other moments
- Convergence of this moment questioned in literature

Falk, Luke ('97)

$$\langle m_X^4 - \langle m_X^2 \rangle^2 \rangle = 0.73 \frac{\bar{\Lambda}^2}{\Lambda_{\rm QCD}^2} - 0.96 \frac{\lambda_1}{\Lambda_{\rm QCD}^2} - 0.56 \frac{\rho_1}{\Lambda_{\rm QCD}^3} + \ldots$$

(in units of GeV^4)

- From dimensional analysis

$$\frac{\langle m_X^4 - \langle m_X^2\rangle^2\rangle}{m_B^4} = \mathcal{O}(1)\,\frac{\bar\Lambda^2}{\bar m_B^2} + \mathcal{O}(1)\,\frac{\lambda_1}{\bar m_B^2} + \mathcal{O}(1)\,\frac{\rho_1}{\bar m_B^3} + \dots$$

- Breakdown of OPE: some coeffs $\gg \mathcal{O}(1)$
- The previous expression is

$$\frac{\langle m_X^4 - \langle m_X^2\rangle^2\rangle}{m_B^4} = 0.1\,\frac{\bar\Lambda^2}{\bar m_B^2} - 0.14\,\frac{\lambda_1}{\bar m_B^2} - 0.86\,\frac{\rho_1}{\bar m_B^3} + \dots$$

- Large cancellation in $\bar\Lambda$ and λ_1 term

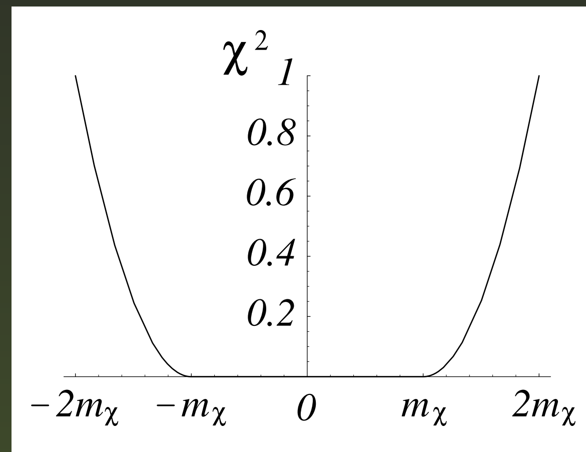
Moment is well behaved, but sensitive to ρ_1

Theoretical Uncertainties

Originate from unknown higher order terms in expansion

- Unknown $1/m_b^3$ matrix elements
  - generic size $\Lambda_{\rm QCD}^3$
  - There is no favorite value
  - In our fits we add

$$\Delta\chi^2(m_\chi, M_\chi) = \begin{cases} 0\,, & |\langle\mathcal{O}\rangle| \le m_\chi^3 \\ \frac{\left[|\langle\mathcal{O}\rangle| - m_\chi^3\right]^2}{M_\chi^6} & |\langle\mathcal{O}\rangle| > m_\chi^3 \end{cases}$$



We vary $0.5\,\mathrm{GeV} < m_\chi < 1\,\mathrm{GeV}$ and take $M_\chi = 0.5\mathrm{GeV}$

- Uncomputed higher order terms
  - Fractional Error
    - $(\alpha_s/4\pi)^2 \sim 0.0003$
    - $(\alpha_s/4\pi)\Lambda^2_{\rm QCD}/m_b^2 \sim 0.0002$
    - $\Lambda^4_{\rm QCD}/(m_b^2 m_c^2) \sim 0.001$
  - We use

$$\sqrt{(0.001)^2 + (\text{last computed}/2)^2}$$

The Result

- One fit including and one fit excluding BABAR data
- This allows to investigate effect of BABAR data

m_χ [GeV]	χ^2	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	5.0	40.8 ± 0.9	4.74 ± 0.10
1.0	3.5	41.1 ± 0.9	4.74 ± 0.11
0.5	12.9	40.8 ± 0.7	4.74 ± 0.10
1.0	8.5	40.9 ± 0.8	4.76 ± 0.11

- BABAR data makes fit considerably worse
- More on this later

Error analysis

- Best estimate of perturbative uncertainties
- Best estimate of uncomputed $1/m^4$ and α_s/m^2 terms
- Very conservative estimate of $1/m^3$ uncertainties
- All publically available experimental uncertainties

Not included:

- Unknown experimental correlations
- Uncertainties from “Duality violations”

More on Theoretical Error

- $1/m_b^3$ uncertainty

m_χ [GeV]	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	40.8 ± 0.9	4.74 ± 0.10
1.0	41.1 ± 0.9	4.74 ± 0.11

- Theoretical correlations

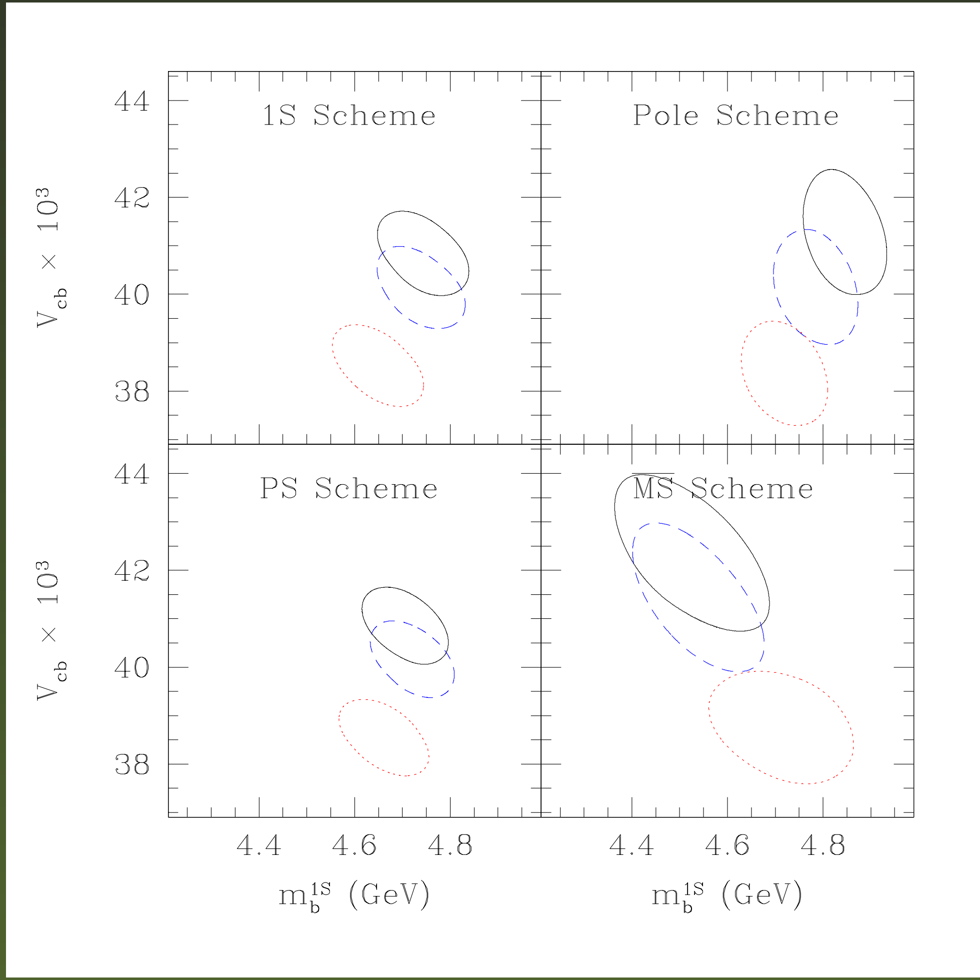
$\delta(\lambda_1)$	$\delta\left(\lambda_1 + \frac{\mathcal{T}_1+3\mathcal{T}_2}{m_b}\right)$
± 0.38	± 0.22

- Theoretical limitations

$\delta(\lvert V_{cb}\rvert) \times 10^3$	$\delta(m_b^{1S})$ [MeV]
± 0.35	± 35

Different mass schemes

tree level, order α_s, order $\alpha_s^2\beta_0$



Better convergence for 1S and PS scheme

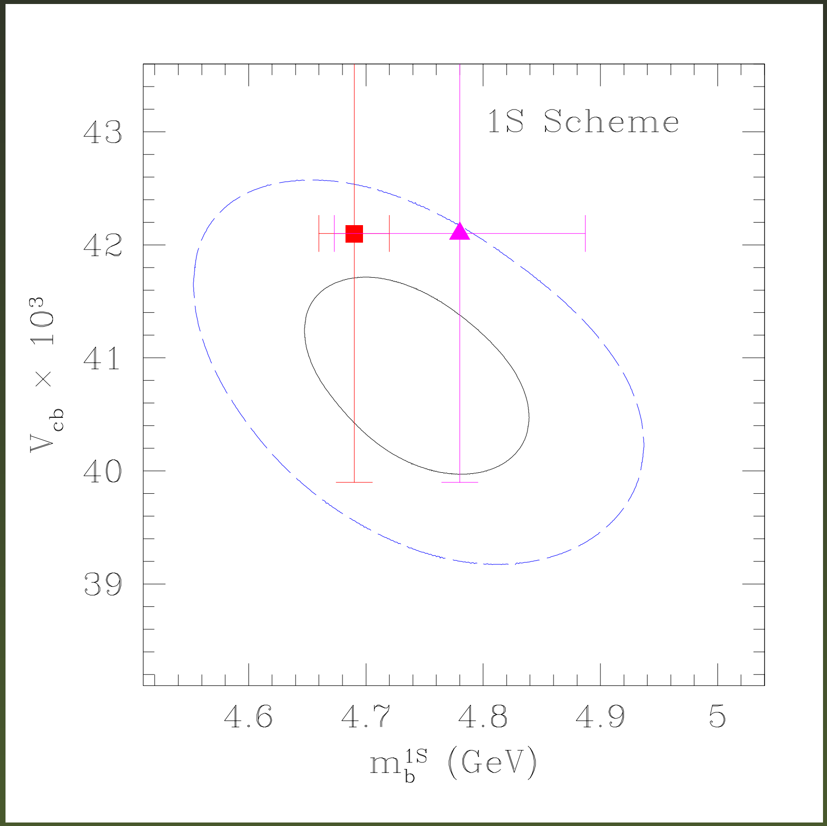
Experimental correlations

How important are experimental correlations?

- Increase all errors (except Γ_{sl}) by 2

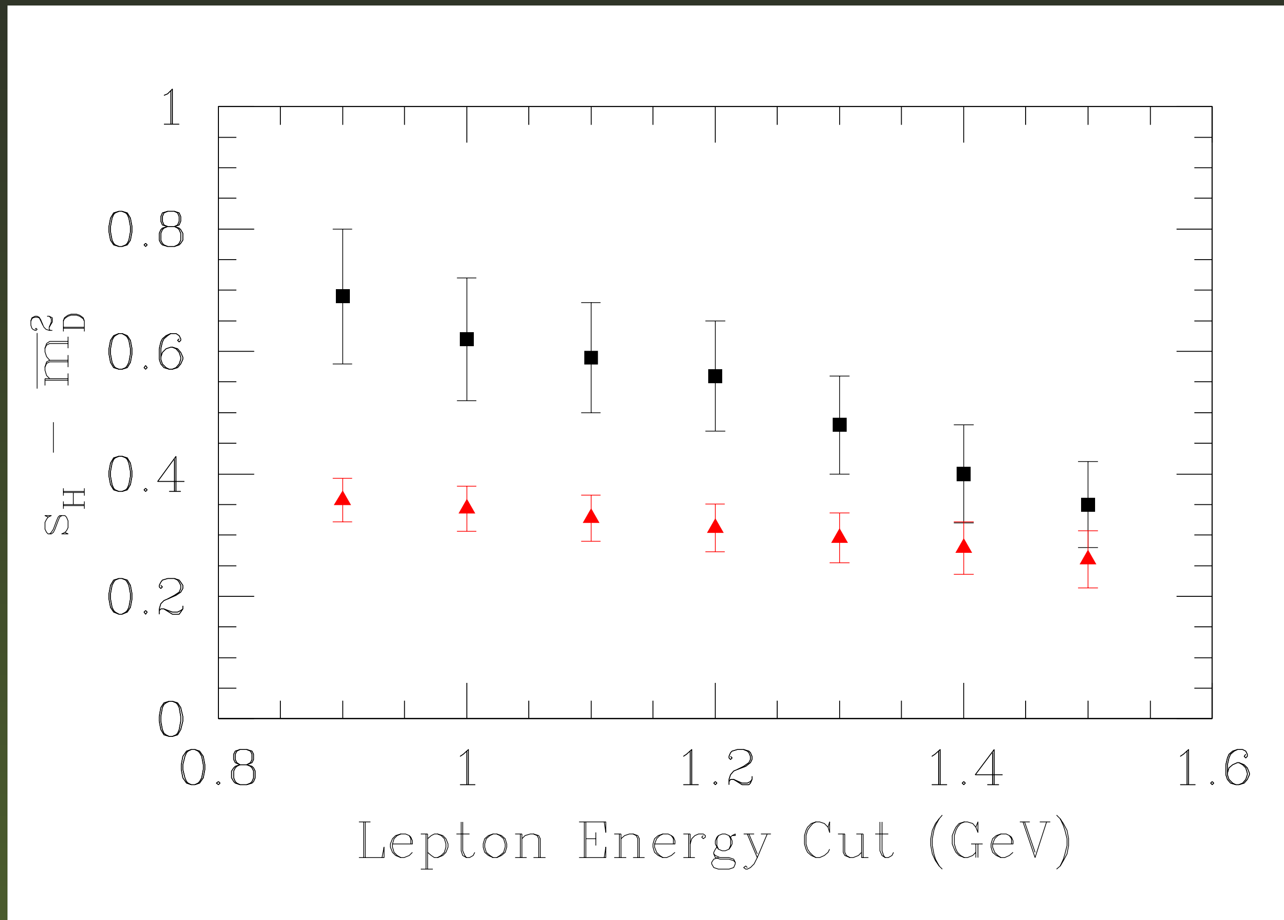
	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
Original Fit	40.8 ± 0.9	4.74 ± 0.10
2 $\times$ errors	40.8 ± 1.2	4.74 ± 0.24

Result once again



$$
\begin{aligned}
|V_{cb}| &= (40.8 \pm 0.9) \times 10^{-3} \\
m_b^{1S} &= (4.74 \pm 0.10)\,\mathrm{GeV} \\
\overline{m}_b(\overline{m}_b) &= 4.22 \pm 0.09\,\mathrm{GeV}
\end{aligned}
$$

Babar hadronic moment

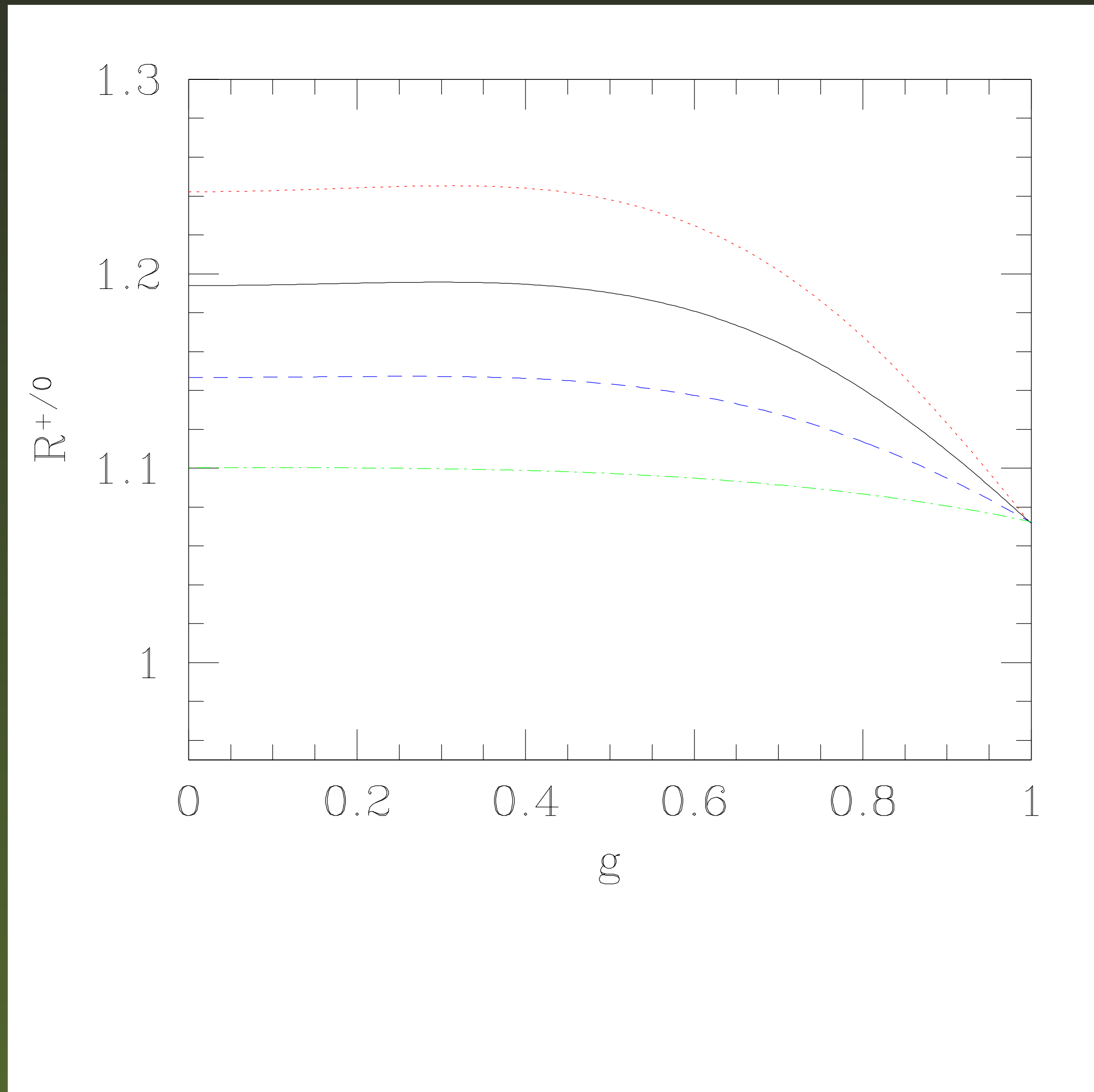


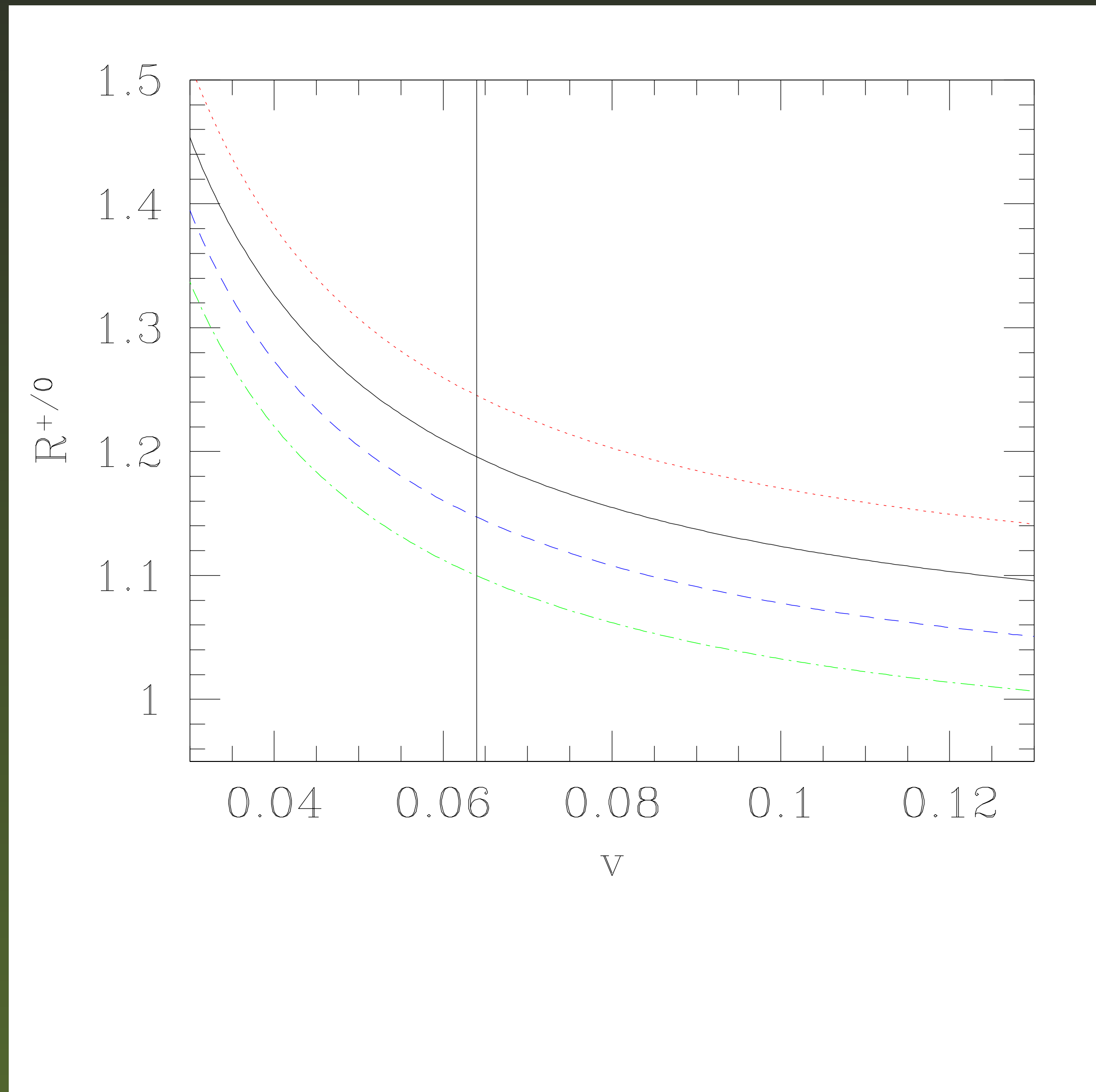
Significant disagreement with our fit results

Assume no non-resonant contribution between D^* and D^{**}. Then find excited D states contribute less than 25% in $B \to X_c e\nu$ decays.

Experimentally, $\sim$ 36%

B^+/B^0 Production Ratio




1.5
1.4
1.3
1.2
1.1
1
$R^{+/0}$
0.04
0.06
0.08
0.1
0.12
v