The Investigation of CP Violation through the Decay of Polarized Tau Leptons*

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Abstract

Under the Assumption that CP violation is caused by exchange of a new boson, we propose to measure the magnitudes and CP-violating phases of the coupling constants of this boson to five different vertices in tau decay. This can be accomplished by studying the decays of polarized tau leptons produced at an $e^+e^-$ collider whose beams are polarized. We point out that CP violation in the tau decay tests most directly the assumption in the standard theory that the imaginary numbers in the mass matrix is the sole cause of CP violation.

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The standard theory assumes that the origin of the CP violation is the imaginary numbers in the mass matrix. These imaginary numbers cause the CKM matrix to have a complex phase that produces the observable CP violating effects. The matrix element in the mass matrix represents the coupling constants between the fermions and the vacuum expectation value of the scalar particle that was eaten by the massless gauge fermions to fatten themselves. Since neutrino are massless in the standard theory these CP violating complex coupling constants must be zero for the mass matrix of the neutrinos and thus the CP violation in the leptonic sector is zero. Maybe CP violation is caused by exchange of a new particle having complex coupling constant instead of arising from the complex coupling constants in the mass matrix. In this case CP violation may not have anything to do with the particle masses, nor does it have anything to do with the number of generations in nature. Thus testing the existence of CP violation in the leptonic decay of the tau directly tests the most basic assumption of the standard model, namely, “the CP violation is caused solely by the existence of complex numbers in the mass matrix.” Beside testing the CP violation in the pure leptonic sector, investigating the CP violation in tau decay using the polarized electron-positron colliding beam machine can also give us the information on the basic structure of CP violation in the first and second generation quarks. Usually the elements $V_{ud}$ and $V_{us}$ of the CKM matrix are assumed to be real because their phases can be transformed away and thus unmeasurable. However the phases and the absolute values of the coupling constants of our hypothetical boson to the ud and us vertices are measurable quantities in our experiment. Thus the way we are proposing to test CP violation can teach us much more than testing the unitarity of the CKM matrix. Non-unitarity of the CKM matrix does not necessarily prove that the complex number in the mass matrix is not the sole cause of CP violation, but the occurrence of CP violation in the decay of tau does.

My proposal can be put into three categories:

1. **For unpolarized Taus [3]:** In this case we measure the branching fraction of a semileptonic decay of $\tau^-$ with two or more hadronic final particles and compare it with that of the charge conjugate decay mode, for example:

$$\Gamma(\tau^-\to\pi^-+\pi^0+\nu_\tau) \neq \Gamma(\tau^+\to\pi^++\pi^0+\nu_\tau^-) \text{ or } \Gamma(\tau^-\to\pi^-+\pi^0+\nu_\tau) \neq \Gamma(\tau^+\to K^+\pi^0+\nu_\tau^-).$$

If any of the inequality holds, it would indicate a CP violation. However, the result of such an experiment is not easily analyzable in terms of the CP violating coupling constant. For example [1] in the 2$\pi$ mode shown above, if CP is violated 2$\pi$’s from $\tau^-$ decay will have different energy-angle distributions from that from $\tau^+$. This results in the difference in the rate of the inelastic final state interactions such as 2$\pi$→4$\pi$. Thus part of the width of 2$\pi$ mode is shuffled into that of 4$\pi$ mode and vice versa differently resulting in different branching fractions for these modes. The total width and as well as the partial widths of those channels without strong final state interactions are CP invariant. Since we do not know anything about the cross section
for $2\pi = 4\pi$, there is no way we can obtain the magnitude and the phase of the complex coupling constant responsible for the violation of CP using this method. Since CP violation discussed here is very indirect, we expect this effect to be small compared with the more direct effect to be discussed next.

2. **Use polarized electron and preferably also positron beams to investigate the semileptonic tau decay modes [1] with at least two final hadrons as considered above:** In this case it is relatively straight forward to interpret the result in term of the strength and the phase of the CP violating coupling constant. Let us assume that CP violation is produced by an interference between two diagrams, A and B, where A is a CP conserving diagram and B is a CP violating diagram due to exchange of a new meson $X$ with a complex coupling constant having a phase $\exp(i\delta_{\tau})$ for $\tau^-$ decay and $\exp(-i\delta_{\tau})$ for $\tau^+$ decay. Let us further assume that the final state interactions for A and B involve angular momenta $j$ and $j'$ with phases $\exp(i\delta_{j})$ and $\exp(i\delta_{j'})$ respectively. Let us write for $\tau^-$ decay

$$A = \exp(i\delta_{\tau})a, \quad (1)$$

$$B = \exp(i\delta_{j} + i\delta_{\tau})b; \quad (2)$$

and for $\tau^+$ decay

$$A' = \exp(i\delta_{\tau})a', \quad (3)$$

$$B' = \exp(i\delta_{j'} - i\delta_{\tau})b'. \quad (4)$$

In the above the strong interaction phases are unchanged because they are invariant under charge conjugation, whereas the CP violating phase has a different sign because of the requirement of Hermiticity in the weak Hamiltonian that is one of the requirements of the validity of TCP theorem [1]. The test of CP consists of comparing the interference between A and B with that between A' and B'.

$$F = A'B + B'A = \cos(\delta_{j'} - \delta_{j} + \delta_{\tau}) (a^*b + b^*a) + \sin(\delta_{j'} - \delta_{j} + \delta_{\tau}) (a^*b - b^*a), \quad (5)$$

$$F' = A'B' + B'A' = \cos(\delta_{j'} - \delta_{j} - \delta_{\tau}) (a'b + b'a) + \sin(\delta_{j'} - \delta_{j} - \delta_{\tau}) (a'b - b'a). \quad (6)$$

In the absence of strong interactions we have $\delta_{j'} = \delta_{\tau} = 0$ and $F'$ is related to $F$ by CP, namely $\delta_{1} = - \delta_{1}$, all the momenta of the particles go into the negative of the momenta of the charge conjugate particles and the polarization vector of the spin $\frac{1}{2}$ particle becomes the polarization vector of its anti-particle without changing sign. Since $\cos(\delta_{\tau}) = \cos(-\delta_{\tau})$, CP is conserved without the strong interaction for the cosine terms and thus the coefficients of cosine terms must be T even from the TCP theorem. On the other hand the coefficients of the sine terms must be T odd using similar argument. Thus for decay mode involving two hadrons, denoted by $q_1$ and $q_2$ for $\tau^-$ decay and $q'_1$ and $q'_2$ for $\tau^+$ decay, we may write:

$$F = C(\cos(\delta_{j'} - \delta_{j} + \delta_{\tau})[W \cdot q_1f_1 + W \cdot q_2f_2 + f_3] + \sin(\delta_{j'} - \delta_{j} + \delta_{\tau})W \cdot (q_1 \times q_2)f_4 \,), \quad (7)$$
\[ F' = C\{\cos(\delta_j - \delta_j')\left[W \cdot q'_1 f_1' + W \cdot q'_2 f_2' + f_3'\right] \} + \sin(\delta_j - \delta_j') W \cdot (q'_1 \times q'_2) f_4' \}, \quad (8) \]

where \( W \) and \( W' \) are the polarization vectors of the \( \tau^- \) and \( \tau^+ \) respectively. We assume CP is conserved in the production of taus, in that case \( W = W' \) in the center of mass system [1]. At energy of the tau/charm factory the production is almost all in the s wave, hence the polarization of the taus are almost all in the direction of incident beam and is independent of the production angle with the magnitude equal [1] to
\[ P = e_z (W_1 + W_2)/(1 + W_1 W_2), \quad (9) \]

where \( e_z \) is the unit vector in the direction of the incident electron, \( W_1 \) and \( W_2 \) are longitudinal polarizations of electron and positron respectively in the z direction.

At energies far above the threshold, for example at \( Z_0 \) pole, the d-wave production becomes comparable to the s wave production. Only s and d wave production of tau pair is allowed for the parity conserving one photon exchange interaction, whereas p-wave is also allowed in the parity non conserving \( Z_0 \) exchange. The conservation of angular momentum prohibits higher partial waves. After integrating with respect to the production angle only the component of the polarization in the z direction comes in but with average magnitude less than the effective beam polarization, \( P \), given above when the energy is far above the threshold. [1]

\( f' \)'s are calculable functions of the dot products of various four momenta of the problem (See reference [1] for examples). In general they are functions of tau production angle but it can be integrated out easily. By reversing the polarization we can isolate the coefficients of \( W \cdot q_1, W \cdot q_2, W \cdot (q_1 \times q_2), W \cdot q'_1, W \cdot q'_2, \) and \( W' \cdot (q'_1 \times q'_2) \). These six equations can be used to solve for three unknowns: \( C, \delta_j - \delta_j', \) and \( \delta_i \) in Equations (7) and (8) as functions of invariant mass of two hadrons. The CP violating phase \( \delta_i \) is the sum of two terms, \( \delta_i \) for the tau-nu vertex and a similar thing for one of the four possible final states: \( \delta_e \) for e-nu, \( \delta\mu \) for mu-nu, \( \delta_{ud} \) for the ud quarks, and \( \delta_{us} \) for the us quarks. The fact that we can obtain the only strong interaction effect, \( \delta_j - \delta_j' \), experimentally is very gratifying because it means that we do not have to rely upon QCD to calculate it.

I would like to make several important observations from the equations given in (7) and (8):

a. Two hadrons must be in two different angular momentum states in order to exhibit any CP violation. Proof: Since \( \delta_j - \delta_j' = 0 \) and \( \cos(\delta_i) = \cos(-\delta_i) \) CP can not be violated in the T-even terms. For T-odd terms if \( a \) and \( b \) represent the same partial wave they must be proportional to each other, hence \( (a^*b - b^*a) = 0 \), Q.E.D.

b. Thus decay channels such as \( \tau - \nu_{\tau} + \rho, \nu_{\tau} + K^+, \nu_{\tau} + \pi, \) etc. can not exhibit CP violation, because each hadronic final state shown above has a definite angular momentum. This shows that some of the CP violation discussed by Goozerat and Nelson [5] can not occur.
c. If we assume that CP violating diagram B is through the exchange of either spin-1 or spin-0 particle, and the diagram A involves only W exchange, then we can have only s-p interference for the CP violating terms. This fact has both advantage and disadvantage. The advantage is that s-p interference gives unique expressions for \( f_1, f_2 \) ... regardless of spin of exchanged particles so there is no ambiguity involved in what kind of expressions to use for these functions (See examples in [1]). The disadvantage is that this means we can not use the energy angular distribution to tell us the spin of the particle exchanged! [4] However if the exchanged particle is a Higgs it may have a larger coupling constant when coupled to heavier particles.

d. The CP violation is manifested by the difference in two cosine and sine functions shown in Equations (7) and (8):

The difference in cosine functions that is responsible for the CP violation in the T-even part is

\[
\cos(\gamma_j' - \gamma_j - \delta_t) - \cos(\gamma_j' - \gamma_j) = 2 \sin(\gamma_j' - \gamma_j) \sin(\delta_t). \tag{10}
\]

The difference in sine functions that is responsible for the CP violation in the T-odd part is

\[
\sin(\gamma_j' - \gamma_j - \delta_t) - \sin(\gamma_j' - \gamma_j) = 2 \cos(\gamma_j' - \gamma_j) \sin(\delta_t). \tag{11}
\]

These two equations show that the CP violation in both cases are induced only by the imaginary part of the coupling constant, i.e. \( \sin(\delta_j) \), as expected. The CP violation is absent in the T-even terms in the absence of strong final state interactions, i.e. \( \delta_j = 0 \) and \( \delta_j = 0 \), in agreement with the TCP theorem. This shows that in the leptonic decay to be treated next, only the T odd terms can appear for the CP violation. From Equation (11) we note that in the T-odd terms, the CP violation is maximum when \( \gamma_j' - \gamma_j = 0 \).

3. The third experiment [2] deals with testing of CP violation in the muon decay of tau. For this we require both polarized tau and detection of transverse polarization of muon. Since there is only one observable momentum in the final state, the momentum of the muon denoted by \( q \), the rotationally invariant quantities we can form are: \( W \cdot q, W_\mu \cdot q \), and \( W \cdot (q \times W_\mu) \).

According to the TCP theorem the first two can not violate CP because they are T even and no strong interactions. The third term is T odd and hence must also violate CP without the final state interaction. This implies that if this term is \( C W \cdot (q \times W_\mu) \) for \( \tau^- \) decay then for \( \tau^+ \) decay it must be \( CW' \cdot (q' \times W'\mu) \). Thus T violation and CP violation can be checked independently in this experiment. This experiment involves only leptons whereas the previous experiments involve leptons and hadrons.

Comments on Polarized beams at BTCF

The longitudinal polarization is the most important tool to select weak interaction events through the parity violating dot product \( P \cdot q \) where \( P \) is defined by Eq. (9) and \( q \) is any momentum of the particle. We can obtain many such dot products as well as T odd products: \( P \cdot (q_1 \times q_2) \) and \( P \cdot (q_\mu \times W_\mu) \) to obtain the structure of weak interaction. In the electron-positron colliding beam the charge conjugate final state is readily available and by comparing the two we obtain parameters for CP violation. At the Beijing Tau/Charm Factory (BTCF) there will be two separate rings for electrons and positrons. The Sokolov-Ternov effect will most likely be used to first transversely polarize both beams with assistance from wigglers to speed
up the polarization time to around 40 minutes for both beams [6]. Siberian snakes will be installed in both rings to rotate the beam polarization before and after the interaction to achieve the longitudinally polarized beams at the interaction region. The polarization achievable for each beam is around 70% without sacrifice to the luminosity if we avoid certain resonance energies of the machine [6]. The experiments I am proposing are not sensitive to the choice of energy except that the cross section must be large and the background must be minimal. The ideal energy is slightly below $\psi'(3685)$ resonance that is slightly below the charm threshold. Below charm threshold all e-hadron, mu-hadron, and e-mu events are tau induced and thus the background is greatly reduced. It should be emphasized that polarizing both electron and positron beams have the following advantages [1]:

1. The effective polarization is increased from $W_1$ to $P = (W_1 + W_2)/(1 + W_1 W_2)$. For example if $W_1 = W_2 = 0.7$ we have $P = 0.94$.

2. All cross sections, not just in the tau production, are increased by a factor $F = 1 + W_1 W_2$. For example $F = 1.49$ if $W_1 = W_2 = 0.7$. This means that it is advisable to have two beams polarized even when the polarization is not required. The longitudinal polarization does not affect any energy-angle distribution unless parity is violated, in other words, unless weak interaction is involved. Of course if a spin 0 particle is produced and decays subsequently by weak interaction, the initial beam polarization will not affect the energy angle of the subsequent decay, because the spin zero particle has no ability to transfer the information of its parent’s angular momentum.

The longitudinally polarized beams can also be used to investigate CP violation in $\Lambda \Lambda$ and $\Xi \Xi$ systems; we may also test whether right handed current and charged Higgs exchange play any role in $\tau - \nu_\tau + \mu + \nu_\mu$ by measuring the longitudinal polarization of muons [5]. Please notice that the CP violation in muon decay of tau deals with the transverse polarization of the muon.

**Future Outlook for CP Violation.**

By year 2005 SLAC, KEK and Cornell probably would have found out whether one of the six unitarity triangles representing the orthogonality between the first and second rows of the CKM matrix closes or not [8]. If it does not close, then the model in the standard model for CP violation is wrong. There are many possible reasons why the CKM matrix not being unitary. One possibility is the one conjectured in my work, namely, the mass matrix has nothing to do with the CP violation and thus CP violation is not prohibited by the smallness of neutrino mass and the number of generations has nothing to do with the CP violation. It has something to do with the exchange of a new particle or particles that couple to other particles with a complex coupling constant. Unitarity of KM matrix can also be violated by the existence of fourth generation fermions and yet CP violation still is caused by the complex coupling constants in the mass matrix and the W boson is still the only boson responsible for all the phenomena, including CP violation, associated with the weak charged current. Testing the CP violation in the tau decay can rule out the second possibility. Hopefully the Tau-Charm Factory will be finished by 2002 and some results on the CP violation will be available by 2005.
On the other hand if the B Factory experiment concludes that one of the unitarity triangle closes or almost closes, then in order to show the unitarity of the CKM matrix one has to check five other unitarity triangles and six normalization conditions which state that sum of the absolute square of each element of any row or column must be unity. This will take a long time to accomplish. In fact B Factories can not do this because we need the knowledge on the CP violation of the top quark.

The X particle responsible for CP violation is coupled to both the tau-nu vertex and the vertex for the final particles. Thus $\delta_t$ is sum of two terms one is due to CP violating phase $\delta_{t}$ in the tau sector and the other is any one of the four possible final states for tau decay: $\delta_e$, $\delta_\mu$, $\delta_{ud}$, and $\delta_{us}$. These five phase angles can not be separated out using the tau decay alone.

We need CP violation from other sources. CP violation in the decays $K^+ \pi^- + \bar{\nu}_\mu + \mu^+$ and the subsequent measurement of muon polarization can yield $\delta_\mu + \delta_{us}$, measurement of CP violation in $\mu^- \nu_\mu + e^+ + e^-$ with subsequent measurement of electron polarization can yield $\delta_\mu + \delta_e$. We have more than enough equations to solve for five unknown phase angles. Measurement of $\delta_e$ is probably the most difficult because it involves measurement of the transverse polarization of an electron. Fortunately the knowledge of $\delta_e$ is not necessary to solve for the other four phase angles.

Suppose we are able to determine all five of the CP violating phases and the coupling constants for all the vertices in the tau decay. We should then be able to construct some theory to correlate all the data and predict the mass and spin of the particle exchanged. This would be analogous to what Weinberg did for the standard theory. If we assume $\delta_t$'s to be roughly $\pi/2$ and the coupling constants to be about the same as the electroweak coupling constants due to some symmetry, and the CP violation has roughly the magnitude of $2 \times 10^{-3}$, the same as the CP violation in $K_L$ into $2\pi$, we can estimate the mass of the particle responsible for CP violation:

$$M_X = \frac{M_W}{\sqrt{2 \times 10^{-3}}} = 1.8 \text{ Tev.} \quad (12)$$

On the other hand if the particle X exists it should also contribute to the total and partial width of Tau. Since the standard model can predict the width to about 2% accuracy, we can obtain the lower bound of the mass of such a particle:

$$M_X > \frac{M_W}{0.02} = 560 \text{ Gev.} \quad (13)$$

If CP violation is transmitted by such a particle, then CP violation we are trying to detect is a 2% effect and we can see the effect with only about one million tau pairs that is achievable within on year of operation of the Tau/Charm Factory. BTCF is designed to produce about 100 million tau pairs per year.

20 years from now we may be investigating the properties of this particle, similarly to the way
we are investigating the properties of W’s and Z₀ today. After that we may finally be able to understand why there is more matter than anti-matter in our universe.

Some of my friends asked me if I had any specific model which predicts the existence and the properties of the hypothetical X particle. My answer is that it is premature to play such a game. Weinberg could not have invented the standard theory of the electroweak interaction without the discovery of parity violation, V-A nature of the coupling, conserved vector current theory, Higgs mechanism, etc. We do not know enough about the phenomenology of CP violation to construct a credible theory yet. Therefore I believe the most useful thing to do now is to devise methods to accumulate more facts on the CP violation independent of any particular model. When devising methods to test CP violation we must remember that its strength is at most about 2% of the weak interaction as discussed in conjunction with Equation (13). Devising methods for testing CP violation that require CP violation to be as strong as electromagnetic interaction is a waste of time. I believe testing the CP violation in the production mechanism of tau is such an example. In the following I give my reasons.

Appendix. CP violation in the production of Tau

I have ignored the possibility of CP violation in the production of taus and considered only the CP violation in the decay. The reason is that I cannot obtain the detectable magnitude of CP violation if we believe that CP violation in the production arises also from the interference between the standard one photon exchange diagram and a CP nonconserving diagram due to exchange of an X particle somewhere in the diagram. There are two classes of diagrams that interfere with the lowest order one photon exchange diagram to produce CP violating effects. The first class is the vertex correction due to exchange of a neutral X between the tau pair resulting in the electrical dipole moment of tau [9]. Now this diagram is similar to the one for calculating the anomalous magnetic moment of tau except that the photon is replaced by an X propagator. Since the contribution of the anomalous magnetic moment to the cross section is approximately \( \alpha / \pi \) [10], if we replace the photon propagator by an X particle propagator there must be another reduction factor of \((M_\tau / M_X)^2\). Using the value of \( M_X \) given by Equation (13) we obtain the contribution of tau electric dipole moment to the cross section to be at most \(2.4 \times 10^{-8}\) of the cross section, which is undetectable using 100 million taus. There are many papers written on the possible CP violation in the tau production due to the possible existence of an intrinsic electrical dipole moment of tau [11]. However I believe that the electric dipole moment of an elementary particle must arise from some T and P non-invariant interactions. Thus such possibility is excluded. In other words if the electric dipole moment of tau can give more than \( \alpha / \pi \) contribution to the cross section it must be due to a force stronger than the electromagnetic force and this is impossible [12].

A better mechanism is the interference between one photon exchange diagram and a diagram consisting of exchange of a neutral vector X between the electron pair and the tau pair. The interference between this and the one photon exchange diagram will contribute approximately to the cross section \((M_e / M_X)^2 < 10^{-5}\) of the cross section which is again undetectable. If the exchanged particle is a scalar particle its contribution will be further suppressed by a factor \( m_e/E \) because two diagrams do not interfere in the limit of zero electron mass. If the particle
exchanged is a Higgs there will be another factor of $m_e/M_w$ because Higgs particle's coupling to any particle is proportional to the mass of the particle. Thus for the Higgs particle exchange the effect is of order $(M_t^2 M_e^2) / (M_w M_H^2) \approx 5 \times 10^{-14}$ which is completely unobservable. There are some papers which claim to obtain null results for detecting the CP violation in Tau production with about one percent accuracy [13]. In my opinion such an experiment can not even measure the effect due to the anomalous magnetic moment which will produce the kind of correlation they are looking for, but its magnitude is $\alpha / \pi = 1/430$. However this does not have anything to do with CP violation, it is just the electromagnetic final state interaction mimicking the CP violation. This effect does not bother us, because the effect is the correlation between the decay product of $\tau^-$ and that of $\tau^+$ which we are not advocating in our experiment. In conclusion I believe that CP violation in the tau production is undetectable even with the designed luminosity of the Tau/Charm Factory. Only CP violation in the tau decay there is a hope of detecting it using the Tau/Charm Factory.

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References and Footnotes

4. This was not obvious in [1]. However if one uses the identity:
   
   $m_n^2 + q_1 \cdot q_2 = P_1 \cdot (q_1 + q_2) - M^2 / 2 = 0$, one can show that the expressions for $f_1, f_2$ ... are independent of spin of the exchanged particle.
8. Vera Luth, a speech made at SLAC 2000 on February 24, 1996.
10. Y.S. Tsai, Phys. Rev. 120, 269 (1960), Eq. (7).
12. Water molecule has an intrinsic electric dipole moment (See for example, Encyclopedia Britannica) of $1.17 \times 10^{-8}$ cm e that does not have anything to do with $T$ and $P$ violations. The water molecule is not rotationally symmetric, hence the usual argument that the electrical dipole moment being a vector must be proportional to the spin of the particle does not apply. Hence it violates neither $P$ nor $T$. Tau being an elementary particle can not have this kind of dipole moment.

13. ALEPH Collaboration, Phys. Lett. B281, 405 (1992); B297, 459 (1992). A null result with one percent accuracy was reported at the Tau/Charm Factory Workshop in Beijing (1996) by Nading Qi of BEPC on the correlation $(P_1 - P_2) \cdot (q_e \times q_{\mu})$, where $P_1$, $P_2$, $q_e$, and $q_{\mu}$ are momenta of incident electron, incident positron, decay electron from $\tau^-$ and decay muon from $\tau^+$ respectively. Such correlation is clearly $T$ odd. Since we have shown that $T$ violation cannot be detected in the pure leptonic decay without detecting the polarization of the decay lepton the $T$ violation necessary for this kind of correlation must come from the $T$ violation in the production. We have shown such a violation is at most $10^{-5}$ and thus cannot be detected even with the future Tau/Charm Factory. It can barely detect this kind of correlation of the magnitude $\alpha / \pi$ due to the final state electromagnetic interaction.