Beam Physics Near the Cathode
\or
Slice Emittance Growth due to Self-Forces

W. S. Graves
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NSLS
Outline

1) Define emittance and emittance growth.
2) Describe what causes growth.
3) Simple calculations to show scaling with charge, gradient.
4) Time dependence.
5) Examples for LCLS beam.
6) L-Band vs S-band
7) PARMELA simulations.
Slice Emittance

Thin slice

Electron Bunch

\[ \varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \]

\[ x' = \frac{dx}{ds}, \quad s = \text{beamline dist.} \]

\[ \langle x^4 \rangle = \frac{\int x^2 f(x, x') \, d^3r \, d^3r'}{\int f(x, x') \, d^3r \, d^3r'} \]

Integrate over short longitudinal distance \( \Delta z \ll L \)
Emittance Growth

\[ \varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle} \]

\[ \varepsilon' = \frac{d\varepsilon}{ds} = \frac{i}{\varepsilon} \left[ \langle x' x'' \rangle \langle xx' \rangle - \langle xx' \rangle \langle xx'' \rangle \right] \]

Note: \[ \frac{d}{ds} \langle x'^2 \rangle = 2 \langle xx' \rangle \]

Moments involving "force" \( \langle x'' \rangle \)
not generally known.

We desire \( \varepsilon' = 0 \) which requires
\[ \langle x'^2 \rangle \langle xx'' \rangle = \langle xx' \rangle \langle xx'' \rangle \]
This is not true in general. It requires forces that vary linearly in \( x \).

\[ x'' = \pm kx \] - OK for applied forces
- Not true for self force.
**Nonlinear Self-Forces**

For long, uniformly populated bunch self-force is linear.

Nonlinearities arise due to:
1) Short pulses \( (2a \ll L) \)
2) Non uniform cathode emission
3) Laser nonuniformities
Non uniformities

Slow risetime on laser pulse.

Desired pulse shape

Hot spots and cold spots due to:
- dirty or damaged laser optics
- polycrystalline metal cathodes
Beam nonuniformities cause growth in slice emittance.

For a given RMS beam size, the uniform distribution requires the minimum amount of work to create.

It takes additional work to create nonuniformities.

Beam self-forces cause the nonuniformities to smooth out. The excess energy is converted to kinetic energy = emittance growth.
**Short Pulses**

Near end of bunch, $F_x$ is no longer linear even for uniform cylinder.

Short bunch (L ≤ 2a) is "all end".
Time scale of emittance growth.

Choose slice far from end, use Gauss’ and Ampère’s laws.

Gauss: \[ \oint \bar{E} \cdot d\mathbf{A} = \frac{Q_{encl}}{\varepsilon_0} \]

\[ E_x = -\frac{en}{2\varepsilon_0} x, \quad x \leq a \]

\[ n = \frac{N}{\pi a^2 L} \]

Ampère: \[ \oint \bar{B} \cdot d\mathbf{S} = \mu_0 I_{encl} \]

\[ B_y = \frac{neun}{2} \beta e x \]
Lorentz: \[ F_x = -eE_x - e\beta c B_y \]

\[ F_x = \frac{e^2 n}{2e_0} (1 - \beta^2) x \]

\[ F_x = \frac{e^2 n}{2e_0 \gamma^2} x \]

\[ F_x = \gamma m \ddot{x} \quad \ddot{x} = \beta^2 c^2 x'' \]

\[ x'' = \frac{F_x}{\gamma \beta^2 m c^2} \]

\[ x'' = \frac{e^2 n}{2e_0 \gamma^2 \beta^3 m c^2} \cdot x \]

\[ x'' = \frac{\omega_p^2}{2} \cdot x \]

where relativistic plasma freq. \[ \omega_p = \sqrt{\frac{e^2 n}{\epsilon_0 \gamma^2 \beta^3 m c^2}} \quad \text{(units of m}^{-1}) \]
\[ x'' = \omega_0^2 x \]
\[ \omega_0 = \frac{\omega}{\sqrt{I}} \]

Distance (time) to convert excess potential energy to transverse kinetic energy is \( \frac{1}{4} \) oscillation period.

\[ \Delta t = \frac{1}{4} \left( \frac{2\pi \sqrt{I}}{\omega_0} \right) \]
Example: LCLS beam - both
\[ \text{1 nC and 0.1 nC} \]

Examine beam near cathode.
- 10 ps flat-top laser pulse
- 1 mm edge radius
- 140 MV/m peak gradient
- init RF phase 45°

Find bunch length (\(\approx\) relativistic length)
\[
F_e = ma = \frac{1}{\sqrt{2}} \cdot 140 \text{ MeV}
\]
\[
a = \frac{140 \text{ MeV}}{\sqrt{2}} \cdot \frac{c^2}{0.511 \text{ MeV}} = \boxed{1.7 \times 10^{17} \text{ m/s}^2}
\]
\[
\Delta z = \frac{1}{2} a t^2 = \frac{1}{2} (1.7 \times 10^{17} \text{ m/s}^2)(10^{-10} \text{ s})^2
\]
\[
\boxed{\Delta z = 0.9 \text{ mm}} (\ll 3 \text{ mm})
\]
Example (cont.)

density \( n = \frac{N}{\pi a^2 L} = \frac{6.25 \times 10^9}{\pi (1 \text{mm})^2 (0.9 \text{mm})} \)

\( n = 2.3 \times 10^{18} \text{ m}^{-3} \)

\( \beta = 1, \quad \gamma = \frac{\beta}{\sqrt{\beta^2 - 1}} = 0.6 \)

\[ \omega_p = \sqrt{\frac{e^2 n}{\gamma^2 \epsilon_0 c^3 \beta^3}} = 475 \text{ m}^{-1} \]

Distance for emittance growth to occur is

\[ \Delta z = \frac{\pi}{\sqrt{2} \omega_p} = 4.7 \text{ mm} \]

Beam is still inside gun.

For same beam dimensions, 0.1 nC
\( \Delta z \) is increased by \( \frac{1}{10} \)

\( \Delta z \approx 15 \text{ mm} \) for 0.1 nC

Numerically integrate envelope eq.
to get better estimate.
Oscillation Phase Advance

1 nC

\[ \gamma \]

0.25

0.2

0.1

0.15

0.1

0.05

0

Distance cm

0

2

4

6

8

10

1.0 nC

\[ \text{4 cm} \]

0.1 nC

0.1

0.08

0.06

0.04

0.02

0

Distance cm

0

2

4

6

8

10

0.1 nC
1 nC, uniform cylinder

- 10 nC
- 115 A
- \( E_{\text{tot}} = 2.2 \text{ mm-mrad} \)
- Gun exit
- Cathode

0.1 nC, uniform cylinder

- 0.1 nC
- 31 Amps
- \( E_{\text{tot}} = 0.6 \text{ mm-mrad} \)
- Cathode
- Gun exit
Comments on Scaling

\[ x'' = \frac{\omega_c^2}{2} x \]

\[ \omega_p \propto \sqrt{\frac{n}{\epsilon_0}} \]

We desire low density + high gradient.

\underline{High gradient} \Rightarrow \text{[S-band]}
- factor of 2 in gradient
- reduces \( \omega_p \) by \( 2^{1/2} \)

\underline{Low density} \Rightarrow \text{[L-band]}
- factor of 2 in frequency
- reduces \( \omega_p \) by \( 2^{3/8} \) for fixed \( Q \)
- can increase current factor of 4 for fixed density.