Measurement of BR (b->1) and BR (b->c->1)

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Measurement of inclusive $\text{BR}(b \to l)$ and $\text{BR}(b \to c \to l)$

- $\text{BR}(b \to l)$
- The method
- Some results
- Electron (muon) ID
- Miscellaneous

**$\text{BR}(b \to l)$**

Definition and theoretical predictions:

$$BR(b \to l) = \frac{\Gamma(X_b \to X l\nu)}{\Gamma(X_b \to \text{all})} \equiv \frac{\#(X_b \to X l\nu)}{\#(X_b)}$$

where $X_b$ is a hadron containing $b$ quark, and $l$ is $e$ or $\mu$ (not $\tau$). $c\bar{c}s$ and $c\tau\nu_\tau$ are phase space suppressed to about 20% of the "light" modes. In the spectator model this gives
With the QCD corrections $BR(b \rightarrow l)$ is estimated to $\geq 12.5\%$. K. Schubert and R. Waldi (hep-ph/9409341) claim $BR(b \rightarrow l) = (11.2 \pm 0.5 \pm 1.7)\%$ when the interfering amplitudes in charged $B$ mesons are taken into account.

Experimental results:

- CLEO: $(10.49 \pm 0.17 \pm 0.43)\%$, Phys. Rev. Lett. 76, 1570, (1996),
- OPAL: $(10.5 \pm 0.6 \pm 0.5)\%$, CERN-PPE/93-106 (Z. Phys. C),
- ALEPH: $(11.01 \pm 0.23_{stat} \pm 0.28_{syst} \pm 0.11_{model})\%$ in a model independent measurement, EPS-95 conf. paper (EPS0404).

The method

The starting point is a simple probabilistic equation:

$$f(E_1^-, Q_2, A_2) = \sum_i f(E_1^-, Q_2, A_2, x_i)$$ (1)
where $E_1^-, Q_2, A_2$ are taken to be random variables:

$$E_1^- = \begin{cases} 
0 & \text{no particle identified as } e^- \text{ in hem. } 1 \\
1 & \text{one particle identified as } e^- \text{ in hem. } 1 
\end{cases}$$

$$Q_2 = \text{jet charge}$$

$$A_2 = -p \cos(\theta) \left\{ \text{inv mass tag in hem 2} \right\}$$

and $x_i$ the contributing processes:

<table>
<thead>
<tr>
<th>hem 2</th>
<th>hem 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$b_1 \rightarrow e^-$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$b_1 \rightarrow b_1 \rightarrow \bar{c}_1 \rightarrow e^-$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$e^-$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$\bar{b}_1 \rightarrow \bar{b}_1 \rightarrow e^-$</td>
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<tr>
<td>$b_2$</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>$e^-$</td>
</tr>
<tr>
<td>non $b\bar{b}$ events</td>
<td></td>
</tr>
</tbody>
</table>

mix, casc

bug

mix

casc

bug
Eq. (1) can be transformed into

$$\frac{\# e^-}{\# \text{tags}} = f(E_1^- | q_1 q_2) \cdot \text{efficiency} \cdot f(E_1 e)$$

$$f^- = \eta(\theta_1 \alpha + \theta_2 \beta + g_1)$$  \hspace{1cm} (2)

$$\text{BR}(b \rightarrow e) \quad \text{BR}(b \rightarrow c \rightarrow e)$$

where $\alpha$ and $\beta$ are functions of $P(b)$ and $\chi$, $P(b)$ being the probability that a $b$ quark is in the same hemisphere as that of the particle identified as electron, and $\chi$ being $1/2$ of the value of the $B$ mixing parameter. Simply

Eq. (2) also applies for tracks identified as positrons $E_1^+$ and muons $M_1^\pm$. 

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The estimators:

We look for estimators of $\theta_1$, BR($b\rightarrow l$), and $\theta_2$, BR($b\rightarrow c\rightarrow l$), that are unbiased and have minimum variance.

The likelihood function:

$$L = \prod \left( \begin{array}{c} n \\ k \end{array} \right) f^k (1 - f)^{n-k} \prod \left( \begin{array}{c} n \\ k' \end{array} \right) f'^{k'} (1 - f')^{n-k'}$$

$$K = \# e^- \quad n = \# \text{tags} \quad K' = \# e^+$$

In the large $n$ limit (normal distribution) estimators of $\theta_1$, $\theta_2$ obtained from

$$\frac{\partial \ln L}{\partial \theta_1} = 0, \quad \frac{\partial \ln L}{\partial \theta_2} = 0,$$

are unbiased and have minimum variance.
Smaller overlapping for large $P(b)$

Larger overlapping for small $P(b)$

$e^-$ $P(b) \approx 0.8$
$e^+ P(b) \approx 0.8$
$e^- P(b) \approx 0.6$
$e^+ P(b) \approx 0.6$

$\Theta_2$ $\Theta_1$

$\Theta_2$ $\Theta_1$

all $P(b)$ and all $e^+ e^-$
Some results

Only "toy" solutions are shown on the next few pages. Backgrounds, efficiencies, mixing, etc., are not correctly taken into account. There are also distortions due to the uncertainty in the jet axis.
Electron sample data

\[ \# e^- \]

\[ \# e^+ \]

\[ P(b) \] probability that b quark is in the same hemisphere as e

93-95 data
Solution

\[ \frac{dBR(b \rightarrow c \rightarrow e)}{dp_L} \]

\[ \frac{dBR(b \rightarrow e)}{dp_L} \]
Electron (muon) efficiency

Efficiency of electron identification:

\[
P(E|eX) = \frac{P(e|EX)}{P(e|X)} P(E|X)
\]

where for each individual track \( E, e, X \) take values on \{0, 1\} indicating whether

- \( E \), a track is identified as electron,
- \( e \), a track is a MC electron,
- \( X \), a track is labeled as electron by another routine.

Strictly, \( P(E|eX) \) is efficiency for the chosen sample \( X \) and depends on the correlation between \( E \) and \( X \). What we really want, however, is

\[
P(E|e) \equiv \eta
\]
(as defined in eq.(2)) which is equivalent to $P(E|eX)$ only when sample $X$ is very pure (which is never the case) or when $E$ and $X$ are completely uncorrelated (which is also never the case). In practice we estimate $P(E|e)$ from $P(E|eX_i)$ for several different $X_i$. Some results follow.
NN output

\( \text{nnout} > 0.98, \gamma \) conversions pure sample

\[ P(e1X) = \frac{P(e1EX)}{P(e1X)} P(E1X) \]
nnout > 0.98, τ events

\[ P(E\text{e}X) = \frac{P(e\text{e}X)}{p(e\text{e}X)} \rho(E\text{e}X) \]
A few relevant points:

- The estimators have to be corrected to include constraints that do not allow any confidence region of the BR's to "cross" $\theta_1 = 0$ and $\theta_2 = 0$ while maintaining unbiasedness and minimum variance.

- We have to finish work on electron ID and to begin one on muon ID.

- We need better $b$ tag.

- The measurement doesn't seem to be very sensitive to $A_b$. The sensitivity to $A_b$ is further reduced with the improvement of the $b$ tag based on jet-charge or similar topological techniques.

- The largest problem (and it is the only one worth stressing) are the backgrounds. We want to include as large portions of the spectra as possible. With the $P_{tot}$ cut of 1 GeV for electrons and 1.6 GeV for muons one can cover about 95% of the prompt and about 70% of the cascade spectra. We still don't know how to reduce the backgrounds at low momenta without affecting the statistics.