Coherent Beamsstrahlung

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The radiation coherently emitted by a high energy bunched beam suffering an arbitrarily large disruption in a collision with an idealized undisrupted beam is calculated. The near-luminal velocity of the beam—such that the emitted radiation moves very slowly with respect to the bunch—implies that only a small part of the bunch radiates coherently and necessitates a careful treatment of the disrupted beam phase space during emission. The angular distribution and spectral density are presented. It is found that most of the radiation is at wave lengths greater than or equal to the bunch length and that the total energy lost by the beam due to coherent effects should be negligible in high energy—high luminosity linear colliders.

Energy Loss and Angular Distribution: A sufficiently dense bunch may be expected to beamsstrahlen coherently. A reasonably exact criterion for and characterization of coherent Beamsstrahlung can be obtained by studying the case of a strong beam—weak beam collision in which a strong beam disrupts a weak beam of identical charge distribution but is itself retarding time very slowly with respect to the bunch—implies that only a small part of the beam radiates coherently and necessitates a careful treatment of the disrupted beam phase space during emission. The angular distribution and spectral density are presented. It is found that most of the radiation is at wave lengths greater than or equal to the bunch length and that the total energy lost by the beam due to coherent effects should be negligible in high energy—high luminosity linear colliders.

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\[-\Delta E \approx \frac{1}{4\pi} \sqrt{2\pi} \sigma \int_0^d t \int_0^\theta d\theta \left\{ \sqrt{\frac{\pi}{2\pi} \sigma} \right\}^2 \left[ \int_0^\theta \frac{d\theta}{\beta} \cos \frac{\theta}{\beta} \right]^2 \]

\[= \frac{\sqrt{2\pi}}{\beta^2} \left( \frac{N\sigma_\perp}{\sqrt{2\pi} \sigma_\perp} \right)^2 \int_0^\theta d\theta \theta^3 \left[ \int_0^\theta \frac{d\theta}{\beta} \cos \frac{\theta}{\beta} \right]^2 \]

\[= 16\sqrt{2\pi} \sigma_\perp \left( \frac{N\sigma_\perp}{\beta^2 / \sqrt{2\pi} \sigma_\perp} \right)^2 \int_0^\theta d\theta \theta^3 \left[ \int_0^\theta \frac{d\theta}{\beta} \cos \frac{\theta}{\beta} \right]^2 \] (7)

\[\times \int_0^\theta \frac{d\theta}{\beta^2 / \sqrt{2\pi} \sigma_\perp} \left( \gamma^2 - (\theta + \gamma \sigma_\perp)^2 \right)^{3/2} (\theta - \gamma \sigma_\perp) \delta(\theta - |\gamma \sigma_\perp|) \] (8)

to leading order in $\theta, \gamma \sigma_\perp, 1/\gamma$, and to all orders in $|\gamma \sigma_\perp|$. For application to large linear colliders we are mostly interested, as we shall see, in the limit $|\gamma \sigma_\perp| \gamma \to \infty$, i.e., where the typical particle deflection angle $\sigma_\perp \gamma$ the opening angle with respect to the particle's velocity into which most of the radiation is emitted. In this case

\[\gamma^2 - 4\gamma \sigma_\perp (\gamma^2 + (\theta + \gamma \sigma_\perp)^2) = \theta(\gamma^2 + \gamma^2 - 4/\gamma^2) \ln(\gamma \sigma_\perp) \] (9)

exhibiting an infinitely strong peak in the angular distribution from a single particle at its instantaneous deflection angle—leading to a sharp cutoff in the total angular distribution, and the formula for the total energy loss

\[-\Delta E \approx 16\sqrt{2\pi} \sigma_\perp \left( \frac{N\sigma_\perp}{\beta^2 / \sqrt{2\pi} \sigma_\perp} \right)^2 \int_0^\theta d\theta \theta^3 \left[ \int_0^\theta \frac{d\theta}{\beta} \cos \frac{\theta}{\beta} \right]^2 \]

\[
= \frac{32(N\sigma_\perp)^2}{\sqrt{2\pi} \sigma_\perp} \left[ \ln \left( 2\sqrt{2/\beta} \gamma \sigma_\perp \left[ \sin \left( \sqrt{2/\beta} \sigma_\perp \right) \right] - \frac{\gamma}{2} + O(\ln \gamma) \right) \right]^2 \] (10)

where $\sigma_\max \equiv \sqrt{2} \sin \sqrt{2/\beta} \sigma_\perp \gamma$, and which is valid for $\sin \sqrt{2/\beta} \sigma_\perp \gamma > 1$. It is also instructive to see the opposite extreme, i.e., where $\gamma \sigma_\perp \gamma \to 0$, in which from (7)

\[-\Delta E \approx 8\sqrt{2\pi} \sigma_\perp \left( \frac{N\sigma_\perp}{\beta^2 / \sqrt{2\pi} \sigma_\perp} \right)^2 \]

\[\times \left( \frac{\sigma_\perp}{\sqrt{2\pi} \sigma_\perp} \right)^2 \int_0^\theta d\theta \theta^3 \left[ \int_0^\theta \frac{d\theta}{\beta} \cos \frac{\theta}{\beta} \right]^2 \]

\[= \frac{\sqrt{2\pi}}{\beta^2} \left( \frac{N\sigma_\perp}{\beta^2 / \sqrt{2\pi} \sigma_\perp} \right)^2 \gamma^4 \left( \frac{\sigma_\perp}{\sqrt{2\pi} \sigma_\perp} \sin \sqrt{2/\beta} \sigma_\perp \right)^2 \] (11)

valid for $\sin \sqrt{2/\beta} \sigma_\perp \gamma \sigma_\perp \gamma \ll 1/\gamma$.

**Spectral Density:** The equations above already incorporate the fact that the pulse of radiation accompanying a single high energy collision has an essentially square form $h(t - R) \times h(\sqrt{2\pi} \sigma_\perp - t + R)$, implying that $E(\omega) \propto 2\sin \sqrt{2\pi} \omega \omega / \omega$, independent of $\theta$ for $\theta \ll 1$, and hence that the spectral energy density

\[-dE \approx \frac{4}{\pi} \sin \sqrt{2\pi} \omega \omega \int_0^\theta d\omega \left[ -\Delta E \right. \left. (\ln(\gamma)) \right] \right] \] (11)

Thus 90.3% of the energy is found at wavelengths $\geq \sqrt{2\pi} \sigma_\perp$, well in accordance with most intuition but notably different from the very much shorter and less-obvious scale over which the bunch radiates coherently.

**Discussion:** The focusing strength of the 'strong' beam is explicitly$^3$ (taking $r_\sigma = e^2 / m_e$)

\[
\frac{1}{\beta^2} = \frac{4\sigma_\perp N}{\gamma \sqrt{2\pi} \sigma_\perp} \left( \frac{1}{\sqrt{2\pi} \sigma_\perp} \right)^2 \] (12)

assuming that it is identical to the weak beam. It is useful to note in particular that then

\[
\frac{\sigma_\perp}{\beta} = \frac{\sqrt{2\pi} \sigma_\perp}{\sqrt{2\pi} \sigma_\perp} \sqrt{D} = \left( \frac{\pi}{2} \right)^{1/4} \frac{\sigma_\perp}{\beta} = \sqrt{\frac{8\sigma_\perp}{\gamma \sigma_\perp}} \] (13)

where $D$, the "disruption" parameter,$^3 \approx (5.6 \times \text{number of hadron oscillations})^2$, and to recall that therefore

\[|\gamma \sigma_\perp| = |\gamma \sigma_\perp| \sim O \left( \sin \sqrt{(\pi/2)^2 D} \frac{\sigma_\perp}{\beta} \right) = O(\sigma_\max / \sqrt{2}) \]

Typical values can be conveniently scaled from 'nominal' SLC parameters $\sigma_\perp / \beta \approx 1.06 \times 10^{-3}$, $\gamma \approx 9 \times 10^5$. It is probably easiest to understand (9) by considering its ratio with respect to the result obtained assuming incoherence$^4$

\[-\Delta E_{\text{inco}} \approx \frac{\sqrt{2\pi} \sigma_\perp}{6} \left( \frac{c\sigma_\perp}{\beta^2} \right)^2 \gamma^4 \left( 1 + \frac{\beta}{\sqrt{2\pi} \sigma_\perp} \sin \sqrt{2\pi} \sigma_\perp \right)^2 \] (14)

For nominal values of the disruption parameter

\[-\Delta E \approx \frac{24N}{\gamma^4} \left( \frac{1}{\sqrt{(\pi/2)^2 D} \sigma_\perp} \right)^2 \ln(\cdots) \] (15)

For the SLC $\ln(\cdots) \approx 4.04$, and the ratio $\approx 0.042$—and evidently would most likely get smaller as the parameters are pushed in the directions desired for large linear colliders (baring the extremely unlikely possibility that the number of particles in a bunch could be increased like the 4th power of the energy). (For very large $D$, (15) should be multiplied by 2, and of course there are comparable corrections for $\beta^2 \approx \gamma \sigma_\perp \gamma$ due to the suppression of high energy photons in the incoherent spectral density.) It thus seems clear that there is no pernicious coherent enhancement of the radiation loss rate. For 'sufficiently large' $N$ the physical interpretation of (15) is plain—a value $< 1$ indicates the presence of destructive interference such that the actual radiation loss is smaller than its 'incoherent part' (note that (7) in principle includes the incoherent contribution—plus 'cross terms'). The $E$-field due to a particular particle flips sign as the particle crosses the beam axis (cf. (6)), engendering considerable destructive interference (from different particles) which increases with $D$ (more axis crossings). However, the present calculation makes the fundamental approximation of treating the bunch as a continuum—an idealization that certainly breaks down if the effective coherently-radiating bunch length, which we have seen to be a very small fraction of the total, turns out in reality to contain less than one particle. The fact that even for fairly small values of the disruption parameter, such that little destructive interference can be taking place, the value of the ratio is $\approx 1$, indicates that within the parameter range of interest the latter situation indeed is the case and the formulae given here actually overstate the amount of coherent radiation.
It is a pleasure to thank Alex Chao (who suggested this work), Albert Hofmann, Bob Noble, Ron Ruth and Jim Spencer for useful discussions.


2. Gaussian units with $c = 1$ are employed.


4. The form quoted is for cylindrical bunches and arbitrary disruption parameter, and was derived by the author but is very similar to previous results.