CP VIOLATION AND $B$ PHYSICS $^1$

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ABSTRACT

This is a quick review of CP non-conservation in $B$ physics. Several methods are described for testing the Kobayashi-Maskawa single phase origin of CP violation in $B$ decays, pointing out some limitations due to hadronic uncertainties. A few characteristic signatures of new physics in $B$ decay asymmetries are listed.

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1 The CKM Matrix

In the standard model of electroweak interactions CP violation is due to a nonzero complex phase \([\theta_{ij} (i < j = 1, 2, 3)]\) in the Cabibbo-Kobayashi-Maskawa (CKM) matrix \(V\), describing the weak couplings of the charged gauge boson to quarks. The unitary matrix \(V\), given by three mixing angles \(\theta_{ij}\) and a phase \(\gamma\), can be approximated by \((s_{ij} \equiv \sin \theta_{ij})\) \([2, 3]\)

\[
V \approx \begin{pmatrix}
1 - \frac{1}{2} s_{12}^2 & s_{12} & s_{13} e^{-i\gamma} \\
-\overline{s}_{12} & 1 - \frac{1}{2} s_{12}^2 & s_{23} \\
s_{12} s_{23} - s_{13} e^{i\gamma} & \overline{s}_{12} s_{23} & 1
\end{pmatrix}.
\]

(1)

Within this approximation, the only complex elements are \(V_{ub}\) with phase \(-\gamma\) and \(V_{td}\), the phase of which is denoted \(-\beta\).

The measured values of the three mixing angles and phase are \([3]\)

\[
s_{12} = 0.220 \pm 0.002, \quad s_{23} = 0.040 \pm 0.003, \quad s_{13} = 0.003 \pm 0.001,
\]

(2)

\[
35^0 \leq \gamma \equiv \text{Arg}(V_{ub}^*) \leq 145^0.
\]

(3)

First evidence for a nonzero phase \(\gamma\) came 35 years ago with the measurement of \(\epsilon\), parameterizing CP violation in \(K^0 - \overline{K}^0\) mixing. The second evidence was obtained recently through the measurement of \(\text{Re}(\epsilon'/\epsilon)\) \([4, 5]\) discussed extensively at this meeting.

Unitarity of \(V\) implies a set of 6 triangle relations. The \(db\) triangle,

\[
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0,
\]

(4)

is unique in having three comparable sides, which were measured in \(b \to ul\nu, b \to cl\nu\) and \(\Delta M_{d,s}\), respectively. Whereas \(V_{tb}\) was measured quite precisely, \(V_{ub}\) and \(V_{td}\) are rather poorly known at present. The three large angles of the triangle lie in the ranges \(35^0 \leq \alpha \leq 120^0, 10^0 \leq \beta \leq 35^0\) and Eq. (3). As we will show in the next sections, certain \(B\) decay asymmetries can constrain these angles considerably beyond present limits.

For comparison with \(K\) physics, note that due to the extremely small \(t\)-quark side of the \(ds\) unitarity triangle

\[
V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0,
\]

(5)

this triangle has an angle of order \(10^{-3}\), which accounts for the smallness of CP violation in \(K\) decays. The area of this triangle, which is equal to the area of the \(db\) triangle \([6]\), can be determined by fixing its tiny height through the rate of \(K_L \to \pi^0 \nu\bar{\nu}\). This demonstrates the complementarity of \(K\) and \(B\) physics in verifying or falsifying the assumption that CP violation originates solely in the single phase of the CKM matrix.

As we will show, the advantage of \(B\) decays in testing the KM hypothesis is the large variety of decay modes. This permits a detailed study of the phase structure of the CKM matrix through various interference phenomena which can measure the two phases \(\gamma\) and \(\beta\). New physics can affect this interference in several ways to be discussed below.
2 CP violation in $B^0 - \bar{B}^0$ mixing

The wrong-sign lepton asymmetry

$$A_u = \frac{\Gamma(\bar{B}^0 \to X\ell^+\nu) - \Gamma(B^0 \to X\ell^-\bar{\nu})}{\Gamma(\bar{B}^0 \to X\ell^+\nu) + \Gamma(B^0 \to X\ell^-\bar{\nu})},$$

measures CP violation in $B^0 - \bar{B}^0$ mixing. Top-quark dominance of $B^0 - \bar{B}^0$ mixing implies that this asymmetry is of order $10^{-3}$ or smaller [7].

$$A_u = 4\text{Re}e_B = \text{Im} \left( \frac{\Gamma_{12}}{|M_{12}|} \right) = \frac{|\Gamma_{12}|}{|M_{12}|} \text{Arg} \left( \frac{\Gamma_{12}}{|M_{12}|} \right) \simeq \left( \frac{m_{\ell}^2}{m_t^2} \right) \left( \frac{m_c^2}{m_b^2} \right) \leq O(10^{-3}).$$

Present limits are at the level of 5% [8].

Writing the neutral $B$ mass eigenstates as

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle,$$

one has $2\text{Re}e_{13} \approx 1 - |q/p| \leq O(10^{-3})$. Thus, to a very high accuracy, the mixing amplitude is a pure phase

$$\frac{q}{p} = e^{2i\text{Arg}(V_{ud})} = e^{-2i\beta}.$$

3 The asymmetry in $B^0(t) \to \psi K_S$

When an initially produced $B^0$ state oscillates in time via the mixing amplitude which carries a phase $e^{-2i\beta}$,

$$|B^0(t)\rangle = |B^0\rangle \cos(\Delta mt/2) + |\bar{B}^0\rangle e^{-2i\beta} \sin(\Delta mt/2),$$

the $B^0$ and $\bar{B}^0$ components decay with equal amplitudes to $\psi K_S$. The interference creates a time-dependent CP asymmetry between this process and the corresponding process starting with a $\bar{B}^0$ [9]

$$A(t) = \frac{\Gamma(B^0(t) \to \psi K_S) - \Gamma(\bar{B}^0(t) \to \psi K_S)}{\Gamma(B^0(t) \to \psi K_S) + \Gamma(\bar{B}^0(t) \to \psi K_S)} = -\sin(2\beta) \sin(\Delta mt).$$

The simplicity of this result, relating a measured asymmetry to an angle of the unitarity triangle, follows from having a single weak phase in the decay amplitude which is dominated by $b \to c\bar{c}s$. This single phase approximation holds to better than 1% [10] and provides a clean measurement of $\beta$.

A recent measurement by the CDF collaboration at the Tevatron [11], $\sin(2\beta) = 0.79 \pm 0.39 \pm 0.16$, has not yet produced a significant nonzero result. It is already encouraging however to note that this result prefers positive values, and is not in conflict with present limits, $0.4 \leq \sin(2\beta) \leq 0.8$. 

3
4 Penguin pollution in $B^0 \to \pi^+\pi^-$

By applying the above argument to $B^0 \to \pi^+\pi^-$, in which the decay amplitude has the phase $\gamma$, one would expect the asymmetry in this process to measure $\sin(2(\beta + \gamma)) = -\sin(2\alpha)$. However, this process involves a second amplitude due to penguin operators which carry a different weak phase than the dominant current-current (tree) amplitude [10, 12]. This leads to a more general form of the time-dependent asymmetry, which includes a new term due to direct CP violation in the decay [10]

$$A(t) = a_{\text{dir}} \cos(\Delta m t) + \sqrt{1 - a_{\text{dir}}^2} \sin 2(\alpha + \theta) \sin(\Delta m t).$$  \quad (12)

Both $a_{\text{dir}}$ and $\theta$, the correction to $\alpha$ in the second term, are given roughly by the ratio of penguin to tree amplitudes, $a_{\text{dir}} \sim 2(\text{Penguin}/\text{Tree}) \sin \delta$, $\theta \sim (\text{Penguin}/\text{Tree}) \cos \delta$, where $\delta$ is an unknown strong phase. A crude estimate of the penguin-to-tree ratio, based on CKM and QCD factors, is 0.1. Recently, flavor SU(3) was applied [13] to relate $B \to \pi\pi$ to $B \to K\pi$ data, finding this ratio to be in the range 0.3±0.1. Precise knowledge of this ratio could provide very useful information about $\alpha$ [10, 14].

One way of eliminating the penguin effect is by measuring also the time-integrated rates of $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$, and their charge-conjugates [15]. The three $B \to \pi\pi$ amplitudes obey an isospin triangle relation,

$$A(B^0 \to \pi^+\pi^-)/\sqrt{2} + A(B^0 \to \pi^0\pi^0) = A(B^+ \to \pi^+\pi^0).$$  \quad (13)

A similar relation holds for the charge-conjugate processes. One uses the different isospin properties of the penguin ($\Delta I = 1/2$) and tree ($\Delta I = 1/2, 3/2$) contributions and the well-defined weak phase ($\gamma$) of the tree amplitude. This enables one to determine the correction to $\sin 2\alpha$ in the second term of Eq.(12) by constructing the two isospin triangles.

Electroweak penguin contributions could spoil this method [16] since they involve $\Delta I = 3/2$ components. This implies that the amplitudes of $B^+ \to \pi^+\pi^0$ and its charge-conjugate differ in phase, which introduces a correction at the level of a few percent in the isospin analysis. It was shown recently [17] that this small correction can be taken into account analytically in the isospin analysis, since the dominant electroweak contributions are related by isospin to the tree amplitude. Other very small corrections can come from isospin breaking in strong interactions [18].

The major difficulty of measuring $\alpha$ without knowing the ratio Penguin/Tree is experimental rather than theoretical. The first signal for $B^0 \to \pi^+\pi^-$ reported this summer [19, 20], $\text{BR}(B^0 \to \pi^+\pi^-) = [0.47^{+0.18}_{-0.15} \pm 0.06] \times 10^{-5}$, is somewhat weaker than expected. Worse than that, the branching ratio into two neutral pions is expected to be at most an order of magnitude smaller. This estimate is based on color-suppression, a feature already observed in CKM-favored $B \to D\pi$ decays. Here it was found that [2], $\text{BR}(B^0 \to \bar{D}^0\pi^0)/\text{BR}(B^0 \to D^-\pi^+) < 0.04$. If the same color-suppression holds in $B \to \pi\pi$, then $\text{BR}(B^0 \to \pi^0\pi^0) < 3 \times 10^{-7}$, which would be too small to be measured with a useful precision. Constructive interference between a color-suppressed current-current amplitude and a penguin amplitude can increase the $\pi^0\pi^0$ rate somewhat. Limits
on this rather rare mode can be used to bound the uncertainty in determining $\sin(2\alpha)$ from $B^0 \to \pi^+\pi^-$ [21]

$$\sin(\delta \alpha) \leq \frac{B(B \to \pi^0\pi^0)}{B(B^\pm \to \pi^\pm\pi^0)}$$  \hspace{1cm} (14)$$

Other ways of treating the penguin problem were discussed in [22].

5 \hspace{1cm} \textbf{$B$ decays to three pions}

The angle $\alpha$ can also be studied in the processes $B \to \pi \rho$ [23], which have already been seen with branching ratios larger than those of $B \to \pi\pi$ [24], $\text{BR}(B^0 \to \pi^\pm\rho^\mp) = (3.5^{+1.1}_{-1.0} \pm 0.5) \times 10^{-5}$, $\text{BR}(B^\pm \to \pi^\pm\rho^0) = (1.5 \pm 0.5 \pm 0.4) \times 10^{-5}$. An effective study of $\alpha$, which can eliminate uncertainties due to penguin corrections, requires

- A separation between $B^0$ and $\bar{B}^0$ decays.
- Time-dependent rate asymmetry measurements in $B \to \pi^\pm\rho^\mp$.
- Measuring the rates of processes involving neutral pions, including the color-suppressed $B^0 \to \pi^0\rho^0$.

This will not be an easy task.

6 \hspace{1cm} \textbf{$\gamma$ from $B \to K\pi$ and other processes}

While discussing $B^\pm$ decays to three charged pions, we note that these decays are of high interest for a different reason [25]. When two of the pions form a mass around the charmonium $\chi_{c0}(3415)$ state, a very large CP asymmetry is expected between $B^+$ and $B^-$ decays. In this case the direct decay amplitude into three pions ($b \to uud$) interferes with a comparable amplitude into $\chi_{c0}\pi^\pm$ ($b \to c\bar{d}$) followed by $\chi_{c0} \to \pi^+\pi^-$. The large asymmetry (proportional to $\sin \gamma$), of order several tens of percent, follows from the $90^\circ$ strong phase obtained when the two pion invariant mass approaches the charmonium mass.

A method for determining the angle $\gamma$ through $B^\pm \to DK^\pm$ decays [26], which in principle is completely free of hadronic uncertainties, faces severe experimental difficulties. It requires measuring separately decays to states involving $D^0$ and $\bar{D}^0$. Tagging the flavor of a neutral $D$ by the charge of the decay lepton suffers from a very large background from $B$ decay leptons, while tagging by hadronic modes involves interference with doubly Cabibbo-suppressed $D$ decays. A few variants of this method were suggested [27], however, due to low statistics, it seems unlikely that these variants can be performed effectively in near future facilities.

Much attention was drawn recently to studies of $\gamma$ in $B \to K\pi$, motivated by measurements of charge-averaged $B \to K\pi$ decay branching ratios [19, 20]

$$\text{BR}(B^\pm \to K\pi^\pm) = (1.82^{+0.46}_{-0.30} \pm 0.16) \times 10^{-5},$$  \hspace{1cm} (15)$$
The first suggestion to constrain $\gamma$ from $B \to K\pi$ was made in [28], where electroweak penguin contributions were neglected. The importance of electroweak penguin terms was noted in [29], which was followed by several ideas about controlling these effects [30]. In the present discussion we will focus briefly on very recent work along these lines [17, 31, 32, 33], simplifying the discussion as much as possible.

Decomposing the $B^+ \to K\pi$ amplitudes into contributions from penguin ($P$), color-favored tree ($T$) and color-suppressed tree ($C$) terms [34],

$$A(B^+ \to K^0\pi^+) = P, \quad A(B^+ \to K^+\pi^0) = -(P + T + C)/\sqrt{2},$$  \hspace{1cm} (16)$$

$P$ has a weak phase $\pi$, while $T$ and $C$ each carry the phase $\gamma$. Some information about the relative magnitudes of these terms can be gained by using SU(3) and comparing these amplitudes to those of $B \to \pi\pi$ [13]. This implies

$$r \equiv \frac{T + C}{P} = 0.24 \pm 0.06.$$  \hspace{1cm} (17)$$

Defining the ratio of charge-averaged rates [31]

$$R_s^{-1} = \frac{2B(B^+ \to K^+\pi^0)}{B(B^+ \to K^+\pi^0)},$$  \hspace{1cm} (18)$$

one has

$$R_s^{-1} = 1 - 2r \cos \delta \cos \gamma + r^2,$$  \hspace{1cm} (19)$$

where $\delta$ is the penguin-tree strong phase-difference. Any deviation of this ratio from one would be a clear signal of interference between $T + C$ and $P$ in $B^+ \to K^+\pi^0$ and could be used to constrain $\gamma$.

So far, electroweak penguin contributions have been neglected. These terms can be included in the above ratio by relating them through flavor SU(3) to the corresponding tree amplitudes. This is possible since the two types of operators have the same (V-A)(V-A) structure and differ only by SU(3). Hence, in the SU(3) limit, the dominant electroweak penguin term and the tree amplitude have the same strong phase, and the ratio of their magnitudes is given simply by a ratio of the corresponding Wilson coefficients multiplied by CKM factors [17, 31]

$$\delta_{EW} \equiv \frac{|\text{EWP}(B^+ \to K^0\pi^+) + \sqrt{2}\text{EWP}(B^+ \to K^+\pi^0)|}{|T + C|},$$  \hspace{1cm} (20)$$

$$= \frac{3}{2} \frac{c_0 + c_{30}}{c_1 + c_2} \left| \frac{V_{ub}^* V_{ts}}{V_{tb} V_{us}} \right| = 0.6 \pm 0.2,$$  \hspace{1cm} (21)$$

where the error comes from $|V_{ub}|$. Consequently, one finds instead of (19)

$$R_s^{-1} = 1 - 2r \cos \delta (\cos \gamma - \delta_{EW}) + O(r^2),$$  \hspace{1cm} (22)$$
implying
\[ |\cos \gamma - \delta_{EW}| \geq \frac{|1 - R_s^{-1}|}{2r} \] (23)

If \( R_s^{-1} \neq 1 \), this constraint can be used to exclude a region around \( \gamma = 50^\circ \). The present value of \( R_s^{-1} \) is consistent with one. Experimental errors must be substantially reduced before drawing any conclusions.

The above constraint is based only on charge-averaged rates. Further information on \( \gamma \) can be obtained by measuring separately \( B^+ \) and \( B^- \) decay rates. The \( B^+ \to K\pi \) rates obey a triangle relation with \( B^+ \to \pi^+\pi^0 \) \([17, 28, 31]\)

\[ \sqrt{2}A(B^+ \to K^+\pi^0) + A(B^+ \to K^0\pi^+) = \tilde{r}_u A(B^+ \to \pi^+\pi^0) \left( 1 - \delta_{EW} e^{-i\gamma} \right) , \] (24)

where \( \tilde{r}_u = (f_K/f_\pi) \tan \theta_c \approx 0.28 \) contains explicit SU(3) breaking. This relation and its charge-conjugate permit a determination of \( \gamma \) which does not rely on \( R_s^{-1} \neq 1 \).

This analysis involves uncertainties due to errors in \( r \) and \( \delta_{EW} \), which are expected to be reduced to the level of 10%. Additional uncertainties follow from SU(3) breaking in (20) and from rescattering effects in \( B^+ \to K^0\pi^+ \) which introduce a term with phase \( \gamma \) in this process. The latter effects can be bounded by the U-spin related rate of \( B^+ \to K^+K^0 \)[35]. Present limits on rescattering corrections are at a level of 20% and can be reduced to 10% in future high statistics experiments. Such rescattering corrections introduce an error of about 10^\circ in determining \( \gamma \)[32]. Summing up all the theoretical uncertainties, and neglecting experimental errors, it is unlikely that this method will determine \( \gamma \) to better than \( \pm 20^\circ \). Nevertheless, this would be a substantial improvement relative to the present bounds (3).

We conclude this section with a simple observation [36], which enables an early detection of a CP asymmetry in \( B \to K\pi \). Using \( A(B^0 \to K^+\pi^-) = -P - T \), the hierarchy among amplitudes [34], \(|P| \gg |T| \gg |C| \), implies \( \text{Asym}(B^\pm \to K^{\pm}\pi^0) \approx \text{Asym}(B \to K^{\pm}\pi^\mp) \). This may be used to gain statistics by measuring the combined asymmetry in these two modes. The magnitude of the asymmetry depends on an unknown final state strong phase. Very recently a 90% confidence level upper limit was reported \( \text{Asym}(B \to K^{\pm}\pi^\mp) < 0.35 \)[19, 37].

7 Signals of new physics

The purpose of future \( B \) physics is to over-constrain the unitarity triangle. \( |V_{ub}| \) can at best be determined to 10% [38] and \( |V_{td}| \) relies on future measurements of the higher order \( B_s^0 - \bar{B}_s^0 \) mixing [11] and \( K^+ \to \pi^+\nu\bar{\nu} \)[39]. Constraining the angles \( \alpha, \beta \) and \( \gamma \) by CP asymmetries is complementary to these CP conserving measurements. The asymmetry measurements involve discrete ambiguities in the angles, which ought to be resolved [40].

Hopefully, these studies will not only sharpen our knowledge of the CKM parameters but will eventually show some inconsistencies. In this case, the first purpose of \( B \) physics will be to identify the source of the inconsistencies in a model-independent way. Let us discuss this scenario briefly by considering a few general possibilities.
Physics beyond the standard model can modify CKM phenomenology and predictions for CP asymmetries by introducing additional contributions in three types of amplitudes:

- $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ mixing amplitudes.
- Penguin decay amplitudes.
- Tree decay amplitudes.

The first case is the most likely possibility, demonstrated by a large variety of models [41]. New mixing terms, which can be large and which often also affect the rates of electroweak penguin decays, modify in a universal way the interpretation of asymmetries in terms of phases of $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ mixing amplitudes. These contributions can be identified either by measuring asymmetries which lie outside the allowed range, or by comparison with mixing-unrelated constraints. On the other hand, new contributions in decay amplitudes [42] are usually small, may vary from one process to another, and can be detected be comparing asymmetries in different processes. Processes in which the KM hypothesis implies extremely small asymmetries are particularly sensitive to new amplitudes.

To conclude this brief discussion, let us list a few examples of signals for new physics.

- $A_{ol} \geq O(10^{-2})$.
- Sizable asymmetries in $b \rightarrow s\gamma$ or $B_s \rightarrow \psi\phi$.
- “Forbidden” values of angles, $|\sin 2\beta - 0.6| > 0.2$, $\sin \gamma < 0.6$.
- Different asymmetries in $B^0(t) \rightarrow \psi K_S$, $\phi K_S$, $\eta' K_S$.
- Contradictory constraints on $\gamma$ from $B \rightarrow K \pi$, $B \rightarrow DK$, $B_s \rightarrow D_s K$.
- Rate enhancement beyond standard model predictions for electroweak penguin decays, $B \rightarrow X_{d,s}\ell^+\ell^-$, $B^0/B_s \rightarrow \ell^+\ell^-$.

8 Conclusion

The CP asymmetry in $B \rightarrow \psi K_S$ is related cleanly to the weak phase $\beta$ and can be used experimentally to measure $\sin 2\beta$. In other cases, such as in $B^0 \rightarrow \pi^+\pi^-$ which measures $\sin 2\alpha$ and $B \rightarrow DK$ which determines $\sin \gamma$, the relations between the asymmetries, supplemented by certain rates, and the corresponding weak phases are free of significant theoretical uncertainties. However, the application of these methods are expected to suffer from experimental difficulties due to the small rates of color-suppressed processes.

While one expects qualitatively that color-suppression is affected by final-state interactions, these long distance phenomena are not understood quantitatively. The case of $B \rightarrow K \pi$ demonstrates the need for a better understanding of these features, and the need for a reliable treatment of SU(3) breaking. That is, whereas the short distance effects of QCD in weak hadronic $B$ decays are well-understood [43], we are in great need
of a theoretical framework for studying long distance effects. An interesting suggestion in this direction was made very recently in [44].

We discussed mainly the very immediate $B$ decay modes, for which CP asymmetries can provide new information on CKM parameters. Asymmetries should be searched in all $B$ decay processes, including those which are plagued by theoretical uncertainties due to unknown final state interactions, and those where the KM framework predicts negligibly small asymmetries. Afterall, our understanding of the origin of CP violation is rather limited and surprises may be right around the corner.

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