ON THE NUCLEON RENORMALIZATION IN THE
UNIFIED FORCE MODELS*

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ABSTRACT

Conceptual problems of unified two-nucleon force models are discussed. The
force models are based on the pion-nucleon vertex and attempt a description of
the nucleon-nucleon interaction below and above pion threshold. The conceptual
problems arise from the nucleon renormalization due to pionic degrees of freedom.
Keeping channels with a single pion only no renormalization procedure can be
given which is consistent in the one-nucleon and in the many-nucleon systems.
The medium dependence of the one-pion exchange potential is illustrated.

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I. INTRODUCTION

There is extensive ongoing experimental study of the two-nucleon (NN) system below and above pion (π) threshold. The experimental effort asks for theoretical models of the two-nucleon interaction which describe the coupled channels without and with pions in a unified manner. Such unified force models have been designed for that purpose [1]. However, the account of two-nucleon scattering, pion-deuteron scattering, pion absorption and pion production by the two-nucleon system cannot be the final goal of these force models. Any model of the two-nucleon interaction is also meant to be the basis for a microscopic theory of phenomena with many nucleons. E.g., the models, which treat pionic degrees of freedom explicitly, should be able (i) to provide a microscopic derivation of the pion optical potential and (ii) to yield nuclear-structure corrections going beyond the classic picture of the nucleus as a system of nucleons only. In case (ii) the corrections which arise are many-nucleon forces and electromagnetic and weak exchange currents.

The unified force models incorporate our knowledge on pion-nucleon scattering and require a mechanism for pion production and pion absorption. There are basically two types of models which conceptually differ by the treatment of the latter mechanism. In one type [2], the pion gets produced or absorbed through the intermediary of a Δ-isobar which in turn is coupled to a purely nucleonic configuration through an instantaneous transition potential. In the second type [3] of models the pion is produced or absorbed on an explicit πNN vertex. Both models usually restrict themselves to channels with one pion at most. The second type of model will be discussed in the next sections. We are interested in the conceptual problems which are connected with nucleon renormalization due to pionic degrees of freedom and which arise when the model is applied to many-nucleon systems.

II. THE NUCLEON RENORMALIZATION IN THE NUCLEAR MEDIUM DUE TO PIONIC DEGREES OF FREEDOM

The force model to be discussed attempts a unified account of the coupled
system of nucleons and pions. We choose a noncovariant time-ordered description in terms of the Schrödinger equation

\[ [H_0 + H_1] |\Psi\rangle = E |\Psi\rangle, \]  

(2.1)

in order to facilitate applications to many-nucleon systems. The Schrödinger equation is defined in the Hilbert space of fig. 1. Besides the nucleonic part \( \mathcal{H}_N \) (projector \( P \), plane-wave basic states in the A-nucleon system

\[ |N^{(0)} k_1 \alpha_1 \ldots ; N^{(0)} k_A \alpha_A\rangle, \]  

where the \( k_i \) denote the momentum and \( \alpha_i \) the spin and isospin quantum numbers of the \( i \) nucleon \( N^{(0)} \) it has a sector \( \mathcal{H}_\pi \) in which a single pion is added (projector \( Q \), basis states

\[ |N^{(0)} k_1 \alpha_1 \ldots ; N^{(0)} k_A \alpha_A; \pi k_\pi \alpha_\pi\rangle, \]  

where \( k_\pi \) denotes the pion momentum and \( \alpha_\pi \) its isospin). The restriction to states with at most one pion is standard and, at present, necessary on technical grounds, but responsible for the problems to be discussed.

The force model relates the two-nucleon interaction back to pion-nucleon dynamics. It bases the mechanism of pion production and pion absorption on the explicit \( \piNN \) vertex. The vertex is important for pion-nucleon scattering in the \( P_{11} \) partial wave. It is contained in the interaction Hamiltonian \( H_1 \) which couples the two sectors \( \mathcal{H}_N \) and \( \mathcal{H}_\pi \) of the Hilbert space. It is diagrammatically shown in fig. 2. It has the form

\[ QH_1 P |N^{(0)} k_i \alpha_i\rangle = |g^{(0)}\rangle_i |\bar{k}_i \alpha_i\rangle \]  

(2.2)

In eq. (2.2) \( |g^{(0)}\rangle_i \) is a ket vector related to the relative motion of the pion-nucleon pair created from nucleon \( i \), whereas \( |\bar{k}_i \alpha_i\rangle \) describes the motion of the pion-nucleon c.m. We use the following nonrelativistic form of the \( \piNN \) vertex

\[ \langle q\alpha | g^{(0)} \rangle = \frac{f^{(0)}}{m_\pi c^2} \frac{1}{\sqrt{(2\pi)^3}} \frac{q}{\sqrt{2\omega_\pi(q)}} \left( \frac{\Lambda^2 - m_\pi^2 c^2}{\Lambda^2 + q^2} \right)^2 \hat{\sigma} \cdot \hat{q} \tau_\alpha, \]  

(2.3)

where \( \hat{q} \) is the relative pion-nucleon momentum, \( \omega_\pi(q) \), \( m_\pi \) and \( \alpha \) are the pion energy, mass and isospin projection and \( \hat{\sigma} \) are the spin Pauli matrices of the nucleon. \( |g^{(0)}\rangle \) is defined with an unrenormalized coupling constant \( f^{(0)} \). A
The dipole form factor is used, \( \Lambda c \) is the corresponding cut-off mass. The \( \pi NN \) vertex provides the nuclear-pole contribution to \( P_{11} \) scattering, but some background piece — often more conveniently given in terms of the Roper resonance — has to be added in order to ensure a quantitative account of the experimental \( P_{11} \) data. This has been carried out for the two fits presented in fig. 3. The nucleon of the Hilbert sector \( \mathcal{H}_N \) — as all nucleons of \( \mathcal{H}_N \) — is a bare entity \( N^{(0)} \) which only due to its coupling with pionic channel receives its dressing as physical nucleon \( N \). Difficulties of the renormalization procedure are the topic of this section. The interaction Hamiltonian \( H_I \) may also contain pion-nucleon partial waves other than \( P_{11} \). (It may furthermore contain two-nucleon potentials, which account for all physically necessary interactions not mediated by the pion.) We do not consider them in this context at all. Just for the clarity of the argument we assume pion-nucleon scattering to proceed solely through the \( P_{11} \) partial wave and even simplify this partial wave by neglecting the background contribution kept for the results of fig. 3. These simplifications do not harm the physics of the subsequent discussion. The part of the Hamiltonian acting on single-nucleon states is

\[
PH_0 P |N^{(0)} \vec{k}_i \alpha_i \rangle = \left[ m_N^{(0)} c^2 + \frac{k_i^2}{2m_N} \right] |N^{(0)} \vec{k}_i \alpha_i \rangle \quad , \tag{2.4a}
\]

\[
QH_0 Q |N^{(0)} \vec{k}_i \alpha_i ; \pi \vec{k}_\pi \alpha_\pi \rangle = \left[ m_N c^2 + \frac{k_i^2}{2m_N} + \sqrt{m_\pi^2 c^4 + k_\pi^2 c^2} \right] \times |N^{(0)} \vec{k}_i \alpha_i ; \pi \vec{k}_\pi \alpha_\pi \rangle \quad \tag{2.4b}
\]

\[
\simeq \left[ \hbar_{\pi N} + \frac{(\vec{k}_i + \vec{k}_\pi)^2}{2m_N} \right] |N^{(0)} \vec{k}_i \alpha_i ; \pi \vec{k}_\pi \alpha_\pi \rangle \quad . \tag{2.4c}
\]

Its action on many nucleon states is now obvious. Equation (2.4a) describes the kinetic energy of the bare nucleon in \( \mathcal{H}_N \) with rest mass \( m_N^{(0)} \). Pionic dressing turns \( m_N^{(0)} \) into the physical rest mass \( m_N \). Since in \( \mathcal{H}_\pi \) the nucleon cannot get any pionic dressing due to the restriction to single-pion states, the kinetic energy of the nucleon there must refer to the already physical rest mass \( m_N \). This fact is borne out by eq. (2.4b). Non-relativistic (relativistic) kinetic energies are
used for nucleons (pions). The step to relative and c.m. motion is approximate and done as in eq. (2.4c). The operator \( h_{\pi N} \) is the kinetic energy of the relative pion-nucleon motion with the inclusion of rest masses.

The superscript zero in \( m_N^{(0)} \) indicates that the mass is still unrenormalized. In \( |g^{(0)}\rangle_i \) of eq. (2.3) it indicates that the coupling constant \( f^{(0)} \) hidden in \( |g^{(0)}\rangle_i \) is the unrenormalized one and will receive dressing through the pion in the same way as the bare mass receives dressing.

In applications of many-nucleon systems the Schrödinger equation projected onto the nucleonic subspace

\[
P[H_0 + \delta H_0(E) + H_1 + \delta H_1(E)] P |\Psi\rangle = E P |\Psi\rangle , \tag{2.5}
\]

or the corresponding free propagator in the nucleonic sector \( \mathcal{H}_N \) of the Hilbert space \( P[E - H_0 - \delta H_0(E)]^{-1} P \) are considered. As long as there is no interaction in the pionic sector \( \mathcal{H}_\pi \), i.e., \( QH_1 Q = 0 \), the formal elimination of the pionic sector only yields one-body \( P\delta H_0(E) P \) and two-body \( P\delta H_1(E) P \) corrections, i.e.,

\[
P\delta H_0(E) P = \left[ PH_1 Q \frac{1}{E - QH_0 Q} QH_1 P \right]_{\text{one-body}} , \tag{2.6a}
\]

\[
P\delta H_1(E) P = \left[ PH_1 Q \frac{1}{E - QH_0 Q} QH_1 P \right]_{\text{two-body}} , \tag{2.6b}
\]

but no few-body corrections when \( A \geq 3 \). The corrections are illustrated in fig. 4. The one-body piece \( P\delta H_0(E) P \) yields the required self-energy correction of the nucleonic mass

\[
\delta H_0 \left( m_N c^2 + \frac{k_i^2}{2m_N} \right) |N^{(0)} k_i \alpha_i \rangle = \left[ m_N c^2 - m_N^{(0)} c^2 \right] |N^{(0)} k_i \alpha_i \rangle \tag{2.7a}
\]

\[
= \left\langle g^{(0)} \left| \frac{1}{m_N c^2 - h_{\pi N}} \right| g^{(0)} \right\rangle |N^{(0)} k_i \alpha_i \rangle \tag{2.7b}
\]
and gives the free nucleon propagator of fig. 5 the form

\[ P[E - H_0 - \delta H_0(E)]^{-1} P |N^{(0)} \tilde{k}_1 \alpha_1 ; \ldots ; N^{(0)} \tilde{k}_A \alpha_A \rangle \]

\[ = Z_A^{1/2} \left[ m_N c^2 + E - A(m_N c^2 - m^{(0)}_N c^2) - P H_0 P \right] \]

\[ \times \frac{1}{E - A(m_N c^2 - m^{(0)}_N c^2) - P H_0 P} \]

\[ \times Z_A^{1/2} \left[ m_N c^2 + E - A(m_N c^2 - m^{(0)}_N c^2) - P H_0 P \right] \]

\[ \times |N^{(0)} \tilde{k}_1 \alpha_1 ; \ldots , N^{(0)} \tilde{k}_A \alpha_A \rangle , \quad (2.8) \]

where

\[ Z_A(m_N c^2 + \epsilon) = \left[ 1 + A \left( g^{(0)}_\epsilon \left| m_N c^2 + \epsilon - \hbar \pi \right| m_N c^2 - \hbar \pi \right) \right]^{-1} \quad (2.9) \]

and \( \epsilon \) being a \( c \)-number for the basic states of \( \mathcal{H}_N \). The propagator has the physically required pole at \( E = \sum_i [m_N c^2 + \frac{k_i^2}{2m_N}] \) where \( \epsilon = 0 \). The residue at the pole, i.e., \( z_A = Z_A(m_N c^2) \), represents the overlap \( \langle N^{(0)} \tilde{k}_1 \alpha_1 ; \ldots ; N^{(0)} \tilde{k}_A \alpha_A | N \tilde{k}_1 \alpha_1 ; \ldots ; N \tilde{k}_A \alpha_A \rangle^2 \) between bare \( N^{(0)} \) and dressed \( N \) nucleonic states- except for an overall \( \delta \)-function. One expects the relation \( z_A = (z_1)^A \). However, this relation is badly violated as fig. 6 illustrates. This violation is due to the truncation of the pionic sector \( \mathcal{H}_\pi \) in the Hilbert space to contain just states with a single pion irrespectively of the number of nucleons present. Thus, a nucleon cannot dress itself as long as another one is doing so. E.g., a nucleon has the bare mass \( m^{(0)}_N \) as long as no pion is present, but its bare mass is turned into the physical mass when another nucleon is dressing itself by a pion. Even without any interaction a nucleon notices the presence of other nucleons. The free propagator does not have necessary clustering properties.

This lack of necessary clustering properties leads to inconsistencies in the application of the discussed unified force model to many-nucleon systems. Figure 7 describes two basic interaction situations for the pion, i.e., pion scattering from
the nucleus and the one-pion exchange potential in nuclear structure. When one
requires the proper behavior of the $P_{11}$ $\pi N$ transition matrix at the nucleonic
pole, the physical $\pi NN$ coupling constant $f$, renormalized in the one-nucleon
system, is

$$f^2 = (f(0))^2 Z_1$$  \hspace{1cm} (2.10)

whereas the corresponding strength $f(A)$ in the $A$-nucleon system turns out to be

$$f(A)^2 = (f(0))^2 A = f^2 \frac{Z_A}{Z_1}$$  \hspace{1cm} (2.11)

Thus, the unified force model is neither able to yield the correct low-density limit
for the pion-nucleus optical potential nor does the one-pion exchange potential in
nuclear structure have the expected strength. The deviation in strength $(f(A)/f)^2$
is shown in fig. 8 as a function of nucleon number $A$. We note that this problem
already exists when the unified force model attempts to connect the coupled two
nucleon systems $NN$ and $NNN$ with the underlying pion-nucleon input. The
physical $\pi NN$ coupling strength $f$ is either fixed at the $P_{11}$ pole of pion-nucleon
scattering and then yields the decreased strength $(f(2))^2 = \frac{Z_1}{Z_2} f^2$ in the two-
nucleon system. Or the strength is fixed in the two-nucleon system and then
the nucleon pole in $P_{11}$ does not have the required strength, but $(f(1))^2 = \frac{Z_1}{Z_2} f^2$.
The inconsistency is especially severe for large cut-off masses. In this case the
pionization of the nucleon is strong and a restriction to states with a single pion
is unnatural. The actual size of the inconsistency can be read off from fig. 8. This
inconsistency does not allow the use of the same hamiltonian for nuclear systems
with different baryon number $A$. In this sense the basic idea of a microscopic of
nuclear phenomena is defeated.

This renormalization problem cannot be resolve in time-ordered noncovariant
perturbation theory as long as one works with pionic channels of finite maximal
pion number. Clearly, when starting with a physical nucleon and considerable
one-pion content, the $A$-nucleon system needs $A$-pion content. However, $A$-pion
processes also take place in the one-nucleon system and are needed for ensuring unitarity for $A$-pion channels. We are aware of the fact that some realistic versions of unified force models contain relativistic improvements by the use of the Blankenbecler-Sugar equation [6] instead of eq. (2.1). However, such an extension does not appear to have cured any of the conceptual problems discussed in this note. Indeed, the $\pi NN$ coupling strength fixed at the $P_{11}$ pole in pion-nucleon channel, has to be multiplied in ref. [7] by the factor $3/2$ in order to provide the description of the reaction $p + p \rightarrow \pi + d$. This fact does have the simple explanation that in two system nucleon system $(f^{(2)})^2 = (Z_2/Z_1) \cdot f^2 \simeq 0.7f^2$. The reduction factor $Z_2/Z_1$ is model dependent and actual quoted value roughly corresponds to the input parametrization of ref. [7]. In Fig. 8 we present the behavior of $(f^{(A)})^2/(f^2) = Z_A/Z_1$ as the function of the number of nucleons $A$ for two different cut-off masses used in vertex (2.3).

III. ONE PION EXCHANGE POTENTIAL

In this section we assume that the coupling strength of the $\pi NN$ vertex is renormalized to the physical value $f$ in the $A$-nucleon system. The renormalization depends on the nucleon number $A$. When renormalizing in the $A$-nucleon system we give up the attempt to relate the properties of the $A$-nucleon system back to the nucleon-nucleon interaction without further adjustment. We thereby avoid the unresolved renormalization problem of section 2. In this section we want to discuss some other aspects of the one-pion exchange in the medium which arises from the unified force models, i.e., the energy dependence of renormalization and of pion propagation.

The one-pion exchange potential $V_{OPE}$ between nucleons $i$ and $j$ takes the form

$$V_{OPE} = \frac{1}{Z_A} \left[ m_N c^2 + E - A(m_N c^2 - m_0(c)^2) - PH_0P \right] P H_1(E) P$$

where

$$A = [Z_A]^2 \left[ m_N c^2 + E - A(m_N c^2 - m_0(c)^2) - PH_0P \right] .$$

(3.1a)
in the A-nucleon system with $Z_A(m_Nc^2 + \epsilon)$ defined in eq. (2.9). In eq. (3.1b) the vertex $|g\rangle_i^j$ is written without superscript zero in contrast to eq. (2.2) indicating that the renormalized $\pi NN$ coupling constant $f$ is used. The one-pion exchange potential of eq. (3.1) appears quite different from the local form which we usually employ and on whose long-range part a lot of physics interpretation is based [8].

The present form of the one-pion exchange potential is energy-dependent due to (i) the wave function renormalization factor $Z_A$ and (ii) the energy denominator of pion and nucleon propagation. When considering diagonal matrix elements of the potential and when replacing the energy of the virtual exchanged pion by its rest mass, the one-pion exchange potential may be approximated by

$$V_{OPE} \simeq \frac{Z_A(m_Nc^2 + \epsilon)}{Z_A} \frac{1}{\epsilon - m^2 c^2} \left[ i \langle g | g \rangle_j + \langle g | g \rangle_i \right]^{3.2}$$

In eq. (3.2), as in eq. (2.9), $\epsilon$ represents $E - A(m_Nc^2 - m_N^{(0)}c^2) - PH_0P$ and takes for diagonal matrix elements definite $c$-number values. They are characteristic for the retardation and therefore for the medium dependence of the potential.

In fig. 9 the deviation from the instantaneous version is studied in the form $V_{OPE}(\epsilon)/V_{OPE}(0)$. The energy dependence arising from the wave function renormalization factor ("dressing") tends to increase the strength of the potential, but the energy dependence of the propagator ("retardation") is the leading effect and the attraction of the nuclear force gets reduced. This trend emphasizes the current deficiencies of nuclear structure with two-nucleon forces, which is unable to yield sufficient binding in few-nucleon systems.
IV. CONCLUSION

This note discusses the force model which attempts to provide a unified description of the nucleon-nucleon interaction below and above pion threshold and which bases the mechanism of pion production and absorption on the $\pi NN$ vertex of fig. 2. The model works in a Hilbert space of at most one pion. The necessary renormalization of nucleon properties due to pionic degrees of freedom makes the force model fail to fulfill its role in a microscopic theory of nuclear phenomena: It cannot relate the properties of the many-nucleon system back to the two-nucleon interaction without readjustment of parameters.

We admit the present note not to be a constructive one. It does not observe a conceptual problem, which is then finally resolved. We are unable to provide a solution. Nevertheless we think it helpful to have formulated the conceptual problems inherent in a force model often employed. We also emphasize that the discussed problems do not arise in those unified force models [2], which account for pion absorption and production through the intermediary of the $\Delta$-isobar excited through a two-body transition potential. These latter models avoid the problem of nucleon renormalization on purpose. Finally, we observe that the nucleon renormalization amounts to energy-dependence of vertices as well as to the pion retardation in the one-pion exchange. This does yield new features for the two-nucleon interaction in the nuclear medium.

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FIGURE CAPTIONS

1. Hilbert space used for the Schrödinger eq. (2.1). It is shown for the three-nucleon system. Besides the purely nucleonic part it has a sector in which a single pion is added. The nucleons are denoted by solid lines, the pions is denoted by the dashed line.

2. The $\pi NN$ vertex.

3. Real part of the $P_{11}$ pion-nucleon scattering phase shift as a function of the pion lab energy $T_\pi$. Besides the nucleon pole both fits require a background contribution, which is parametrized in terms of a Roper resonance $R$ as in ref. [4]. The fit of the dashed curve is the one of ref. [4]. The parameters of the fit for the solid curve are $\Lambda c = 1200$ MeV, $f_{NR}^{(0)} / 4\pi(\hbar c)^3 = 0.77$, $\Lambda Rc = 1440$ MeV and $m_R^{(0)} c^2 = 1630$ MeV. The parameters $f^{(0)}$ and $m_N^{(0)}$ are eliminated by the conditions, that at the pole $m_N c^2$ of the physical nucleon the coupling to the pion is the physical one, i.e., $f^2 / 4\pi(\hbar c)^3 = 0.082$. The experimental data are taken from ref. [5]. For the following results in figs. 6, 8 and 9 the coupling to the Roper resonance will be omitted and a spread of results due to different cut-off masses $\Lambda c$ will be discussed.

4. One- (left) and two-body (right) corrections of the effective nucleonic hamiltonian due to the elimination of the pion sector $H_\pi$ of the Hilbert space.

5. Propagator of free physical nucleons.

6. Ratio $Z_A/(Z_1)^A$ as a function of nucleon number $A$. The deviation from the expected value one measures the extent by which the renormalization of the nucleon cannot be carried over to many-nucleon systems.

7. Typical processes contributing to pion scattering from a nucleus (left) and to pion exchange in the nucleus (right).

8. Ratio $(f^{(A)})^2/f^2$ as a function of nucleon number $A$. The deviation from the expected value one measures the extent by which the renormalization of the nucleon cannot be carried over to many-nucleon systems.
9. Energy-dependence of the one-pion exchange potential in the approximation of eq. (3.2). The ratio \( V_{OPE}(\epsilon_{o ff}) / V_{OPE}(0) \) is shown. \( \epsilon_{o ff} \) in the figure stands for \( \epsilon \) in eq. (3.2). The dependence on the energy denominator is labelled "retardation" and is the same for all nucleon numbers \( A \). The energy dependence due to the renormalization is labelled "dressing" and varies with nucleon number \( A \). The upper and lower limiting curves refer to the nucleon number \( A = \infty \) and \( A = 2 \), the dashed-dotted curves refer to \( A = 3 \). The total ratio \( V_{OPE}(\epsilon_{o ff}) / V_{OPE}(0) \) is given as solid curve "total" for the \( A = 3 \) system, in which \( \epsilon_{o ff} \approx -50 \text{ MeV} \) is a typical value. In such a case the one-pion exchange potential is decreased by about 20% compared with its instantaneous form due to its energy dependence.
Fig. 3
Fig. 6
Fig. 8
Fig. 9

$V_{OPE}(\epsilon_{\text{off}})/V_{OPE}(0)$

$\Delta = 1200 \text{ MeV/c}$