Tau anomalous moments

by T. Barklow
Direct Measurement of the Neutral Weak Dipole Moments of the $\tau$ Lepton

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INTRODUCTION

Why measure weak dipole moments of tau?

- Test SM tenet that fundamental fermions are pointlike objects without structure.

- Perform general search for new physics which couples more strongly to 3rd generation.

SM Radiative Corrections:

\[ \Delta a^w_\tau \approx 2 \times 10^{-6} \]
\[ \Delta d^w_\tau \approx 3 \times 10^{-37} e \cdot cm \]

where

\( a^w_\tau \equiv \text{neutral weak anom. magnetic dipole mom.} \)

\( d^w_\tau \equiv \text{neutral weak electric dipole moment} \)

\( d^w_\tau \neq 0 \) would indicate new source of CP violation
We directly measure $a^w_\tau$ and $d^w_\tau$ by analyzing the $e^-$, $\mu^-$, $\pi^-$, and $\rho^-$ decays of $\tau$'s produced in $Z$ boson decay.

General $Z\tau^+\tau^-$ coupling:

$$\Gamma^\mu_{Z\tau\tau} = i \frac{g}{2 \cos \theta_W} \left[ \gamma^\mu (g_V^\tau - g_A^\tau \gamma_5) + \frac{i}{2m_\tau} \sigma^{\mu\nu} q_\nu (\kappa - i \tilde{\kappa} \gamma_5) \right]$$

$$a^w_\tau = \frac{\kappa}{2 \cos \theta_W \sin \theta_W} \approx 1.188 \kappa$$

$$d^w_\tau = \frac{e \tilde{\kappa}}{4 \cos \theta_W \sin \theta_W m_\tau} \approx 6.586 \times 10^{-15} \tilde{\kappa} \ (e \cdot cm)$$
$a_{\tau}^{w}$ and $d_{\tau}^{w}$ enhance the transverse polarization of $\tau$'s.

Initial electron beam polarization plays a big role.

In the limit $|\kappa|, |\bar{\kappa}| \ll m_\tau / \sqrt{s} \ll 1$ and $g_{V}^{\tau}, g_{e}^{V} \to 0$ (expressions from E. Torrence thesis)

\[
T_{x}^{\pm} = \frac{2 \sqrt{s}}{m_\tau} \frac{\sin \Theta}{1 + \cos^2 \Theta} \left[ -\text{Re}(\kappa) \cos \Theta \pm \text{Im}(\bar{\kappa}) P_e \right]
\]

\[
T_{y}^{\pm} = \frac{2 \sqrt{s}}{m_\tau} \frac{\sin \Theta}{1 + \cos^2 \Theta} \left[ \text{Im}(\kappa) P_e \pm \text{Re}(\bar{\kappa}) \cos \Theta \right]
\]

\[
T_{z}^{\pm} = \frac{2 P_e \cos \Theta}{1 + \cos^2 \Theta}
\]

Terms proportional to $\text{Im}(\kappa) g_{V}^{\tau} \cos \Theta$ and $\text{Im}(\bar{\kappa}) g_{V}^{\tau} \cos \Theta$ give an experiment with $P_e = 0$ some sensitivity to $\text{Im}(\kappa)$ and $\text{Im}(\bar{\kappa})$. 
Transverse Polarization

\[ \cos(\theta) \]

\[ \text{Re}[\rho] \]

\[ \text{Im}[\rho] \]

\[ \% 80^\circ = \rho \]

\[ \% -80^\circ = \rho \]

Assume \( \text{Re}[\rho] = \frac{(2\pi)^2}{\mu_0} \) \( \text{Im}[\rho] \) on measurements.

Effect of \( \text{Re} \) on \( \rho \) at \( \beta \) not measured.
Instead of measuring $T_{x}^{\pm}$ and $T_{y}^{\pm}$ we fit directly for $a_{\tau}^{w}$ and $d_{\tau}^{w}$ using an unbinned maximum likelihood fit.

Experimental input to the likelihood fit:

- Electron beam polarization.

- Laboratory frame four-vectors of the $e^{-}, \mu^{-}, \pi^{+}$ or $\rho^{+}$.

- $\rho^{+}$ rest frame four-vectors of the $\pi^{+}$ and $\pi^{0}$ produced in $\rho^{+}$ decay.
MONTE CARLO SIMULATION OF 
\[ Z \rightarrow \tau^+\tau^- \]

KORALZ – Contains electroweak radiative corrections but does not simulate transverse spin correlations.

KORALB – When run on the Z resonance does not contain radiative corrections to \( \tau^+\tau^- \) production but does contain transverse spin correlations.

We have augmented KORALB so that it simulates \( \tau^+\tau^- \) production with non-zero values for \( a^w_\tau \) and \( d^w_\tau \).
LAC CLUSTER ANALYSIS

Since the \( \pi^0 \) and \( \pi^+ \) from \( \rho^+ \) decay often deposit their energy in the same LAC cluster, we have modified the usual definition of SLD associated and unassociated LAC clusters.

Using Monte Carlo events with tau decays to a single \( \pi^+ \) we parameterize the longitudinal and lateral response of the LAC to a single \( \pi^+ \).

This response is then used to estimate the fraction of energy in the electromagnetic layers of a cluster that is due to its associated \( \pi^+ \). The remainder of the energy is assumed to be due to direct photons and is added to the list of unassociated clusters.
Another issue involving overlapping LAC clusters is that the two $\pi^0$'s in the decay $a_1^+ \rightarrow \pi^+\pi^0\pi^0$ often coalesce, making it difficult to distinguish $\rho^+$ and $a_1^+$ decays.

To ameliorate this problem we use the cluster width in $\zeta \equiv \cos \theta$ and the azimuthal angle about the beam axis, $\phi$, to impart a mass to a single LAC cluster.

Assume that a LAC cluster is formed by $N_{cl}$ photons and let $E_{cl}$ be the cluster energy. The momentum of the cluster, $\vec{P}_{cl}$, is then

$$\vec{P}_{cl} = E_{cl}\vec{\beta}, \quad \vec{\beta} = \frac{1}{N_{cl}} \sum_i \hat{n}_i,$$

where $\hat{n}_i$ is the direction of photon $i$. 
The magnitude of the cluster velocity is

\[ \beta = \frac{1}{N_{cl}} \sum_{i} \hat{n}_i \cdot \hat{n}_0 = 1 - \left[ \frac{\sin^2 \theta}{2} \left( \frac{\sigma_\zeta^2}{\sin^4 \theta} + \sigma_\phi^2 \right) - \sigma_0^2 \right] \]

where

\( \hat{n}_0 \) is the direction of the cluster.

\( \sigma_\zeta \) and \( \sigma_\phi \) are the LAC cluster widths in \( \zeta \) and \( \phi \) respectively.

\( \sigma_0 \) is a correction factor used to account for the nonzero cluster widths of single photons.
\[ \beta = \frac{\sqrt{\frac{3}{2}}}{2} \left( \frac{E^2}{p^2} + \frac{Q^2}{p^2} \right) \]
TAU EVENT SELECTION

We require:

- $2 \leq N_{\text{qual}} \leq 4$
- Acollinearity angle $< 18^\circ$
- Acollinearity angle $> 0.3^\circ$ if $N_{\text{qual}} = 2$
- $|\cos \theta_{\text{thrust}}| < 0.60$
- At least one hemisphere must be identified as an $e^-, \mu^-, \pi^+, \text{ or } \rho^+$.

Quality charged track:

- $|\cos \theta| < 0.80$
- 2-d transverse impact parameter $< 2 \text{ mm}$
- 3-d impact parameter $< 5 \text{ mm}$
TAU DECAY MODE IDENTIFICATION

An identified hemisphere must have one and only one primary charged track.

The primary charged track must be a quality track with charge equal to the hemisphere charge.

The primary charged track, any secondary charged tracks, and the neutral clusters in an identified hemisphere must satisfy the condition $\chi^2_{P\gamma} < 6$ where

$$\chi^2_{P\gamma} = \frac{M_{chg}^2}{\sigma_{chg}^2} + \frac{M_{e\gamma}^2}{\sigma_{e\gamma}^2} + \frac{(E_{had}/E_P)^2}{\sigma_{had}^2}$$
\[ x^2_{P\gamma} = \frac{M_{chg}^2}{\sigma_{chg}^2} + \frac{M_{e\gamma}^2}{\sigma_{e\gamma}^2} + \frac{(E_{had}/E_P)^2}{\sigma_{had}^2} \]

- \( M_{chg} \) is the invariant mass of the vector sum of any secondary charged tracks.

- \( M_{e\gamma} \) is the invariant mass of the vector sum of any secondary charged tracks and the electromagnetic layer components of any unassociated LAC clusters.

- \( E_{had} \) is the sum of the energies of the hadronic layer components of both associated and unassociated LAC clusters.

- \( E_P \) is the CDC momentum of the primary charged track.

\( \sigma_{chg}^2, \sigma_{e\gamma}^2 \) and \( \sigma_{had}^2 \) are the variances of the distributions for \( M_{chg}, M_{e\gamma} \) and \( E_{had}/E_P \), respectively, for tau decays to \( e^-, \mu^-, \pi^+, \) and \( \rho^+ \).
\[ \chi^2_{P\gamma} = \frac{M_{\text{chg}}^2}{\sigma_{\text{chg}}^2} + \frac{M_{e\gamma}^2}{\sigma_{e\gamma}^2} + \frac{(E_{\text{had}}/E_P)^2}{\sigma_{\text{had}}^2} \]

The 1st term in the expression for \( \chi^2_{P\gamma} \) ensures that secondary charged tracks all come from a single photon.

The 2nd term ensures that any direct neutral electromagnetic energy in the hemisphere is due either to a single photon or to a \( \pi^0 \).

The 3rd term is used to reject decays such as \( \tau^- \rightarrow \pi^+ K^0 \nu_\tau \).
We calculate the hemisphere energy and mass, $E_{\text{hemi}}$ and $M_{\text{hemi}}$, by summing together the four-vectors of the primary charged track and the electromagnetic layer components of those unassociated LAC clusters which are within $37^\circ$ of the primary charged track.

The energy and mass obtained by summing together only the unassociated cluster four-vectors are denoted by $E_{\text{neu}}$ and $M_{\text{neu}}$, respectively.

Electrons and muons are identified in the standard way.

Electrons and muons furthermore must have their hemisphere energy in the range $5 < E_{\text{hemi}} < 36$ GeV and their hemisphere mass must satisfy $M_{\text{hemi}} < 0.26$ GeV for hemispheres with $E_{\text{neu}} > 0.5$ GeV.
A hemisphere is identified as a single $\pi^+$ decay if all the following are satisfied:

- It is not identified as an electron or muon
- $M_{\text{hemi}} < 0.26$ GeV for hemispheres with $E_{\text{neu}} > 0.5$ GeV
- $E_{\text{had}}/E_P < 1.7$
- $5 < E_{\text{hemi}} < 44$ GeV

A hemisphere is identified as a $\rho^+$ if all the following are satisfied:

- It is not identified as an electron, muon, or $\pi^+$.
- $E_{\text{had}}/E_P < 2.5$
- $M_{\text{neu}} < 0.48$ GeV
- $0.35 < M_{\text{hemi}} < 1.20$ GeV
- $5 < E_{\text{hemi}} < 44$ GeV
Applying the above criteria to the 1993–1998 SLD data sample we select a total of 6736 tau pairs, with 2016 electron, 2577 muon, 1847 pion, 3799 rho, and 3233 unidentified hemispheres.

Table 1: τ decay identification efficiency and purity

<table>
<thead>
<tr>
<th>Mode</th>
<th>Efficiency</th>
<th>Purity</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^- \bar{\nu}<em>e \nu</em>\tau$</td>
<td>0.74</td>
<td>0.98</td>
<td>2016</td>
</tr>
<tr>
<td>$\mu^- \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>0.93</td>
<td>0.98</td>
<td>2577</td>
</tr>
<tr>
<td>$\pi^+ \nu_\tau$</td>
<td>0.76</td>
<td>0.76</td>
<td>1847</td>
</tr>
<tr>
<td>$\rho^+ \nu_\tau$</td>
<td>0.70</td>
<td>0.83</td>
<td>3799</td>
</tr>
</tbody>
</table>

The electron and muon samples are very clean.

The single pion sample receives roughly equal background from misidentified electron, muons and rho's.

Most of the contamination of the rho sample comes from misidentified $a_1 \rightarrow \pi^+ \pi^0 \pi^0$ decays.

The non-τ background from two-photon, Bhabha's, muon pairs, and hadronic events is estimated to be 0.38%, 0.87%, 0.2%, and 0.01% respectively.
Momentum distributions for hemispheres identified as \(e^-, \mu^-, \pi^+, \) and \(\rho^+\)

Correctly and incorrectly identified decays are summed together in the MC curves.

Single normalization factor is used to scale MC to the data so that these plots also check absolute normalization of efficiency calculations.
LIKELIHOOD FIT

The identification and misidentification efficiencies for tau decay modes as a function of kinematic variables are incorporated into the probability density function (PDF) of the likelihood fit.

Detector resolution, radiative corrections, and non-$\tau$ background are not included in the fit, however.

The PDF is then a sum of lowest-order multi-differential cross-section expressions for $\tau^+\tau^-$ production and decay modulated by identification or misidentification efficiency functions.
$p + \text{ backgrounds summed}$

$p_{3zabs} p_{3bzabs}, \rho_{0+}$

$|p|$
The multi-differential cross-section is calculated using helicity amplitude expressions for $\tau^+\tau^-$ production and $\tau$ decay.

In order to calculate the multi-differential cross-section the $\tau^-$ direction must be specified.

No attempt is made to measure the tau direction, however.

For events with two semi-hadronic decays there is a discrete two-fold ambiguity in the tau direction which is summed over in the likelihood fit.

For events with a leptonic decay or with an unidentified hemisphere the ambiguity is continuous and an integration is performed over possible tau directions.
KORALB - Correct Fit 95% CL.
Applying our likelihood fit to the data we obtain initial estimates of

\[
\begin{align*}
Re(a_T^w) &= (1.16 \pm 0.99) \times 10^{-3} \\
Im(a_T^w) &= (-0.27 \pm 0.62) \times 10^{-3} \\
Re(d_T^w) &= (0.18 \pm 0.61) \times 10^{-17} e \cdot cm \\
Im(d_T^w) &= (-0.14 \pm 0.35) \times 10^{-17} e \cdot cm
\end{align*}
\]

where the errors are statistical.

Our final estimates are obtained by making corrections to these values in order to account for effects not included in the PDF of the likelihood fit.
SHIFTS IN DIPOLE MOMENTS

The absence of a detector resolution function, QED radiative corrections and non-$\tau$ background in the PDF produces shifts in the fitted values of the anomalous moments with respect to their true values.

The KORALZ Monte Carlo is used to calculate the shifts due to the combined effects of detector resolution and QED radiative corrections.

The KORALB Monte Carlo is used to calculate the shifts due to detector resolution alone.

Shifts due to non-$\tau$ background are calculated by adding a Monte Carlo sample of the background to the KORALZ Monte Carlo sample.
Table 2: Shifts in fitted values of $a^w_\tau$ and $d^w_\tau$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$Re(a^w_\tau)$</th>
<th>$Im(a^w_\tau)$</th>
<th>$Re(d^w_\tau)$</th>
<th>$Im(d^w_\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(10^{-3})$</td>
<td>$(10^{-3})$</td>
<td>$(10^{-17} e \cdot cm)$</td>
<td>$(10^{-17} e \cdot cm)$</td>
</tr>
<tr>
<td>detector effects</td>
<td>-0.637</td>
<td>-0.064</td>
<td>-0.023</td>
<td>0.052</td>
</tr>
<tr>
<td>QED rad. corr.</td>
<td>1.221</td>
<td>-0.234</td>
<td>0.031</td>
<td>0.075</td>
</tr>
<tr>
<td>Non-$\tau$ events</td>
<td>0.321</td>
<td>0.046</td>
<td>-0.005</td>
<td>-0.011</td>
</tr>
<tr>
<td>Total</td>
<td>0.905</td>
<td>-0.252</td>
<td>0.003</td>
<td>0.116</td>
</tr>
</tbody>
</table>
SYSTEMATIC ERRORS

With the exception of identification and misidentification detection efficiencies the PDF is free of systematic uncertainties.

The fitted value of a dipole moment depends on detection efficiency only through terms proportional to unity plus the ratio of the misidentification and identification efficiencies.

Our systematic error is dominated by the uncertainties in our dipole moment shifts.
For the detector systematic errors we conservatively use the entire detector effects shifts, taking into account the KORALB Monte Carlo statistical errors.

Specifically, we assume that the detector effects shift is equal to the sum in quadrature of the KORALB statistical error and the detector systematic error.

If the detector systematic error extracted under this assumption is less than the KORALB statistical error then we set the detector systematic error equal to the KORALB statistical error.

The systematic errors in the calculation of the shifts due to QED radiative corrections and non-τ background are estimated using the Monte Carlo statistical errors for the KORALZ and non-τ background Monte Carlo samples.
Table 3: Systematic errors in $a^w_\tau$ and $d^w_\tau$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$Re(a^w_\tau)$ $(10^{-3})$</th>
<th>$Im(a^w_\tau)$ $(10^{-3})$</th>
<th>$Re(d^w_\tau)$ $(10^{-17}e \cdot cm)$</th>
<th>$Im(d^w_\tau)$ $(10^{-17}e \cdot cm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>detector effects</td>
<td>0.585</td>
<td>0.120</td>
<td>0.134</td>
<td>0.067</td>
</tr>
<tr>
<td>QED rad. corr.</td>
<td>0.421</td>
<td>0.185</td>
<td>0.224</td>
<td>0.099</td>
</tr>
<tr>
<td>Non-$\tau$ events</td>
<td>0.063</td>
<td>0.029</td>
<td>0.036</td>
<td>0.016</td>
</tr>
<tr>
<td>'Total</td>
<td>0.723</td>
<td>0.222</td>
<td>0.263</td>
<td>0.121</td>
</tr>
</tbody>
</table>
FINAL RESULTS

Applying our calculated shifts to the initial estimates of $a_T^w$ and $d_T^w$ we obtain our final results:

$$Re(a_T^w) = (0.26 \pm 0.99 \pm 0.72) \times 10^{-3}$$
$$Im(a_T^w) = (-0.02 \pm 0.62 \pm 0.22) \times 10^{-3}$$
$$Re(d_T^w) = (0.18 \pm 0.61 \pm 0.26) \times 10^{-17} e \cdot cm$$
$$Im(d_T^w) = (-0.26 \pm 0.35 \pm 0.12) \times 10^{-17} e \cdot cm$$

where the first error is statistical and the second error is systematic.

The results are consistent with the standard model expectation of $\approx 0$.
Weak Electric Dipole Measurements

- **ALEPH**: $-0.29 \pm 2.59 \pm 0.88$
- **DELPHI**: $-1.48 \pm 2.64 \pm 0.27$
- **OPAL**: $0.72 \pm 2.46 \pm 0.24$
- **L3**: $-4.4 \pm 8.8 \pm 13.3$
- **SLD**: $1.8 \pm 6.1 \pm 2.6$

- **DELPHI**: $-4.4 \pm 7.7 \pm 1.3$
- **OPAL**: $3.5 \pm 5.7 \pm 0.8$
- **SLD**: $-2.6 \pm 3.5 \pm 1.2$
Weak Magnetic Dipole Measurements

\[ \Re(a^Z) \times 10^{-3} \]

- L3: 0.0 ± 1.6 ± 2.3
- SLD: 0.3 ± 1.0 ± 0.7

\[ \Im(a^Z) \times 10^{-3} \]

- L3: -1.00 ± 3.6 ± 4.3
- SLD: -0.02 ± 0.62 ± 0.22
Our 95\% confidence level limits are

\[
|Re(a_\tau^w)| < 2.47 \times 10^{-3}
\]
\[
|Im(a_\tau^w)| < 1.25 \times 10^{-3}
\]
\[
|Re(d_\tau^w)| < 1.35 \times 10^{-17} e \cdot cm
\]
\[
|Im(d_\tau^w)| < 0.87 \times 10^{-17} e \cdot cm
\]

With the exception of \(Re(d_\tau^w)\), all of our limits are improvements over the best previously published limits of

\[
|Re(a_\tau^w)| < 4.50 \times 10^{-3}
\]
\[
|Im(a_\tau^w)| < 9.90 \times 10^{-3}
\]
\[
|Re(d_\tau^w)| < 0.56 \times 10^{-17} e \cdot cm
\]
\[
|Im(d_\tau^w)| < 1.50 \times 10^{-17} e \cdot cm
\]