Comparative Surveys and Successive Realignments: Useful Tools and Significant Estimates

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Scope of this review

- Only long linear/curvilinear networks or “objets” will be considered here
  - In which the “absolute” geometry is significantly biased by cumulated errors ⇒ global geometrical comparisons (congruence models) are no longer valid
- “Ground” motion? “Universal” laws?
  - Inhomogeneous complex {structures+ground}, with different sectors of coherent behaviour + singularities
Vertical Surveys (levelling) : random errors

“random walk” errors and final discrepancy («misclosure»)

\[ \sigma_{\text{rms}} = \sigma (L_{\text{km}})^{\frac{1}{2}} \]

(random walk)

Vertical Surveys (levelling) : systematic errors

- Repetitive, proportional to Height Differences
  - (geometrical) scale errors of staves ⇒ calibrate
  - “zero” of staves ⇒ compare/alternate or adjust
- Repetitive, proportional to other factors
  - Earth’s magnetic field ⇒ now cured by shielding
- Irregular
  - temperature and strong field effects on electronics
Horizontal Surveys: combined effects of all errors

- The overall changes of curvature are induced by errors (random & systematic) in distances and/or in radial measurements, and also by those of control points;
- The general pattern - i.e. the polynomial order of the trend curve - depends on the scheme and quality of the measurements (redundancy, accuracy, overlaps);
- Local dispersion (smoothness) depends on resulting short range errors;
Horizontal Surveys: how to avoid some errors and deformations...

- Take all possible cares for fighting systematic errors: suppress their sources or find parry (if any) ⇒ sequential checks and calibrations, forced-centring, cancellation by symmetries in the measurement scheme, record of observation conditions and related data, environmental control (if manageable), etc.
- Make Monte Carlo simulations in which all possible systematic errors are progressively combined with random ones, analyse geometrical and statistical effects, compare the resulting empirical estimates to the “theoretical” ones (issued from variance & covariance analysis) and observe their change under such various successive constraints: this is the best way to analyse the real computation of actual measurements and to identify “warning signals”...
- Find adequate actions for avoiding some common sources of deformation - like conflicting measurements of unbalanced weights, weak parts in measurement structure & scheme, overconstrained fit on control points, etc.

Comparing successive surveys: general problem

Successive surveys of a perfectly rigid and stable object:
if random errors only, all “wrong” lines have the same likelihood to be true...

Stable traverse or arc AB of length $L_{km}$

No deformation signal here, only measurement noise (random errors);
Beware of biased conclusions...
Comparing successive surveys:
what is it looked for?

- Seeing & correcting true deformations:
  "ad hoc" local comparisons

- Restoring a functional alignment and correcting singular movements:
  smoothing process

- Assessing the alignment decay:
  statistics on alignment changes

Comparing surveys at epochs $T_i, T_j$:
do not be lured by error laws...

- Local geometrical & statistical approach
  - locate the deformation area, look for stable points each side
  - $\Rightarrow$ translate / rotate / evaluate / valid or reject

- Local algebraic approach
  - Express both trend curves and then "map" survey $T_j$ on survey $T_i$ from polynomial difference

- Long range deformations or movements
  - Very delicate to assess: stochastic analysis + tests
Comparing successive surveys: “datum-free” deformation pattern

- 1st derivative $\frac{\delta x}{\delta L}$ on successive pairs
  - azimuth or slope difference of a segment during the time interval of two epochs: “angular” movement (1)

- 2nd derivative / interval $\Delta L$
  - change of shape (rad/m) $\Rightarrow$ deformation gradient (2)

- Plot (1) and (2) $\Rightarrow$ cf. work and paper of F. JIN

- Find “ad hoc” analysis tools of deformation patterns
  - analysis through a sliding window for locating singularities and characterising/comparing homogeneous sectors $\Rightarrow$ future work

Correcting singular movements and restoring a functional alignment

- “smoothing” process around trend curves
  - over selected areas or within a sliding window
  - parametric, non parametric - or mixture of both

- acceptance bandwidth and corrections
  - all points beyond the chosen threshold are brought back to the trend curve $\Rightarrow$ iterative optimisation of the alignment

- Final quality of the smoothing
  - mathematics, window size, acceptance bandwidth
Assessment of the alignment decay
from successive sequences [survey + smoothing]

- **Alignment Smoothness Quality:**
  - ASQ (mm, $1\sigma$) = dispersion around the trend curve - at alignment/realignment epoch $T$
  - Additional estimates: min-max, adjacent max, rms difference on triplets or quintuplets

- **Alignment Decay Speed:**
  - ADS (mm/year or m/s): arithmetical difference of ASQ ($1\sigma$) between two epochs ($T + \Delta t - T$)

Direct statistical estimates
from successive vertical or radial data sets

- **ATL law**
  - Biased by $F(L)$ errors on measurements $\Rightarrow$ the interpretation is confusing

- **Is a normalised $AT(L)$ law conceivable?**
  - Movements are not correlated with $L$, except locally in deformation bumps or dips;
  - Dispersion changes depend on various physical and geomechanical factors ... but long homogeneous sectors may have the same behaviour;
Initial assumptions and levelling models for simulations

- Spatial data: straight line, length 10 km
  - 201 points, interval ΔL=50 m, dx=0 at initial state
- Dynamic model M (10 data sets correlated with time):
  - 10 epochs “i” (each = 1 year), dispersion growth of <dx>:
    \[ \sigma^2 = + (0.1)^2 \text{ per year} \Rightarrow \text{NOT CORRELATED WITH L} \]
- Noise model N (10 independent data sets):
  - Levelling “random walk” errors in 10 independent sets
    \[ \Rightarrow \text{one pseudo measurement per year.} \]
- Observation model: \( O = M + N \Rightarrow \Delta T_{\text{max}} = 9 \text{ years} \)

Loops or adjusted traverses: change in N(L) noise models

Open traverse: random walk, i.e. parabolic error law (rms.)
Closed loop or adjusted traverse: elliptic error law (rms.)

Midway reduction factor (Mm/Mm') \( \approx 0.7 \) for AB=10 km
(\( \text{NB. Curves very close over the 2 first kilometres} \))
Observed successive states

\[ O = M_{\text{movement}} + N_{\text{noise}} \]

- Successive dispersion values of M with \( d\sigma^2 = + (0.1)^2 \) per year:
  - State at T0: \( \sigma_0^2 = 0.1^2 \text{mm}^2 \) (dispersion of the initial alignment)
  - Survey T1: \( \sigma_1^2 = \sigma_0^2 \text{mm}^2 + d\sigma^2 \text{mm}^2 = 2 \times 0.1^2 \)
  - Survey T2: \( \sigma_2^2 = \sigma_0^2 \text{mm}^2 + d\sigma^2 \text{mm}^2 + d\sigma^2 \text{mm}^2 = 3 \times 0.1^2 \)
  - Survey Tn: \( \sigma_n^2 = \sigma_0^2 \text{mm}^2 + n \times d\sigma^2 \text{mm}^2 = (n+1) \times 0.1^2 \)
- For any point l or k in each survey, considered at epochs T(i+j) and T(i), the \( x \) value observed is made of:

\[ x_i + \text{Movements (} j \text{ additive moves)} + \text{Noise (measurement errors)} \]

Deriving the dispersion growth:

**tentative breakdown of ATL (1)**

- **Movements alone in points k and l between 2 epochs**
  - Geometry: \( dx_{i,k}(i+j-i) = dx_{i,k}(i+j) - dx_{i,k}(i) = x_i(i+j) - x_i(i) - x_k(i+j) + x_k(i) \)
  - Statistics: \( \sum <dx_{i,k}^2>(i+j-i) = j \times d\sigma^2 + j \times d\sigma^2 = 2j \times d\sigma^2 \)
  - because random movements in k and l are only correlated with time
- **Noise alone in k and l** (with same measurement scheme and error laws)
  - Geometry: \( dx_{i,k}(i+j-i) = dx_{i,k}(i+j) - dx_{i,k}(i) = \{ x_i(i+j) - x_k(i+j) \} - \{ x_i(i) - x_k(i) \} \)
  - Statistics: \( \sum <dx^2>(i+j-i) = \sigma^2_{\Delta L} + \sigma^2_{\Delta L} = 2 \times \sigma^2_{\Delta L} \)
  - because measurement errors in k and l are only correlated with space;
Deriving the dispersion growth: tentative breakdown of ATL (2)

- Processing the Observation model $O = M + N$ on all pairs $(k,l)$ vs. $(i+j,j)$

  **Geometry:** $O(i+j-i) = O(i+j) - O(i) = \{M(i+j) - M(i)\} + 2x\{N(\Delta L)\}$

  **Statistics:** $\Sigma<dx^2>_{obs. (i+j-i)} = ATL = 2x\times\sigma^2 + \Sigma2x\sigma^2_{(\Delta L)}$

- Cumulated squared differences over total time:
  
  \[ ATL = 2xM + 2xN = 2x(M+N) \Rightarrow M = 0.5 \times (ATL - 2xN) \]

  *NB. Not yet fully verified numerically with simulations…*

- Derived parameters of $ATL = 2(M+N) = 2x[\sigma^2(T) + N(L)]$
  
  $ATL / T = 2(\sigma^2 + N(L)/T) \Rightarrow$ supposed to be a function of $L$

  $ATL / L = AT = 2(\sigma^2(T)/L + 0.2\times10^2) \Rightarrow$ supposed to be a function of $T$

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Application to levelling data:

**ATL from $O_{observed} = M(T) + N(L)$**

![Graph showing ATL from O=M+N model](image)
Comments on ATL applied to geometrical levelling data

- ATL plot gives a $F(L&T)$ picture of the dispersion growth, altered by the addition of random errors and by an overall scale factor (value to be checked);

- For getting $2M$, one has to derive $(ATL - 2N)$ one way or the other... The error law $N(L)$ is known and values can be taken into account for each difference observed;

- In our assumptions, the ratio $K(T) = ATL/2N(L)$ is a constant factor of proportionality, easy to average while summing ATL values...

\[ K(T) = \frac{2[M(T)+N(L)]}{N(L)} = \frac{\text{(signal + noise)}}{\text{(noise)}} \]

$\Rightarrow$ accumulating + averaging $ATL/\Delta L$ and plotting as $F(\Delta T)$ would already be a less confusing approach for levelling data.

Normalising the process for getting the time-dependent diffusion

- Covariance function $1/2N(\Delta L) \ (s/n \ ratio)$ for normalising & averaging:

\[ \frac{\sum dx^2/2\sigma_{\Delta L}^2}{\text{number of occurrences for each independent } \Delta T \ (10-1, 9-1, \text{etc.})} \Rightarrow \text{No more correlation with } L \text{ or } R, \text{ average } K(T) \text{ value obtained} \]

$NB. \ An \ additional \ weighting \ (according \ to \ \Delta T=j \ in \ the \ loops) \ may \ be \ considered \ for \ optimising \ the \ process...$

- For keeping rigorous in this normalisation, the $N(\Delta L)$ errors might be derived from the variance-covariance matrix of the final least-squares adjustment

- From $ATL=2(M+N)$ and $K=ATL/2N$, one finally gets $M=(K - 1)N$

- Then the time-dependent diffusion (i.e. the alignment decay) can be finally derived at the end of the process: $\alpha(T) = M/\Delta T$
ATL versus a normalised $N_{ATL}$ process on levelling (with $N(L)=0.4^2xL$)

Time-dependent diffusion $\alpha(T)$ (alignment decay) versus ATL

Zoom around the origin showing the various parameters

(NB. Observe that $ATL/L$ is nearly equivalent to $2\alpha(T)$)
Conclusions

- Maintenance surveys are a necessary process for restoring a smooth geometry whenever required, and all derived parameters (like alignment decay) are reliable and useful;

- Direct statistical estimates on integrated data sets (keeping memory of movements) are a bit confusing and more delicate to handle - but they can give sufficient approximate values and trends in global assessments.
  - F(L) biases must be removed by normalising the process.
  - One would also gain in considering and analysing systematic occurrences is these data
  - For the future, in a fully automatic alignment control, they can be an interesting tool for linear colliders.