Abstract

The implications of the recent measurement of the left-right asymmetry, $A_{LR}$, by the SLD Collaboration for theories with extended gauge sectors is examined. We show that it is possible to arrange for large, negative values of $S$, based on an analysis of leptonic data, without serious side effects for other observables in certain classes of models. The implications of such scenarios for future measurements on the $Z$ peak, at the Tevatron, and for atomic parity violation experiments are examined.

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The left-right polarization asymmetry, $A_{LR}$, provides one of the most sensitive probes of the standard model (SM) leptonic couplings of the $Z$ boson. At tree level, $A_{LR}$ is directly related to the ratio of the vector to the axial vector coupling of the electron and is independent of the nature of the fermions produced in the final state, \textit{i.e.},

$$A_{LR} = \frac{2y}{1+y^2}, \quad (1)$$

where $y = v_e/a_e = 1 - 4x_{\text{eff}}$ in the SM. Recently, the SLD Collaboration\cite{1} has announced a new, high-precision determination of $A_{LR}$ based on their 1993 data sample, $A_{LR} = 0.1656 \pm 0.0073 \pm 0.0032$, which should be compared to the earlier result\cite{2} from their 1992 data, $A_{LR} = 0.100 \pm 0.044 \pm 0.004$. (Both determinations essentially make use of only the hadronic final states in $e^+e^-$ annihilation.) Within the SM, box and vertex corrections can be directly evaluated so that the radiatively corrected values for $x_{\text{eff}}$ can be extracted from both determinations: $x_{\text{eff}} = 0.2288 \pm 0.0009 \pm 0.004$ (1993) and $x_{\text{eff}} = 0.2378 \pm 0.0055 \pm 0.0005$ (1992) which when folded together yield $x_{\text{eff}} = 0.2290 \pm 0.0010$ if the errors are combined in quadrature. The surprise here is that this value of $x_{\text{eff}}$ is quite different than that obtained from LEP data alone, $x_{\text{eff}} = 0.2324 \pm 0.0005$, or when LEP data is combined with $W$ boson mass determinations and low energy neutral current data, $x_{\text{eff}} = 0.2325 \pm 0.0005$\cite{3}, both of which are several $\sigma$ away from the SLD value. The implications of this apparent conflict are at the moment unclear as the LEP value is ‘supported’ by the latest set of preliminary $W$ mass measurements by both CDF and D0\cite{4} (which now yield a new world average value of $M_W = 80.21 \pm 0.18$ GeV). On the otherhand, the $A_{LR}$ measurement is extremely sensitive to $x_{\text{eff}}$, is exceptionally clean, and has totally different systematics than the LEP experiments. (We note in passing that for top-quark masses in the range 160-180 GeV, near the central value extracted from the radiative corrections analyses of LEP data, $x_{\text{eff}}$ is numerically equal to $\sin^2\theta_{\text{MS}}(M_Z)$ to a very high precision, as defined
in the minimal way\cite{5}, i.e., when the additional Marciano-Rosner subtraction scheme\cite{5} is not employed.)

One approach is to combine the \(A_{LR}\) measurement with the other existing data and see what we can learn. If we take the combined LEP asymmetry and leptonic width results together with the latest \(M_W\) determination and the value of \(A_{LR}\) (as they are all rather insensitive to the value of \(\alpha_s(M_Z)\)), a Peskin-Takeuchi\cite{7} type analysis can be performed which yields the central values \(T \simeq -0.38 \pm 0.34\) and \(S \simeq -0.58 \pm 0.30\) assuming \(m_t = 165\) GeV and \(M_{Higgs} = 300\) GeV\cite{8}. In what follows, we will adopt these values as input into our analysis as they conform to the most recent complete fit to electroweak data performed in Ref.\cite{3}. (For larger values of the top-quark mass, \(m_t = 175\) GeV, say, the central value for \(T\) decreases by about 0.20 and \(S\) increases only very slightly for fixed \(M_{Higgs}\). \(S\) and \(T\) would then have comparably negative values which are about 2\(\sigma\) from zero.) We should note, however, that in the absence of the new \(A_{LR}\) result from SLD, both \(S\) and \(T\) would be essentially zero as we might have anticipated based on the well-known excellent fit of the SM to the previously existing data. The SM is certainly very far from being excluded by this new analysis (as the SM prediction still lies well within the 90\% CL ellipsoid), however, we are led to contemplate what kind of new physics would push the fit for the leptonic data down into the negative \(S,T\) quadrant without much influence elsewhere.

In this paper, we examine several extended electroweak gauge models, which predict the existence of a new \(Z'\) (and in some cases, a \(W'\)) gauge boson(s), in the light of the recent \(A_{LR}\) measurement by SLD. In particular, we are interested in identifying models that move us closer to the central values of \(S\) and \(T\) above without overly disturbing other electroweak observables such as \(R_{inv} = \Gamma_{inv}/\Gamma_{\ell}, R_h = \Gamma_{had}/\Gamma_{\ell}, R_b = \Gamma_b/\Gamma_{had}, M_W,\) and \(A_b,\) the b-quark asymmetry parameter. The present analysis only uses these separate quantities as additional constraints on the parameters of potential \(Z'\) models. We remind the reader
that the constraints on the existence of a new $Z'$ from radiative corrections analyses made before the announcement of the SLD result were very strong\cite{9}, in some cases requiring $M_{Z'} > 0.5 - 1$ TeV. This merely reflected the observation that since the SM fit the data so well there was little room remaining for significant shifts in observables due to new physics. The new SLD result for $A_{LR}$, now provides a bit more breathing space for a relatively light $Z'$, possibly in the mass range that can be explored at the Tevatron in the future, e.g., 500-800 GeV. In this analysis, we are trying to identify if regions exist in a given extended gauge model’s parameter space that allow large, negative $S$ (as defined by the leptonic data) without upsetting the values of the other observables. These regions, if they exist for a given model, will be ones where $v_e$ experiences a significant increase in magnitude while the other $Z$ couplings will be little affected. The ‘successful’ regions of parameter space we find below are meant only to be suggestive as they depend upon the specific values of the input parameters, e.g., $m_t$, that we employ in this analysis. As we will see below, it is not always possible for models to produce a significant shift in $v_e$ without there being sizeable changes in the other couplings.

Direct searches for a $Z'$ at the Tevatron by CDF have resulted in a preliminary lower limit of 495 GeV, from a partial analysis of the electron data from the 1992-93 run Ia\cite{10}, assuming SM couplings and that the $Z'$ decays to SM particles only. We note that the full analysis of the data from this run, including muons, may increase this limit by about 90 GeV. In addition, run Ib has already commenced which will increase these limits even further. In any of the more realistic extended electroweak models (EEMs), the $Z'$ couplings are sufficiently different from those of a SM $Z$ so that the actual mass limits could be significantly higher or lower than the quoted 495 GeV result. For example, in $E_6$ models\cite{11}, under identical assumptions, the corresponding bounds hover near 400-420 GeV for all values of the $E_6$ mixing parameter $-\pi/2 \leq \theta \leq \pi/2$ while in the Left-Right
Symmetric Model (LRM)\cite{12} the limit is 445 GeV assuming the ratio of $SU(2)_R$ to $SU(2)_L$
gauge couplings, $\kappa = g_R/g_L = 1$. Similarly, in the Alternative LRM of Ma et al. (ALRM)\cite{13},
one finds a $Z'$ mass limit of nearly 550 GeV but in the Un-Unified Model (UUM)\cite{14} of Georgi
et al., the limits vary from 400 to 600 GeV depending upon the value of the model parameter,
$0.22 \leq s_\phi \leq 0.99$. As we will see below, it is quite easy for the $Z''$'s of interest to us to satisfy
these direct search constraints. We will assume for simplicity that in models where a $W'$
is present, it plays no role in low-energy processes and does not mix with the SM $W$. We
note that while the above list of extended models is reasonably representative it is far from
exhaustive as the literature on this class of extensions to the SM is quite robust. For the
explicit expressions of the various fermion couplings in each of the models described above,
we refer the reader to the original literature.

We first wish to explore these various models to find out which, if any, have parameter
spaces that allow us to move toward the negative $S$ and $T$ regions discussed above. While a
general Peskin-Takeuchi analysis cannot be applied to an extended gauge model as a whole,
since the $Z'$ can induce significant flavour-dependent modifications in fermionic couplings,
a restricted analysis of this kind is possible if we limit ourselves to leptonic observables at
the $Z$-pole and $M_{W'}$. The reasoning here is clear; since there are only three observables
under consideration one is completely free to parameterize their potential deviations from
SM predictions in terms of three variables which can be identified as $S, T$ and $U$. This
has been pointed out most clearly by the work of Altarelli et al.\cite{15}, and this particular
approach has been employed by other authors to constrain some other extended models\cite{16}
not discussed in the present work. This particular set of observables has the added advantage
of being quite insensitive to the precise value of $\alpha_s(M_Z)$. The influence of a $Z'$ and small
$Z - Z'$ mixing (through an angle $\phi$) has three direct effects on these observables which can
be summarized as follows. Due to mixing, the $Z$ and $Z'$ form the mass eigenstates $Z_{1,2}$
with $M_1 < M_Z$, the SM $Z$ mass. However, using the observed mass (i.e., $M_1$) as an input parameter $(i)$ modifies the traditional $W - Z$ mass relation by introducing an effective $\rho$-parameter, $\rho = 1 + \delta \rho$, where $\delta \rho$ is of order $r = M_1^2/M_Z^2$ in models where $SU(2)_L$ breaking is performed solely by isodoublets (as will be the case for all the models we examine below).

This $\rho$ parameter also produces an overall rescaling of the $Z$ partial widths, as calculated in the `$G_F$'-scheme, when the measured $Z_1$ mass is used as input. $(ii)$ The SM vector and axial-vector couplings $v, a$ are directly modified by the small admixture of the corresponding $Z'$ couplings $v', a'$. $(iii)$ If one uses the observed $Z_1$ mass to define the weak mixing angle, the shift from the SM $Z$ mass due to mixing induces a corresponding change in the value of $\sin^2 \theta_w$ that should be employed in the evaluation of fermionic couplings.

In almost all models of interest, including those discussed here, $\phi, r,$ and $\delta \rho$ are directly related to each other via a model-dependent parameter, $\gamma$, which is of order unity and is sensitive to the details of the symmetry breaking scheme of the extended model. In terms of the elements of the $Z - Z'$ mass matrix, $\gamma$ is defined by writing the matrix in the form

$$M^2 = \begin{pmatrix} M_Z^2 & \gamma M_Z^2 \\ \gamma M_Z^2 & M_{Z'}^2 \end{pmatrix},$$

which exploits the fact that the vacuum expectation values (vev’s) contributing to both the $M_{11}^2$ and $M_{12}^2$ elements are the same. The particular values of $\gamma$ for the models above have been discussed in detail elsewhere\textsuperscript{[17]} and we simply give the relevant expressions below:

$$\gamma_{LRM} = -(\kappa^2 - (1 + \kappa^2) x_w)^{1/2},$$

$$\gamma_{ALRM} = \frac{x_w t_{\beta}^2 - (1 - 2 x_w)}{(1 - 2 x_w)^{1/2}(1 + t_{\beta}^2)},$$

(3)
\[ \gamma_{ER5M} = -2\sqrt{\frac{5x_w}{3}} \left[ \left( \frac{c_\beta}{\sqrt{6}} - \frac{s_\beta}{\sqrt{10}} \right) t_\beta^2 - \left( \frac{c_\beta}{\sqrt{6}} + \frac{s_\beta}{\sqrt{10}} \right) \right] (1 + t_\beta^2)^{-1}, \]
\[ \gamma_{UUM} = \frac{-(1 - x_w)^{1/2} s_\phi}{(1 - s_\phi^2)^{1/2}}, \]

where \( x_w = \sin^2 \theta_w, \) \( t_\beta = \tan \beta = v_t/v_b, \) the usual ratio of vacuum expectation values responsible for the top and bottom quark masses, and \( s_\theta(c_\theta) = \sin(\theta)\cos(\theta) \) being the sine and cosine of the ER5M mixing angle discussed above. Note that if \( \theta = -90^\circ \) (model \( \chi \)) then \( \gamma_{\chi} = -(2x_w/3)^{1/2} \) is independent of the value of \( \tan \beta. \) To lowest order in \( r, \) one then finds the simple result \( \phi = -\gamma r, \) \[ (4) \]
\( \delta \rho = \gamma^2 r. \)

To similar leading order in \( \phi \) (or \( r \)), Altarelli et al.\[^{15}\] then obtain the following relations for the shifted values of \( S, T, \) and \( U: \)
\[ \Delta T \simeq \alpha^{-1}(\delta \rho - 4a' \phi), \]
\[ \Delta S \simeq 2\phi \alpha^{-1}[ (1 - 2x) v' - (1 + 2x) a' ], \]
\[ \Delta U \simeq 4\phi \alpha^{-1}( v' + 3a' ), \]

where \( x \) is simply the value of \( \sin^2 \theta_w \) one would obtain in the SM limit, which we take to be 0.2325 in light of the discussion above and the assumed values of \( m_t \) and \( M_{Higgs} \) we use as input into this analysis. \( \alpha^{-1} \simeq 128 \) and \( v' \) and \( a' \) are the charged lepton couplings to the \( Z' \) normalized as in SM. Of course, in the results presented below, we use only exact expressions which include all the higher order terms in \( \phi \) (or \( r \)) and not the suggestive approximate forms given above. These approximate expressions do, however, reproduce the exact results at the level of 5\% or so for the cases of interest and show us precisely which combinations of the
properties of the $Z'$ are being probed. The exact expressions are cumbersome and not very enlightening and thus we do not reproduce them here.

To demonstrate that an arbitrary extended model will not put us into the $S - T$ range of interest, we first consider the case of the ALRM. The couplings in this model are free of independent parameters and $\gamma$ depends solely on the ratio of the two Higgs doublet vev’s (which as mentioned above is traditionally denoted by $\tan\beta$) that are present in the model. The only additional parameter we need to consider is the mass of the $Z'$ itself. As $\tan\beta$ is varied in this model, for fixed $M_{Z'}$, a curve is traced out in the $S - T$ plane as is shown in Fig. 1. Here we see that this model populates the wrong part of the $S - T$ plane and never reaches sufficiently large negative values of $S$, close to the central value of the leptonic $S - T$ fit described above. This model demonstrates that it is not obvious that a given extended model can actually produce the desired range of $S, T$. In fact, many other extended models tend to favour $S > 0$, an example of which is the universality violating model discussed in Ref. [16].

However, some models can produce negative values of $S$ and $T$ (especially if they have greater parameter flexibility) an excellent example being those based on the gauge group $E_6$, i.e., the Effective Rank-5 Models(ER5M). In Figs. 2a and 2b, we present the values of $S$ and $T$ as functions of the $E_6$ parameter $\theta$ assuming a representative $Z'$ mass of 750 GeV and a range of $\tan\beta$ values. Here we see that once $\tan\beta$ exceeds 5-10 the curves become quite indistinguishable. Figs. 3a and 3b show the resulting $S - T$ plots for this model assuming $M_{Z'} = 500$ and 750 GeV, respectively, for the same set of $\tan\beta$ values. As $\theta$ is varied, the curves form closed ellipses which share a common point at $\theta = \pm 90^\circ$ where the results are $\tan\beta$ independent. Also shown on these figures is the ‘data point’ corresponding to the $S - T$ fit described above. Similarly, Fig. 3c shows the case with $\tan\beta = 20$ held fixed but with $Z'$ masses varying between 500 and 1500 GeV. For low masses, it is clear that
sufficiently negative values of $S$ and $T$ are easily reached but this becomes more difficult as $M_{Z'}$ increases beyond $\sim 1$ TeV. In fact, for $M_{Z'} = 500(600, 750, 850)$ GeV, the best fit is provided by $\theta = 24^\circ(19.5^\circ, 9.5^\circ, 0.5^\circ)$ with the corresponding $\tan\beta$ values of $3(4, 8.5, 100)$. ($\tan\beta = 100$ was assumed to be the maximum allowed value but the difference in the fit between $\tan\beta = 20$ and 100 is very minimal.) For larger masses, the best fit value for $\theta$ becomes negative (as we are pushed to the lower left end of the ellipse’s major axis corresponding to increasingly negative values of $\theta$) while $\tan\beta$ assumes its maximally allowed value, hence, the choice of a large $\tan\beta$ in Fig. 3c. These best fit values are suggestive of the region of the model’s parameter space that is preferred by the $S - T$ analysis. We thus conclude that the ER5M with a $Z'$ in the 500-850 GeV can provide reasonable $S, T$ values but larger masses would have somewhat higher $\chi^2$. This entire mass range is clearly accessible at the Tevatron for integrated luminosities greater than 100-500 $pb^{-1}$.

Interestingly, the regions in the ER5M parameter space which yield negative values for both $S$ and $T$ near the central values of the fit lead to very small fractional changes in all of the SM $Z$ fermionic couplings except for the electron’s vector coupling. Specifically, for the case of a 750 GeV $Z'$ with $\theta$ and $\tan\beta$ values in the neighborhood of the ranges quoted above, we typically find that $v_d, a_d, v_b, a_b,$ and $a_e$ are only modified at the level of $0.2\%$, $a_u$ and $v_\nu = a_\nu$ by $0.5\%$, and $v_u$ by $0.7\%$. Shifts of similar magnitude are also encountered for the lower mass 500 and 600 GeV $Z'$ cases assuming the specific values of $\theta$ and $\tan\beta$ listed above.

We must be sure to check that for the above parameter choices, other electroweak observables are not overly affected or perhaps lead to improvements in comparison to the data since combinations of the the small changes in the individual couplings may conspire together to cause a significant deviation. Based on the apparent shifts in the couplings discussed in the previous paragraph, however, we expect to be quite safe. To prove that this
is indeed the case, we show the ratio of the predictions of the ER5M to those of the SM for $R_h, R_b, R_{inv}, A_b$ as well as the predicted fractional shift in $M_W$ in Figs. 4a-e as a function of $\theta$ with $M_{Z'} = 750$ GeV. In order to be specific, we will assume $\alpha_s(M_Z) = 0.123$ when performing our numerical evaluations. For values of $\theta$ (or $\tan\beta$) sufficiently far away from the range which yields negative $S$ and $T$ as discussed above, we see that significant shifts may occur in any or all of these observables. However, for the specific range of parameters of interest to us very little influence from $Z - Z'$ mixing is noted. Typically, the largest deviations we find are an increase in $R_{inv}$ by $\simeq 0.67\%$ (i.e., $\Delta N_\nu = 0.0022$), a decrease in $R_b$ by $\simeq 0.39\%$, an increase in $R_h$ by $\simeq 0.4-0.5\%$, and an upward shift in $M_W$ by about 80 MeV, all of which are at the level being probed by current experiment. The existing 95% CL upper limits on the allowed variation in these quantities (in the directions that they are shifted within the model) are approximately $1.39\%, 1.34\%, 0.47\%$ and 350 MeV, respectively, for $m_t = 165$ GeV and $M_{Higgs} = 300$ GeV with $\alpha_s(M_Z) = 0.123$ held fixed. Shifts of similar magnitude are also found for lighter (and somewhat heavier) $Z'$ masses for parameters tuned near the above choices. We can thus conclude that the ER5M has sufficient freedom such that for a reasonable range of $Z'$ masses (500-1000 GeV, say) we can find values of the parameters $\theta$ and $\tan\beta$ that lead to large negative $S$, through a significant shift in $v_e$, without similar drastic changes in the other couplings or direct observables. However, the predicted shifts are not so small as to render them unobservable in the near future and the $Z'$ masses are not so large as to make direct production of a new neutral gauge boson arising from this model impossible to observe at the Tevatron.

Next, we turn our attention to the LRM with the variable $\kappa = g_R/g_L$ as the only free parameter in addition to $M_{Z'}$. As such, there is clearly much less freedom in the model. Fig. 5 shows the $S-T$ plane for this model assuming the $Z'$ mass lies in the 500-1500 GeV range; possible effects from the $W'_R$ are ignored. The ‘curves’ are essentially straight lines
that penetrate into the negative $S$, negative $T$ quadrant as the value of $\kappa$ is varied. As $\kappa$ is increased, we move further down and to the left along the curve for each value of the $Z'$ mass. Clearly, somewhat heavier (> 750 GeV) $Z'$ masses are favored by the $S - T$ region fit to the present data. For $0.55 < \kappa < 2$, we find that a $Z'$ in the 0.8-3.0 TeV mass range will yield results for $S$ and $T$ quite close to the central values from the fit. (We restrict our attention to this $\kappa$ region as we expect on general grounds that this ratio should not be too different from unity as suggested by grand unified models. Finiteness of the $Z'$ couplings in this models also requires that $\kappa^2 > \frac{1}{1-x_w} \simeq 0.55$.) As $M_{Z'}$ increases, the best fit values of $\kappa$ also increases so that for larger masses, the restricted range of $\kappa$ we employ is insufficient to reach close to the $S - T$ central values. For a $Z'$ mass of 800(1000, 1200, 1500, 2000, 3000) GeV, the range of $\kappa$ values with the best $\chi^2$'s is centered correspondingly at 0.82(0.89, 0.98, 1.13, 1.39, 1.95). As in the ER5M case, we must also test that these values of the model parameters do not significantly modify the other observables. (We still will implicitly assume that the existence of the $W'$ has no influence here.) Figs. 6a-e show the $\kappa$ sensitivity of the observables discussed above all of which are found to slightly increase in magnitude in comparison to their SM predictions. When the best fit values of the $\kappa$'s are employed we see that all the variations are safely small provided $M_{Z'}$ is greater than about $\sim 1$ TeV or so. Thus for $Z'$ masses in the 1-3 TeV range with the above $\kappa$ values the LRM will yield only small modifications to nonleptonic observables and will still produce negative values of $S$ and $T$ in the range of interest.

The last model we consider is the UUM where we will again assume for simplicity that $W'$ effects can be ignored. The mass matrix parameter $\gamma$ in this model is completely determined by the value of $s_\phi$ so there is less parameter freedom than in the $E_6$ ER5M scenario. Fig. 7 shows the $S - T$ plane for this model where the general behaviour $S \simeq T < 0$ is observed. (This would be a particularly nice prediction if the top-quark mass were 175 GeV
as discussed above.) Here, the curves for different \( Z' \) masses lie atop one another and as the free parameter \( s_\phi \) increases we move away from the region of the origin out towards negative \( S, T \). We arrive near the central part of the \( S, T \) region of interest with \( s_\phi = 0.61(0.76, 0.84, 0.89, 0.92) \) assuming \( M_{Z'} = 1(1.5, 2, 2.5, 3) \) TeV. However, even for sizeable \( Z' \) masses we find that some of the other observables are significantly altered. While the resulting shifts in \( R_b, A_b, \) and \( R_{inv} \) are found to be quite small, below \( \sim 0.1\% \), in all the cases above, we find that \( R_h \) is significantly increased by \( 1.1 - 1.7\% \), with the magnitude of the shift decreasing very slowly with larger \( Z' \) mass. Similarly, the \( W \) mass is shifted upwards by about 160 MeV in all cases as larger values of \( M_{Z'} \) are compensated for by the correspondingly increasing values of \( s_\phi \) required by the \( S - T \) analysis. While we might be able to defeat any potential \( W \) mass shift problem by allowing a small \( W - W' \) mixing, the rather large increase in \( R_h \) is too big to be tolerated by existing data even when we allow for the uncertainty in \( \alpha_s(M_Z) \).

To reduce the upward shift in \( R_h \) to a manageable level, below \( \lesssim 0.4\% - 0.5\% \), would require increasing the \( Z' \) mass to the multi-TeV range (beyond what can be easily probed by the LHC) and fine-tuning \( s_\phi \) to values extremely close to unity. This would force the \( Z' \) to be strongly coupled as discussed in the last two papers in Ref.\[14\]. Thus, unless we allow for an extremely massive, strongly coupled \( Z' \) and (possibly) significant \( W - W' \) mixing, the UUM does not provide adequately for the possible shift in the \( S, T \) parameters while leaving other observables essentially unaffected.

Besides the \( W \) mass measurement, direct \( Z' \) production, and the refinement of the data for on-resonance observables, how can we probe the physics of these models above in the near future? One possibility is to further improve the measurements\[18\] of the weak charge, \( Q_W \), as determined by atomic parity violation experiments. The sensitivity of \( Q_W \) to the existence of a \( Z' \) has been discussed in the recent literature by several authors\[19\]. As above, we take the SM value of \( Q_W \) to be that given by the choice \( m_t = 165 \) GeV and \( M_{higgs} = 300 \)
GeV corresponding to \( \sin^2\theta_{\text{MS}}(M_Z) = 0.2325 \). For Cesium, the current experimental value of \( Q_W \) is \(-71.04 \pm 1.58 \pm 0.88\), the SM predicts \(-73.25\), yielding \( \Delta Q_W = Q_W^{\text{exp}} - Q_W^{\text{SM}} = 2.21 \pm 1.81 \). Future experiments are expected to reduce these errors by a factor of order 5-6 making such measurements competitive with \( M_W \) determinations in probing electroweak corrections.

How large and what sign is the predicted shift in the value of \( Q_W, \delta Q_W \), due to the existence of a \( Z' \) for the models discussed above? (We note in passing that the effective value of the \( S \) parameter extracted from atomic parity violation measurements in Cesium is essentially given by \( S_{\text{eff}} = -\delta Q_W/0.795 \).) Figs. 8a-b show the \( E_6 \) model predictions for a \( Z' \) of mass 500 or 750 GeV, respectively, as a function of \( \theta \), assuming different \( \tan\beta \) values. For the 500 GeV case, we see \( \delta Q_W \) is very small and positive near \( \theta = 24^\circ \) if \( \tan\beta = 3 \), while for the corresponding 750 GeV example, \( \delta Q_W \simeq 0.25 \). For the LRM scenario, Fig. 9 shows that \( Z' \) masses in the 1-2.5 TeV range yield small, positive predictions for \( \delta Q_W \simeq 0.2 \) when the values of \( \kappa \) found above are used. Thus we see a general pattern arising, which suggests that our ‘successful’ models predict small, positive increases in \( Q_W \) (i.e., small, negative values of \( S_{\text{eff}} \simeq -0.3 \)) which should be observable during the next round of atomic parity violation experiments.

In this paper, we have examined the possibility that extended electroweak gauge models can allow for a large and negative value of \( S \), as determined by LEP/SLC leptonic decay and asymmetry data, while not significantly affecting other observables. What is required is a region of the model’s parameter space where the charged lepton vector coupling, \( v_e \), is significantly increased while the corresponding fractional deviations in all other couplings are obliged to remain small. This requirement is far from trivial and cannot be realized in most models; even the ‘successful’ models only do so over a relatively narrow range of parameters.

Specifically, our results can be summarized as follows:
(i) The Alternative Left-Right Model was found to lead to values of $S$ and $T$ which populated the wrong regions of the $S - T$ plane, e.g., when $S$ was sufficiently negative, $T$ was very large and positive.

(ii) The Un-unified model, while easily obtaining $S, T$ values of interest over a wide range of $Z'$ masses, always resulted in too large an increase in $R_h$ by a factor of 2-3 beyond what the existing data can tolerate. This was found to be true even for extremely large $Z'$ masses. This situation might be avoided if the model parameter $s_\phi$ were tuned extremely close to unity but then we would pay the price of having extreme fine-tuning and a strongly coupled new gauge sector.

(iii) The Left-Right Model was found to easily satisfy all of the necessary constraints for $Z'$ masses in the 1-3 TeV range with appropriately chosen values of the parameter $\kappa = g_R/g_L$, assuming that the $W'$ did not influence low energy physics. These $Z''$s would clearly be beyond the range accessible to the Tevatron and must await discovery at the LHC and NLC.

(iv) The $E_6$ models, which perhaps have the most flexibility amongst those models examined here, were also found to be able to satisfy all the necessary conditions for $Z'$ masses in the 500-1000 GeV range for values of the model parameters $0^\circ \leq \theta \leq 24^\circ$ and $\tan \beta > 3$. This $Z'$ mass range is accessible at the Tevatron for integrated luminosities in excess of 100 pb$^{-1}$ which may be achieved in the not too distant future.

(v) In addition to $Z$-pole, $W$-mass, and direct $Z'$ searches, perhaps one of the best signatures for the models discussed here is a small positive increase in the value for the weak charge in Cesium, $\delta Q_w \simeq 0.2 - 0.3$, in comparison to the SM prediction. In the usual language, this would correspond to extracting an effective value of the $S$ parameter from these measurements in the range $-0.25$ to $-0.4$. Such a shift was observed to occur for both
the LRM and $E_6$ models in the parameter ranges of interest. Future experiments searching for atomic parity violation should be sensitive to such effects.

Hopefully precision measurements may soon begin to yield some evidence for new physics beyond the Standard Model.

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References


[8] The author would like to thank Michael Peskin for providing him with the results of this analysis.

[9] See, for example, W. Hollik and M. Swartz, talks given at the *XVI International Symposium on Lepton-Photon Interactions*, Cornell University, August 1993.


Figure Captions

Figure 1. $S - T$ plot for the ALRM assuming $Z'$ masses of 500(dots), 750(dashes), 1000(dash-dots), 1250(solid), or 1500(square-dots) GeV. The value of the parameter $tan\beta$ varies along each curve. The $S, T$ origin was assumed to correspond to $m_t = 165$ GeV and $M_{Higgs} = 300$ GeV within the SM.

Figure 2. Values of the parameters (a)$T$ and (b)$S$ in the ER5M case as functions of the parameter $\theta$ assuming $M_{Z'} = 750$ GeV. From top to bottom, the curves correspond to $tan\beta = 1, 1.5, 2, 3, 5, 10, 40$.

Figure 3. $S - T$ plots for the ER5M assuming a $Z'$ mass of (a)500 or (b)750 GeV for the same set of $tan\beta$ values as in Fig. 2. The plotted ‘data’ point corresponds to the $S - T$ fit to the LEP and SLC leptonic data. From right to left, the ellipses correspond to $tan\beta = 1, 1.5, 2, 3, 5, 10, 40$. (c) Same as (a) and (b) but with $tan\beta = 20$ for increasing $Z'$ masses of 500(dots), 750(dashes), 1000(dash-dots), 1250(solid), and 1500(heavy solid) GeV.

Figure 4. The ratio of the predicted values for (a)$R_h$, (b)$R_b$, (c)$R_{inv}$, and (d)$A_b$ in the ER5M compared to the SM for a 750 GeV $Z'$. (e)The corresponding fractional shift in the $W$ mass. The curves are for the same values of $tan\beta$ as shown in Figs. 2a-b, with the smallest value of $tan\beta$ corresponding to the lowest dotted curve.

Figure 5. $S - T$ plot for the LRM for the same $Z'$ mass values displayed in Fig. 3c together with the ‘data’ point representing the $S, T$ fit.

Figure 6. Same as Fig. 4, but for the LRM as a function of the parameter $\kappa$. From top to bottom the results shown are for $Z'$ masses of 0.8, 0.9, 1, 1.2, 1.5, and 2 TeV.
Figure 7. $S-T$ plot for the UUM; the curves corresponding to the different $Z'$ masses discussed in the text (1 – 3 TeV) lie atop one another.

Figure 8. $\delta Q_W$ as predicted in the $E_6$ model case as a function of $\theta$ for a $Z'$ mass of (a) 500 and (b) 750 GeV. From bottom to top, the curves on the right-hand side of the figure correspond to the same values of $\tan\beta$ as in Fig. 2.

Figure 9. Same as Fig. 8, but for the LRM as a function of $\kappa$. From top to bottom, the curves correspond to a $Z'$ mass of 1, 1.25, 1.5, 1.75, 2, 2.25, and 2.5 TeV, respectively.