$B \to X_s \ell^+ \ell^-$: Exclusive Modes

- FCNC decays such as $B \to (K, K^*, \rho)\ell^+\ell^-$, $B \to (K^*, \rho)\gamma$, etc.; as well as Semileptonic modes $B \to (\pi, \rho)\ell\nu$, ..., are affected by large theoretical uncertainties from hadronic form-factors.
- Model-independent statements:
  - *Symmetries*: in the HQL and LEL.
  - *Lattice Computations*: practical at low recoil energies.
- Question: How do we make use of these ?
  The case of $B \to K^* \ell^+ \ell^-$.

Heavy Quark Symmetry and $B \to K^* \ell^+ \ell^-$

Amplitude in terms of form-factors:

- "Semileptonic" type (from $\mathcal{O}_9, \mathcal{O}_{10}$):

$$\langle K^*(\mathbf{k}, \epsilon)|\bar{s}_L \gamma_\mu b_L|B(\mathbf{p})\rangle => \begin{cases} V(q^2), A_1(q^2) \\ A_2(q^2), A_0(q^2) \end{cases}$$

- "Photonic" type (from $\mathcal{O}_7$):

$$\langle K^*(\mathbf{k}, \epsilon)|\bar{s}_L \sigma_{\mu\nu} b_R|B(\mathbf{p})\rangle => \{T_1(q^2), T_2(q^2), T_3(q^2)$$

- But in the Heavy Quark Limit, $\{T_i\} \leftrightarrow \{V,\ A_1,\ A_2,\ A_0\}$. E.g. measurement of $B \to \rho \ell \nu$ => prediction for $B \to K^* \ell^+ \ell^-$ (assuming $SU(3)$).

The lepton $\boxed{\text{Forward-Backward Asymmetry}}$

$$A_{FB}(s) \equiv \frac{\int_{-1}^{0} dx \, \frac{d\Gamma}{dx ds} - \int_{0}^{1} dx \, \frac{d\Gamma}{dx ds}}{\frac{d\Gamma}{ds}}$$

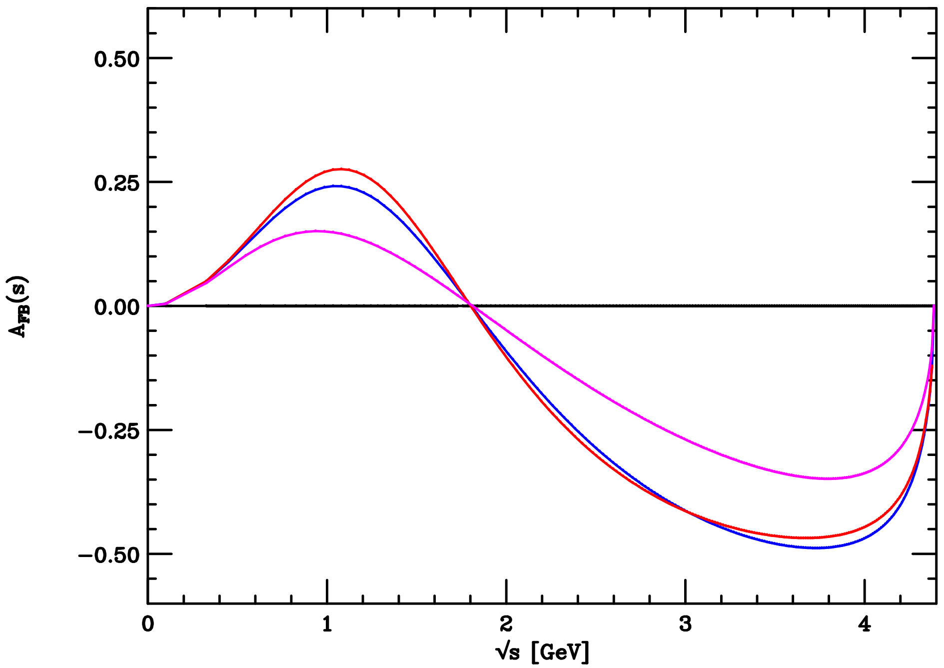
where $x \equiv \cos\theta$, θ angle between $\bar{B}$ and ℓ^+ in the $\ell^+\ell^-$ rest frame, and $s = q^2 = m^2_{\ell^+\ell^-}$.

Using the HQS relations, $\boxed{A_{FB}(m_{\ell\ell}) = 0}$ =>

$$\frac{\bar{C}_9}{\bar{C}_7} = -\frac{m_b}{s_0}\left\{\frac{2m_B\,k^2}{(m_B + m_{K^*})^2} R_V + \frac{(m_B + m_{K^*})^2}{2m_B R_V} + 2(m_B - E_{K^*})\right\}$$

with $R_V \equiv V/A_1$ evaluated at s_0.

$\ast$ The ratio R_V is very stable across models!



$\longrightarrow$ BSW*: Bauer, Stech and Wirbel, Z. Phys. C29, 637 (1985); Stech, Phys. Lett. B354, 447 (1995).

$\longrightarrow$ LCSR: Ball and Braun, Phys. Rev. D55, 5561 (1997).

$\longrightarrow$ MNS: Melikhov, Nikitin and Simula, Phys. Lett. B410, 210 (1997).

Large Energy Limit

(*Dugan and Grinstein '91, Charles et al. '99.*) => Additional relations among matrix elements. All 7 form-factors can be written in terms of 2 functions: $\xi_\perp(m_Q, v\cdot p)$ and $\xi_\|(m_Q, v\cdot p)$.

For $B\to K^*\ell^+\ell^-$, the "semileptonic" form-factors are:

$$
\begin{aligned}
V(q^2) &= \left(1+\frac{m_{K^*}}{m_B}\right)\xi_\perp \\
A_1(q^2) &= \frac{2E}{m_B+m_{K^*}}\xi_\perp \\
A_2(q^2) &= \left(1+\frac{m_{K^*}}{m_B}\right)\left\{\xi_\perp-\frac{m_{K^*}}{E}\xi_\|\right\}\ , \\
A_0(q^2) &= \left(1-\frac{m_{K^*}^2}{m_B E}\right)\xi_\|+\frac{m_{K^*}}{m_B}\xi_\perp\ ,
\end{aligned}
$$

and the "photonic" form-factors obey:

$$
\begin{aligned}
T_1(q^2) &= \xi_\perp \ , \\
T_2(q^2) &= \left(1 - \frac{q^2}{m_B^2 - m_{K^*}^2}\right) \xi_\perp \ , \\
T_3(q^2) &= \xi_\perp - \frac{m_{K^*}}{E}\left(1 - \frac{m_{K^*}^2}{m_B^2}\right) \xi_\parallel \ .
\end{aligned}
$$

In LEL we now have

$$R_V = \frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_{K^*})^2}{2m_B\, E_{K^*}}$$

An effective field theory formulation => include collinear gluons
SCET (*Bauer, Fleming, Pirjol, Stewart, ...'00-'02*)

R_V in the Large Energy Limit

What is the physical interpretation?

Transverse Helicity Amplitudes in $B \to V\ell\ell'$

$$H_\pm = \mathcal{F}\left(V(q^2) \mp \frac{(m_B+m_V)^2}{2m_B\, k_V} A_1(q^2)\right)$$

=> The LEL prediction

$$R_V = \frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_{K^*})^2}{2m_B\, E_V}$$

gives

$$H_+ = 0 + \mathcal{O}(\frac{m_V^2}{m_B^2}) + \ldots$$

In the HQ and LE limits the helicity of the fast light quark is inherited by the vector meson! This is built in in many model calculations.

$V(0)$ and $A_1(0)$ from $B \to K^*\gamma$ and LEL (*G.B., G. Hiller, '00*)

- "Semileptonic" form-factors $V(q^2)$ and $A_1(q^2)$ in $B \to K^*\ell^+\ell^-$ (same as the ones in $B \to \rho\ell\nu$ up to $SU(3)$ corrections.)

- $B \to K^*\gamma$ is determined by the "tensor" ($\sigma_{\mu\nu}$) form-factor $T_1(0)$:

$$\frac{Br(B \to K^*\gamma)}{\tau_B\,|V_{cb}V_{cs}^*|^2} = \frac{\alpha\,G_F^2}{32\pi^4}\,m_b^2\,m_B^3\,(1 - \frac{m_{K^*}^2}{m_B^2})^3\;|A_7|^{\,2}\;|T_1(0)|^2$$

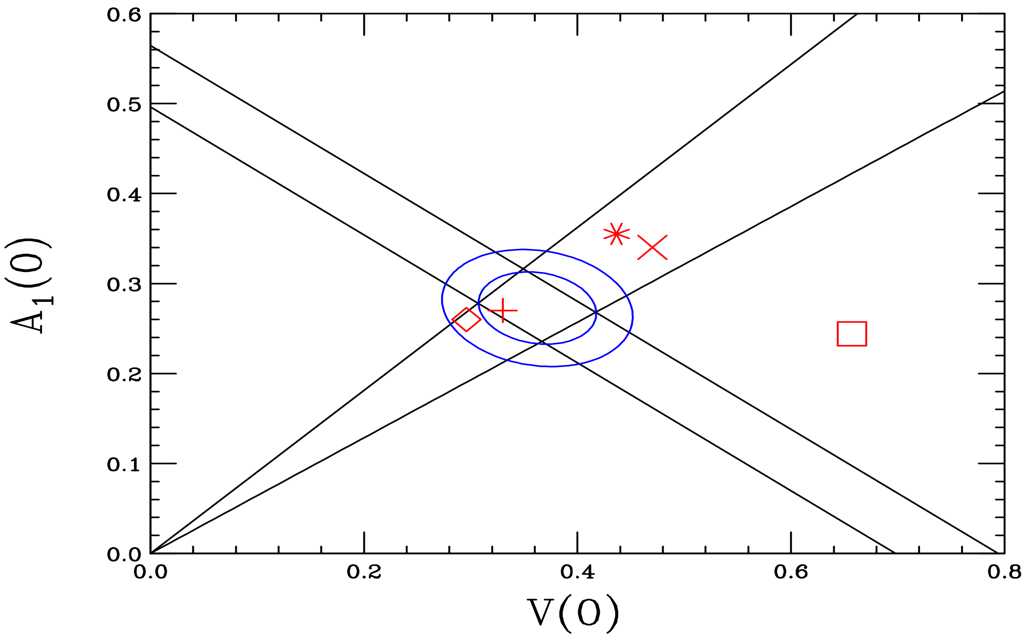
- Heavy Quark *spin* Symmetry (eg. $\not{v}b(x) = b(x)$) relates them through (evaluating at $q^2 = 0$)

$$\boxed{A_1(0) = \frac{2m_B}{m_B + m_{K^*}}\,T_1(0) - \frac{m_B - m_{K^*}}{m_B + m_{K^*}}\,V(0)}$$

=> band on the $(V(0), A_1(0))$ plane.

- LEL imposes a value on the ratio $\boxed{\frac{V(0)}{A_1(0)} = R_V = \frac{(m_B + m_{K^*})^2}{2m_B\,E_{K^*}}}$

$V(0)$ and $A_1(0)$ from $B \to K^*\gamma$ in the LEL



- $+$ BSW (Bauer, Stech, Wirbel '85)
- $\diamond$ Isgur, Scora '95
- $*$ Melikhov, Stech '00
- $\times$ Ali, Ball, Handoko, Hiller '00 (LCSR)
- $\square$ Ligeti, Wise '99 (HQS+$D \to K^*$ +monopole)

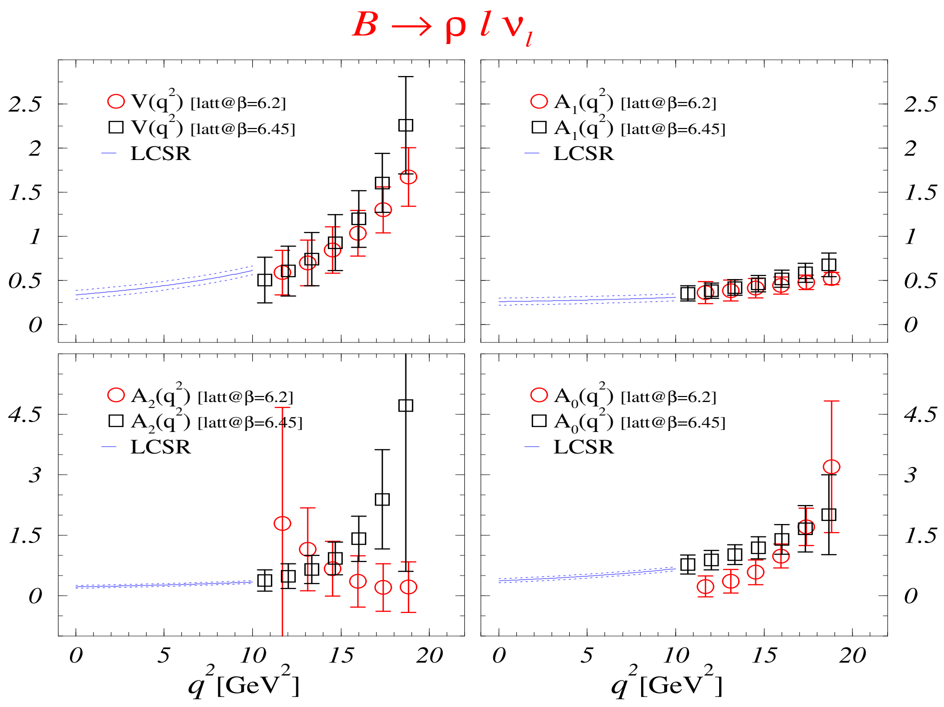
The fit gives

$$\boxed{V(0) = 0.36 \pm 0.04 \quad ; \qquad A_1(0) = 0.27 \pm 0.03}$$

Lattice Calculations

Lattice computations: at low recoil or large q^2.

E.g.: SPQcdR Collaboration (*hep-lat/0209116*)



Strategy

- For $V(q^2)$ and $A_1(q^2)$ we can interpolate between lattice at low recoil and LEL/HQL at high recoil.
- Parametrization used should respect HQL scaling rules at low recoil. For V it must include B^*-pole dominance at low recoil.
- $A_2(q^2)$? It enters in the longitudinal polarization. Can we use $B \to \rho\ell\nu$? $SU(3)$ breaking might not be too large if one extracts ratios of form-factors.

For example:

$$V(q^2) = \frac{a}{(1 - q^2/m_{B^*}^2)} \left(\frac{M_1^2 - m_{B^*}^2}{M_1^2 - q^2} \right)$$

with $M_1 \simeq (6 - 6.2)$ GeV, $a \simeq 1.4$.

Conclusions

- We should generate sensible parameterizations of the form-factors to be used as interpolations between low and high recoil. We can already do this for $V(q^2)$ and $A_1(q^2)$.
- We should carefully attach errors to this procedure (e.g. propagation of lattice errors, $1/M$, $1/E$ corrections, α_s, etc.).