Beam phase space determination at 50 GeV in the SLC linac

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Abstract

The procedure for determining the six-dimensional phase space volume of high energy beams in the SLAC Linear Collider (SLC) requires multiple transverse profile and lattice measurements. The standard use of these measurements to determine the emittance and betatron functions is reviewed, and complications from anomalous dispersion are included. Anomalous dispersion influences betatron matching of the beams to downstream systems and affects the measurement of energy spectra. Techniques to measure the full beam sigma matrix and distortions of the beams entering the linac, alignment of the beam phase space at the end of the linac is affected by size quadrupole focusing lattice, transverse and longitudinal wake fields, and mismatched beams in the quadrupole lattice [2].

Measurements of the transverse phase space of the beams are obtained using intercepting and non-intercepting profile monitors. These measurements and the subsequent calculations are the subjects of this paper. The bunch length is determined by the ring-to-linac transport system and is measured using a streak camera and quartz radiator [3].

Beam Condition

Both electrons and positrons enter the linac from the ring-to-linac transport lines with horizontal and vertical emittances of 0.9 x 10^-8 rad-m and 2.2 x 10^-8 rad-m, respectively, at 1.15 GeV after extraction from the damping ring and bunch length compressed. The design emittance values are 1.3 x 10^-8 rad-m. Typical values of the bunch length are 0.6 mm, the beam intensity 0.7 x 10^13 particles, and the repetition rate 30 Hz. The energy spectrum is about 1%. The residual dispersion in the beam at the entrance to the linac is measured and corrected to be less than 1 cm in each plane. Both beams in the linac are accelerated on the same RF pulse and have their trajectories corrected simultaneously with dipole correctors located 12 m apart. The spacing is closer in the first 300 m. The typical residual trajectory has rms value of 250 microns for both beams in both planes. An example is shown in Fig. 1.

Special feedback on the incoming and exiting beam launch parameters (z, z', y, y') of both beams occur once per minute. The electron energy is maintained at 47 GeV to within 50 MeV with a one minute per cycle feedback. The positron energy and the electron and positron energy spectra are maintained by the accelerator operators. These manual operations are performed every one to four hours.

One Beam Size Measurement

A measurement of the transverse beam dimension is obtained by inserting a fluorescent profile monitor into the beam's path and viewing the size by a video television camera. A photograph of a single beam pulse is shown in Fig. 2. The video signal is digitized, the background is subtracted, and a few large position offsets are due to broken electronic modules.
The emittance $\varepsilon$ can be calculated from

$$\varepsilon = \frac{\sigma_x \sigma_y}{\beta}$$

where $\beta$ is the betatron function calculated from the present quadrupole lattice [2]. Horizontal $x$ and vertical $y$ measurements can be taken. The beam image of Fig. 2 gives a calculated emittance value of $\varepsilon = 2.7 \times 10^{-10}$ rad-m assuming a $\beta$ of 60 m. The design is $3.3 \times 10^{-10}$ rad-m. This result is comforting except for the fact that the beam size varies by a factor of two throughout the day and thus, the apparent emittance by a factor of four. More detailed measurements and analysis shed light on this question. Finally, one can also calculate the effective $\beta$ in the beam by assuming the emittance is known.

$$\beta = \sigma_i^2 / \varepsilon.$$
is believed to be caused by anomalous dispersion generated by steering the beam to off-axis quadrupoles and position monitors in the linac as is illustrated in Fig. 6. At the beam intensities of \(0.7 \times 10^{10}\) transverse wake field effects are not expected [2].

Effect of Momentum Dispersion on Emittance

If the beam has a momentum dispersion (transverse position-energy correlation) then the standard emittance measurement above will be corrupted. In order to include the effects, the position dispersion \(d\), angular dispersion \(d'\) and the energy spectrum \(\delta(\equiv \sigma_E/E)\) must be included in the calculation. The beam size calculation becomes

\[
\sigma^2 = \epsilon + d^2 + \delta^2 \cdot \frac{\epsilon}{\delta}.
\]

(5)

The dispersion at position B can be calculated from the dispersion functions \(d\) and \(d'\) at A and the R matrix from A to B, assuming linear transport theory:

\[
d_B = R_{11}d_A + R_{12}d'_A.
\]

(6)

Inserting Eqs. 5 and 6 into Eq. 4 and following the above analysis procedure to determine \(\epsilon\), \(\beta\), and \(\alpha\), an effective emittance as would be measured using the above method can be calculated showing how \(d\), \(d'\), and \(\delta\) contribute:

\[
\epsilon_{eff} = \epsilon \left[ 1 + \frac{\delta^2[\beta_Ad^2_A + \delta^2 \gamma_A + 2d_A\delta_A\alpha_A]}{\epsilon} \right].
\]

(7)

Also, the effect on the calculation of \(\beta\) at position A is obtained:

\[
\beta_{eff} = \frac{\epsilon\beta_A + \frac{\delta^2 d^3_A}{\epsilon}}{\epsilon_{eff}}.
\]

(8)

Full Phase Space Measurements

All the unknowns in the beam in a given plane \(\epsilon\), \(\beta\), \(\alpha\), \(\delta\), \(d\), and \(d'\) can be determined by making three beam measurements each in non-dispersive and in dispersive beam transport regions. In the SLC the configuration is shown in Fig. 4. Beam size versus quadrupole setting data are taken with quadrupole-profile monitor pairs A-B and C-D. The region C-D has a non-zero dispersive R matrix as generated by the beam separating dipole. The A-B region has zero dispersion in the R matrix. The dispersion \(d\) near the quadrupole C and profile monitor D is about 7 cm. The beam energy and energy spectrum are traditionally measured here. The analysis below uses all the simultaneous measurements to determine the beam quantities at position A. Quadrupole C is part of a dipole alternating gradient magnet which makes it difficult to vary more than about 10% in strength. The profile monitor D is a non-intercepting synchrotron radiation monitor [4]. These measurements cannot be made at present in the vertical plane as there is no vertical bending to produce vertical dispersion.

The data analysis uses the SIGMA matrix of TRANSPORT [6] to calculate the beam size at the appropriate measurement point as computed from the known R matrices and initial beam conditions. Due to the number of elements in the transport line the equations are quite large but can be represented in a simple form:

\[
\sigma_{XX} = f(A,C)\left[ \epsilon_\beta + d^2 \epsilon_\beta + d^2 \epsilon_\beta + d^2 \epsilon_\beta \right]
\]

(9)

where \(f\), \(g\), \(h\), \(s\), \(t\), and \(u\) are long but simple expressions of R matrix elements between positions A and B or A or D. Once
the quadrupole values of \( A \) and \( C \) are determined, \( f \) through \( u \) are uniquely given. The expressions in the square brackets are just the elements of the SIGMA matrix expressed in terms of \( \epsilon, \beta, \alpha, d, d', \) and \( \delta. \) The matrix elements associated with functions \( f \) through \( u \) are \( \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}, \sigma_{30}, \) and \( \sigma_{20}, \) respectively.

The careful, but not difficult, choice of three independent quadrupole settings for \( A \) and \( C \) leads to six independent sets of functions, \( f, g, h, s, t, \) and \( u \) and thus, six independent beam size measurements. Again, Kramer's Rule can be used to determine the six quantities in square brackets in Eq. 9. Those six quantities can be subsequently analyzed to reveal \( \epsilon, \beta_A, \alpha_A, \delta, d_A, \) and \( d'_A. \)

This technique has become available only recently and has been used only a few times due to the difficulty in taking the measurements. The results of a data set taken in March 1988 are shown in Table 1. The results indicate that the beam has the correct emittance but that it has a large dispersion component and a large betatron mismatch. It is not known whether these results are stable day-to-day. However, this technique shows promise to reveal the full parameters of the beams.

### Table 1. Full beam parameter calculation at 47 GeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Three Measurements</th>
<th>Six Measurements</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_r )</td>
<td>( 7.8 \times 10^{-10} )</td>
<td>( 3.1 \times 10^{-10} )</td>
<td>( 3 \times 10^{-10} )</td>
</tr>
<tr>
<td>( \beta_A )</td>
<td>105</td>
<td>131</td>
<td>60</td>
</tr>
<tr>
<td>( \alpha_A )</td>
<td>0.12</td>
<td>-0.47</td>
<td>-3</td>
</tr>
<tr>
<td>( d_A )</td>
<td>0</td>
<td>-0.056</td>
<td>0</td>
</tr>
<tr>
<td>( d'_A )</td>
<td>0</td>
<td>0.0002</td>
<td>0</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-</td>
<td>0.34</td>
<td>0.2</td>
</tr>
</tbody>
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In the future this technique will be improved by reducing the measurement errors as many additions and subtractions have to be made in the analysis, by reducing the time required for data collection, by improving hardware (quadrupole \( C \)), and by finding a method to make measurements in the vertical plane.

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### References


