VI. SEPARATION OF R AND THE STRUCTURE FUNCTIONS

VI. A. Interpolation of the Cross Sections

The separation of $W_1$ and $W_2$ (or equivalently $\sigma_L$ and $\sigma_T$) at fixed $(\nu, Q^2)$ required differential cross sections $\frac{d^2\sigma}{d\Omega dE'}(\nu, Q^2, \theta)$ for at least two values of $\theta$. According to Eq. (I.2), $\sigma_L$ is the slope and $\sigma_T$ the $\varepsilon = 0$ intercept of a linear fit to

$$\sum(\nu, Q^2, \theta) = \frac{1}{\Gamma} \frac{d^2\sigma}{d\Omega dE'} = \sigma_T(\nu, Q^2) + \varepsilon(\nu, Q^2, \theta)\sigma_L(\nu, Q^2)$$

(VI.1)

The structure functions and $R$ are readily calculated from $\sigma_L$ and $\sigma_T$ according to Eqs. (I.3) and (I.4). There were, however only a few kinematic points $(\nu, Q^2)$ at which the differential cross sections had been directly measured for two or more values of $\theta$. Consequently, values of $\Sigma$ and its error were obtained by interpolation of the cross sections measured at each angle to selected kinematic points $(\nu, Q^2)$ that fell within the overlaps of two or more of the data triangles measured in experiments A, B, and C. The kinematic region of $Q^2 - W^2$ space spanned by these overlaps of the measured data triangles is shown in Figure (27). An array of 75 kinematic points $(\nu, Q^2)$, chosen to reflect the distribution of measured cross sections, was used in a systematic study of $R$ and the structure functions. As shown in Figure (27), these points lie at the intersections of contours of constant $x(0.1 \leq x \leq 0.8)$ and constant $Q^2(1 \leq Q^2 \leq 16 \text{ GeV}^2)$ with $W > 1.8 \text{ GeV}$. A subset
Fig. 27. The kinematic region of $Q^2 - W^2$ space available for the separation of $R$ and the structure functions. Separations were made at the 75 kinematic points $(v, Q^2)$ shown.
of this $x - Q^2$ array, containing 51 $(v, Q^2)$ points with
$0.2 \leq x \leq 0.8$ and $2 \leq Q^2 \leq 16$ GeV$^2$, was used in a parallel study
wherein only cross sections from experiments A and B were
used to extract $R$ and the structure functions. Only the re-
results from the full $x - Q^2$ array are reported here in detail.
The results obtained for the restricted $x - Q^2$ array were
consistent with those of the full $x - Q^2$ array. Previous
separations of $R$ and the structure functions using cross sec-
tions from experiments A and C have been reported earlier.$^{(24, 27)}$
These previous results are consistent with the present results
but are superseded by them.

The e-p and e-d cross sections from Table (V) were used to prepare interpolations at five different values
of the scattering angle. As mentioned earlier, all
cross sections from experiment B were multiplied by the normali-
ization factor $N_{AB} = 1.010$. In this way, triangles of cross
section data were assembled at $\theta = 15^\circ, 18^\circ, 19^\circ, 26^\circ,$ and $34^\circ$.
In order to extend the accessible kinematic region to $x < 0.2$
and to extend the ranges of $Q^2$ and $\varepsilon$ available for $x \geq 0.2$, cross
sections measured at $6^\circ$ and $10^\circ$ in experiment C were also used
in this analysis. These cross sections had been radiatively
corrected$^{(27)}$ by the same method as had been used for experi-
ments A and B. Prior to the interpolations, they were multiplied
by $N_{AC} = 1.019$ to normalize them to those of experiment A.

Values of $\Sigma(v,Q^2,\theta)$ and its random error were obtained by an interpolation scheme, similar to the method used in the radiative corrections, that made no a priori assumptions about the behavior of $R$. Because this scheme effectively averaged 16 cross section measurements for each $(v,Q^2,\theta)$, the values of $\Sigma(v,Q^2,\theta)$ and its errors were correlated for neighboring kinematic points $(v,Q^2)$. In practice, these correlations were difficult to remove, and the distribution of kinematic points $(v,Q^2)$ was chosen to minimize them. As many as five values of $\Sigma$ for five values of $\varepsilon$ were available at a given kinematic point $(v,Q^2)$. In general, the errors of the separated quantities varied inversely as the range $\Delta\varepsilon$ of the variable $\varepsilon$ spanned by the cross sections for fixed $(v,Q^2)$. In the present separations, $\Delta\varepsilon$ ranged from 0.16 to 0.57, while $\varepsilon$ itself ranged from 0.24 to 0.98.

VI.B. Separation of $R_p$ and $R_d$

The quantities $\sigma_L$ and $\sigma_T$ were available as the parameters of a linear least square fit to $\Sigma(v,Q^2,\theta)$ versus $e(v,Q^2,\theta)$ at each kinematic point $(v,Q^2)$. Sample fits are shown in Figure (28); in general, the confidence level for these fits was quite good. In only a few instances did $\chi^2$ deviate from the number of degrees of freedom $n_D$ of the fit by more than $(2n_D)^{1/2}$. Values of $R = \sigma_L/\sigma_T$ are presented for the proton in Table (XII) along
Fig. 28. Sample least square fits to $\Sigma(\nu, Q^2, \theta)$ vs. $\varepsilon(\nu, Q^2, \theta)$ in comparison with data. $\Sigma, \varepsilon$ and the fitting formulas are found in equations I.2 and VI.1. The quantities $R$ and $\sigma_T$ are available from the fitting parameters and from them, $\sigma_L$. 
Table XII. Separated values of $R'_p$, $R'_d$, and $\delta$ with their random errors and systematic uncertainties. The quantities $\Delta R_p$ and $\Delta R_d$ are discussed in the text.
with statistical errors and estimates of the systematic uncertainty $\Delta R_p$. The five contributions to the total systematic uncertainty $\Delta R_p$ are listed separately in Table (XII). The uncertainty $\Delta R_{p1}$ arising from the uncertainty of 0.010 in $N_{AB}^p$ was estimated by repeating the separations using instead a normalization factor $N_{AB}^p = 1.020$. A similar procedure was used to estimate the uncertainty $\Delta R_{p2}$ arising from the uncertainty of 0.017 in $N_{AC}^p$. The uncertainty $\Delta R_{p3}$ arising from a possible $E'$ dependence of the spectrometer acceptance was estimated (25) by using a redefined acceptance that varied by at most 1% from its nominal value (see Appendix 1). The uncertainty $\Delta R_{p4}$ due to relative uncertainties in detector efficiencies was estimated by using redefined efficiencies that varied from their nominal values by at most 1% (at $E' = 2$ GeV). The radiative correction uncertainty $\Delta R_{p5}$ was estimated by varying all proton cross sections by an amount $\Delta \sigma$ determined according to equation (IV.7). These five contributions were added in quadrature to obtain the total uncertainty $\Delta R_p$ reported in Table (XII). The present values of $R_p$ are consistent with those reported earlier (25, 27); much more accurate data are presented for $\omega_2$ than were available before.

Values of $R_d$ are also listed in Table (XII); they were extracted from the interpolated deuteron cross sections using the same procedure as used for the proton. The five contributions to the systematic uncertainty in $R_d$ were calculated
in the same manner as used for $R_P$, except that uncertainties of 0.007 and 0.024 in the deuteron normalization factors $N_{AB}^d$ and $N_{AC}^d$ were used. They were added in quadrature to obtain the total uncertainty $\Delta R_d$ listed.

The weighted averages of $R_P$ and $R_d$ over the full $x - Q^2$ array provide a rough comparison of these quantities. We find $\bar{R}_P = 0.138 \pm 0.011$, with a total systematic uncertainty $\Delta \bar{R}_P = 0.056$, and $\bar{R}_d = 0.175 \pm 0.009$, with a total systematic uncertainty $\Delta \bar{R}_d = 0.060$. Within the normalization uncertainty of experiment C alone, $\bar{R}_d$ is consistent with being equal to $\bar{R}_P$. When the weighted averages are taken over the restricted $x - Q^2$ array only, using data from experiments A and B, we find $\bar{R}_P = 0.136 \pm 0.017$ and $\bar{R}_d = 0.137 \pm 0.013$.

A more detailed and accurate comparison of $R_P$, $R_d$, and $R_n$ was achieved by extracting the quantity $\delta = R_d - R_p$ from the ratio of differential cross sections $\sigma_d/\sigma_p$ in a method that exploited the expected small systematic uncertainty in this ratio. From Eq. (I.2) we get $^{26}$$^\text{(VI.2)}$

$$G_d/G_P = \frac{G_{T_d} + \epsilon' G_{L_d}}{G_{T_P} + \epsilon' G_{L_P}} = \frac{1 + \epsilon' \bar{R}_d}{1 + \epsilon' \bar{R}_P} = \frac{1 + \epsilon' \delta}{1 + \epsilon' \bar{R}_P}$$

where $T = \sigma_{T_d}/\sigma_{T_P}$ and $\epsilon' = \epsilon/(1 + \epsilon \bar{R}_P)$. The physical meaning of Equation (VI.2) is clear: a difference between $R_d$ and $R_P$ results in a slope in $\sigma_d/\sigma_p$ plotted versus $\epsilon'$ (or, essentially versus $\epsilon$). The connection between $R_n$ and $\delta$ is achieved through an expression $^{20}$ that exploits the observation that the smearing correction is empirically the same for $W_1$ and $W_2$ (see
Appendix III

\[ R_d = R_p \left( \frac{1}{1 + Z} \right) + R_n \left( \frac{Z}{1 + Z} \right) \]  \hspace{1cm} (VI.3 a)

\[ R_n = R_d + \delta / Z \]  \hspace{1cm} (VI.3 b)

where \( Z = \frac{W^n_{1s}}{W^p_{1s}} \) is the ratio of smeared \( W^n_{1s} \) to smeared \( W^p_{1s} \).

In practice, Eq. (VI.3 b) is not very useful if \( \delta \neq 0 \), for \( Z \) is also an unknown. But if \( \delta = 0 \), which we find to be consistent with our overall results, then \( R_n = R_d \) and \( R_n = R_p \).

In this manner we can compare \( R_p, R_d, \) and \( R_n \), independent of the assumptions about \( R_n \) needed to calculate \( \sigma_n \) from \( \sigma_d \) in the impulse approximation.

At each of the 75 kinematic points \((v, Q^2)\), the quantity \( \delta \) was extracted as one of the two parameters of a least square fit of the form of Eq. (VI.2) to interpolated values of \( \sigma_d / \sigma_p \) versus \( \varepsilon' \). The interpolations program was almost identical to the one used to interpolate \( E \). At each \((v, Q^2)\) point, the value of \( R_p \) in \( \varepsilon' = \varepsilon / (1 + \varepsilon R_p) \) was taken to be that listed in Table (XII). Values of \( \delta \) and its random error from these fits are reproduced in Table (XII) along with estimates of the total systematic uncertainty \( \Delta \delta \). One contribution to this uncertainty arose from the ambiguity in the appropriate choice of \( R_p \) used to calculate \( \varepsilon' \) and ranged from 0.0 to 0.02 in \( \delta \). Another uncertainty arose from the uncertainty of 1.3% in the
ratic of deuteron to proton normalization factors $N_{d}^{d}/N_{d}^{p}$
and ranged from 0.01 to 0.12 in $\delta$. A third uncertainty
in $\delta$ arose from taking the normalization factor $N_{d}^{d}$ to be
equal to $N_{d}^{p}$, which had been calculated by a comparison of
elastic e-p cross sections; this uncertainty ranged from
0.02 to 0.23 in $\delta$. The quadratic sum of these three un-
certainties is presented in Table (XII) as $\Delta \delta$ and is, in
general, much smaller than the random error in $\delta$.

The result $\delta = 0$ is consistent with all the data listed
in Table (XII). Values of $\delta$ are typically less than one
standard deviation, and in only two instances more than two
standard deviations, different from zero. Weighted averages
of $\delta$ for each of the 11 values of $x$ are presented in Figure
(29) along with their random errors. Systematic uncertain-
ties in these averages range from 0.03 to 0.08 and are largest
for the range $0.15 \leq x \leq 0.33$. No statistically significant
development from zero can be seen anywhere in these data. When
the normalization factor $N_{d}^{d}$ was taken to be unity instead of
1.019, the average values of $\delta$ in the range $0.10 \leq x \leq 0.50$
were all within one standard deviation of zero. The average
of $\delta$ over the full $x - Q^{2}$ array, $\bar{\delta} = 0.031 \pm 0.015$, has a total
systematic uncertainty of $\Delta \delta = 0.036$ and is consistent with zero.

If $\delta$ is calculated using only cross sections from experiments
A and B, its average over the restricted $x - Q^{2}$ array is
$\delta = -0.001 \pm 0.022$. The only suggestion of some non-zero be-
Fig. 29. Average values of the quantity $\delta = R_d - R_p$ for each of the 11 values of $x$ studied. Errors shown are purely random. The systematic error in $\delta$ is 0.036.
behavior of ε occurs for $W \lesssim 2.5$ GeV and $x \gtrsim 0.60$, where $R_d$ is consistently smaller than $R_p$. Present estimates of the off-mass-shell corrections to the deuteron smearing ratios (see Reference (63) and Appendix (III)) are much smaller than the errors in $R_d$ and cannot explain this effect. Except for this possible difference at low $W$, which could be influenced by tails from the nucleon resonances, we conclude that $R_d = R_p$, and hence that $R_n = R_p$, over the full range of the $x - Q^2$ array.

**VI.C. Kinematic Variation of $R_p$ and $R_d$**

The behavior of $R$ in the Bjorken limit is an important test of constituent models (6, 21) of nucleon structure. In conventional field theories with only spin-1/2 charged constituents, $R$ should vanish as $1/Q^2$ in the Bjorken limit (21, 22). More recently, field theories with asymptotic freedom (18) predict that $R$ should vanish as $1/\log Q^2$. In both cases, the presence of charged spin-0 constituents would be reflected in a non-vanishing contribution to $R$, i.e., $R$ itself should scale (23).

The kinematic variation of $R$ was, however, difficult to ascertain because of large random errors and systematic uncertainties in the present data. Consequently, two approaches to the study of the kinematic variation of $R_p$ and $R_d$ were used. In the first approach, universal fits were made to the entire body of data for $R_p$ or $R_d$. In the second approach, individual
fits to $R_p$ or $R_d$ were attempted at each of the 11 values of $x$ at which these quantities were available. The interpretation of these fits is discussed in this section.

The results of four least square fits to all the data for $R_p$ and $R_d$ are presented in Table (XIII). Included in the table are the best fit parameters, their random errors and systematic uncertainties, and the $\chi^2$ sum for each fit. Systematic uncertainties in the fit parameters arising from the five uncertainties in $R_p$ or $R_d$ were added in quadrature to produce the numbers listed under $\Delta$ in Table (XIII). When only the $R_p$ or $R_d$ data for $W \geq 2.0$ GeV were used in these fits, the best fit parameters shifted by less than one standard deviation.

The $\chi^2$ for the universal fits to $R_p$ was consistently smaller than the $\chi^2$ for the corresponding fits to the $R_d$ data. This fact probably reflects the fact that the random errors for $R_d$ are smaller, relative to the systematic uncertainties, than those for $R_p$. In addition to the fits listed in Table (XIII), fits of the forms $R = cQ^2$, $R = cQ^2(1-x)^2$, $R = Q^2/\nu^2$ were attempted. These functions provided very poor fits to the data, and are consequently not listed. Except at low $x \lesssim 0.2$, the data for $R_p$ and $R_d$ are inconsistent with a linear rise in $Q^2$, as required by simple vector dominance models\(^{(13)}\) of inelastic $e-N$ scattering. A constant value still fits the $R_p$ data quite well. The best-fit value $R_p = 0.14 \pm 0.07$ is consistent with
Table XIII. Universal fits to $R_p$ and $R_d$. The best fit parameters for each fit function are listed along with the total $\chi^2$ of the fits to 75 data points. The quantity $\Delta$ represents the systematic uncertainty in each parameter.

<table>
<thead>
<tr>
<th>Fit function</th>
<th>Best-fit parameter</th>
<th>Proton $\chi^2$</th>
<th>Deuteron $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R=c$</td>
<td>$c =0.138$ 0.011</td>
<td>0.056</td>
<td>$c =0.175$ 0.009</td>
</tr>
<tr>
<td>$R=\frac{Q^2}{\nu^2}(c+d)^2$</td>
<td>$c =0.392$ 0.100</td>
<td>0.152</td>
<td>$c =0.334$ 0.080</td>
</tr>
<tr>
<td></td>
<td>$d =-0.073$ 0.012</td>
<td>0.041</td>
<td>$d =-0.108$ 0.010</td>
</tr>
<tr>
<td>$R=\frac{CQ^2}{(Q^2+d^2)^2}$</td>
<td>$c^2=0.861$ 0.202a</td>
<td>0.363a</td>
<td>$c^2=1.281$ 0.167a</td>
</tr>
<tr>
<td></td>
<td>$d^2=0.988$ 0.388a</td>
<td>0.229a</td>
<td>$d^2=1.158$ 0.241a</td>
</tr>
<tr>
<td>$R=\frac{c}{1+d\cdot\ln\left(\frac{Q^2}{M^2}\right)}$</td>
<td>$c =0.294$ 0.063</td>
<td>0.165</td>
<td>$c =0.355$ 0.045</td>
</tr>
<tr>
<td></td>
<td>$d =-0.808$ 0.358</td>
<td>0.237</td>
<td>$d =-0.665$ 0.184</td>
</tr>
</tbody>
</table>

$^a$ in units of GeV$^2$
the values \( R_p = 0.18 \pm 0.10 \) and \( R_d = 0.16 \pm 0.10 \) reported in earlier determinations\(^{(7, 24)}\) of this quantity over different kinematic ranges. On the basis of \( \chi^2 \), a constant fit to the \( R_d \) data fares rather poorly, but this may reflect only the influence of systematic uncertainties, particularly in the deuteron normalization factor \( N_{AC}^d \). The strict Callan-Gross relation\(^{(21)}\) \( R = Q^2/v^2 \) fits both proton and deuteron data very poorly, and the form \( R = cQ^2/v^2 \) is only marginally better. However, a more general spin-1/2 prediction\(^{(22, 23)}\) \( R = g(x)Q^2/v^2 \) provides an excellent representation of the \( R_p \) data and a fair representation of the \( R_d \) data. Such a deviation from simple \( Q^2/v^2 \) behavior at large \( \omega \) has been predicted from Regge arguments\(^{(22)}\) in the framework of light-cone algebras\(^{(21)}\), and deduced\(^{(67)}\) from \( \rho \)-electroproduction data.\(^{(68)}\) The fitting function\(^{(69)}\) \( R = cQ^2/(Q^2 + d^2)^2 \) insures that \( R \to 0 \) as \( Q^2 \to 0 \), as required by gauge invariance, and vanishes as \( 1/Q^2 \) in the Bjorken limit. It provides excellent fits to both the proton and deuteron data. A similar\(^{(69)}\) fit, \( R = cQ^2/(Q^2 + d^2) \), that vanishes as \( Q^2 \to 0 \) and approaches a constant in the Bjorken limit, fits the \( R_p \) and \( R_d \) data with equally good \( \chi^2 \). However, the best fit values of \( d^2 \) are negative producing singularities in \( R_p \) and \( R_d \) at \( Q^2 = -d^2 \), and the fit is not included in Table (XIII). The final fit is derived from \( R = \frac{\alpha}{\ln(Q^2/\beta^2)} \), with \( d = \left( \frac{\ln M^2}{\beta^2} \right)^{-1} \) and \( c = \alpha^2d \). While
this fit is necessarily singular at $Q^2 - \beta^2$, or at $Q^2 - 0.255$ GeV$^2$
for the proton and $Q^2 = 0.196$ GeV$^2$ for the deuteron, the model
is intended to apply in the limit of high $Q^2$. This function
fits the data equally as well as $R = cQ^2/(Q^2 + d^2)^2$, and the
present data cannot distinguish between an asymptotic $1/Q^2$
and $1/\log Q^2$ behavior of $R$ in the Bjorken limit. Although
these two functional forms fit the data better than the constant
fit, we cannot rule out a non-vanishing contribution to $R$, at
least not on the basis of the universal fits to all the present
data. For a sample of data restricted to $x \geq 0.25$, the constant,
the asymptotic $1/Q^2$ and the $1/\log Q^2$ functions all fit $R_p$ equally
well, while the constant fit is still a poor representation of
the data for $R_d$.

The $x - Q^2$ array permitted a study of the $Q^2$-dependence of
$R_p$ and $R_d$ for fixed values of $x$ in the range $0.1 \leq x \leq 0.8$. This
approach allowed unbiased tests of functional forms that could
not be fitted satisfactorily to the overall $x$-dependence of $R$,
and consequently allowed more stringent tests of the behavior
of $R_p$ and $R_d$ in the Bjorken limit for various regions of $x$. The
data for $R_p$ and $R_d$ are plotted versus $Q^2$ in Figure (30) for the
11 fixed values of $x$ available. The three curves plotted at
each $x$ in these figures represent best fits of the functional
forms $R = c(x)$, $R = a^2(x)/\log(Q^2/\beta^2)$, and $R = c(x)Q^2/(Q^2 + d^2)^2$,
corresponding to three of the universal fits reported in
Table (XIII).
Fig. 30. The values $R_p$ and $R_d$ plotted against $x$ for the 11 values of $x$ studied. Errors shown are purely random. The dashed lines represent constant fits to $R_p$ and $R_d$ at each value of $x$. The solid lines and dotted lines represent fixed-$x$ fits of the form $R = c(x)Q^2/(Q^2+d^2)$ and $R = a^2(x)/\ln(Q^2/s^2)$ at each value of $x$. 
The two parameters $\beta^2$ and $d^2$ were set equal to the corresponding parameters of the universal fits in Table (XIII). The best fit parameters of these fits are plotted versus $x$ in Figure (31), and the total $\chi^2$ for the 11 fixed-$x$ fits (64 degrees of freedom) of each function are also given. The solid lines in this figure represent the values of the best-fit parameters of the corresponding universal fits from Table (XIII). Fixed-$x$ fits of other functional forms were also attempted. In particular, a form $R = c(x)/Q^2$ fits the $R_p$ data well for $x \geq 0.25$ but has less than 20% confidence for $x \leq 0.2$. The form $R = c(x)Q^2$ is consistent with the data for $x \leq 0.2$, but is a very poor fit at higher $x$. Over the full range of $x$, it is difficult to distinguish among the constant, the asymptotic $1/Q^2$, and the $1/\log Q^2$ fits to $R$. The relatively large values of $\chi^2$ obtained in the constant universal fits can be seen to be the result of a slow variation of $R$ with $x$. For both the proton and deuteron, $R$ varies from about 0.3 at low values of $x$ to about 0.1 at the high values of $x$ reported. On the other hand, the success of the universal $1/\log Q^2$ fit can be attributed to the fact that it accommodates, perhaps fortuitously, this $x$-variation of $R_p$ and $R_d$ quite well. The modified $1/Q^2$ universal fit also represents the low-$x$, low-$Q^2$ behavior of $R_p$ and $R_d$ fairly well, and provides an equally good fit as $1/\log Q^2$ to all the data. In summary, the present data for $R_p$ and $R_d$ are consistent with
Fig. 31. Best-fit parameters of fixed $x$ fits to the $R_p$ and $R_d$ data. Errors shown are purely random.
either a constant, a \(1/Q^2\), or a \(1/\log Q^2\) dependence in the Bjorken limit. The present errors for \(R\) do not allow us to distinguish among these three functional forms.

The \(x - Q^2\) array also permitted a study of the kinematic variation of \(vR_P\) and \(vR_d\) for fixed values of \(x\). Light cone algebras with only spin-1/2 charged constituents predict\(^{21, 22}\) that \(vR\) should scale, i.e., \(vR(x, Q^2) = a(x)\). If there are charged spin-0 partons in the nucleon\(^{23}\), then \(vR(x, Q^2) = a(x) + vb(x)\), where \(b(x)\) is the ratio of spin-0 to spin-1/2 contributions\(^{69}\) to \(vW_2\), in the limit of large \(Q^2\). Other non-spin-1/2 contributions\(^{67}\) to \(vW_2\) would also result in a non-zero value of \(b(x)\), as would also be expected in asymptotically-free field theories.\(^{70}\)

In Figures (32,33) \(vR_P\) and \(vR_d\) are plotted versus \(Q^2\) for fixed values of \(x\) between 0.1 and 0.8. The solid lines represent least square fits of the form \(vR = a + bv = a + b\frac{x}{2Mx}Q^2\).

Best fit values of \(b(x)\) and its random errors and systematic uncertainties are given in Table (XIV) for the eleven values of \(x\) studied. The five contributions to the systematic uncertainty in \(R_P\) and \(R_d\) also give uncertainties in the parameter \(b\). The quadratic sum of the five such uncertainties in \(b\) is reported in Table (XIV) as \(\Delta b\), the systematic uncertainty in \(b\).

When these fits were restricted to \(W \geq 2.0\) GeV, the best-fit values of \(b\) shifted by less than one standard deviation,
Fig. 32. The quantity $v_{R_p}$ plotted against $Q^2$ for the 11 fixed values of $x$ studied.
Fig. 33. The quantity \( v_{Rd} \) plotted against \( Q^2 \) for the 11 fixed values of \( x \) studied.
Table XIV. Best-fit parameters $b$ and their random errors and systematic uncertainties from least-square fits of the form $v_R = a + bv$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$b_p$</th>
<th>$\Delta b_p$</th>
<th>$b_d$</th>
<th>$\Delta b_d$</th>
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</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.679 $\pm$ 0.330</td>
<td>0.130</td>
<td>0.478 $\pm$ 0.231</td>
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<td>0.15</td>
<td>0.278 $\pm$ 0.166</td>
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</tr>
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<td>0.014 $\pm$ 0.084</td>
<td>0.033</td>
<td>0.108 $\pm$ 0.071</td>
<td>0.037</td>
</tr>
<tr>
<td>0.33</td>
<td>0.003 $\pm$ 0.098</td>
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<td>0.195 $\pm$ 0.086</td>
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</tr>
<tr>
<td>0.40</td>
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<td>0.032</td>
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<td>0.036</td>
</tr>
<tr>
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<td>0.034</td>
<td>0.234 $\pm$ 0.062</td>
<td>0.039</td>
</tr>
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<td>0.036</td>
<td>0.148 $\pm$ 0.096</td>
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</tr>
<tr>
<td>0.67</td>
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<td>0.114 $\pm$ 0.116</td>
<td>0.040</td>
</tr>
<tr>
<td>0.75</td>
<td>0.009 $\pm$ 0.221</td>
<td>0.031</td>
<td>0.233 $\pm$ 0.198</td>
<td>0.033</td>
</tr>
<tr>
<td>0.80</td>
<td>0.496 $\pm$ 0.642</td>
<td>0.049</td>
<td>0.169 $\pm$ 0.562</td>
<td>0.045</td>
</tr>
</tbody>
</table>
except at $x = 0.5$, where $b_p$ shifted from $0.123 \pm 0.075$ to $0.023 \pm 0.114$, and $b_d$ shifted from $0.234 \pm 0.062$ to $0.172 \pm 0.089$. When fits of the form $v_R = a + bv$ were made to data for the $x - Q^2$ array restricted to experiments A and B, the results for $b_p$ and $b_d$ agreed with those of Table (XIV) within their random errors. For $0.25 \leq x \leq 0.80$, $b_p$ is small and consistent with zero, within the random errors quoted. The average of $b_p$ over this range of $x$ is $\bar{b}_p = 0.035 \pm 0.036$ with an estimated systematic uncertainty of 0.033. Over this same range of $x$, $b_d$ is frequently inconsistent with zero, within two standard deviations. Its average value over this range is $\bar{b}_d = 0.161 \pm 0.030$, with a systematic uncertainty of 0.037. The present results are consistent with the scaling of $v_{R_p}$ in this range of $x$, indicative of purely spin-1/2 constituents, in a parton model of the proton. The error in $b$, however, allows up to about a 10% spin-0 contribution to $v_{R_2}$. The results are not consistent with scaling of $v_{R_d}$. They are also consistent with about a 25% spin-0 contribution to $v_{W_2}$. These spin-0 contributions would lead to non-vanishing values of $R_p$ and $R_d$ in the Bjorken limit.\(^{(23)}\) Asymptotically-free field theories\(^{(18)}\) are also consistent with these results, as they predict\(^{(70)}\) a small increment above exact scaling behavior for $v_R$. Large values of $b$ are encountered for
x \leq 0.2, but a considerable portion of the data at these values of x is for \(Q^2 \leq 2.0 \text{ GeV}^2\), and the observed slope may represent only the low \(Q^2\) turn-on (59) of \(\nu W_2\). One could argue that the Fermi motion of the nucleons within the deuteron might lead to a non-zero value of \(b_d\), while \(b_p\) remained equal to zero. But as discussed in Appendix (III), the approximate equality of the smearing ratios for \(W_1\) and \(W_2\) implies that smearing should have little effect upon \(R_d\). Off-mass shell corrections to these smearing ratios are expected to reduce \(R_d\) at low \(Q^2\) but these effects are estimated to increase \(b_d\) by about 0.01. It is presently unclear whether the behavior of \(\nu R_d\) at fixed \(x\) is indicative of a non-spin-1/2 contribution to inelastic e-d scattering or is due to some aspect of deuteron binding not now understood.

Recently, the Callan-Gross relation \(R = Q^2/\nu^2\) (i.e., \(F_2 = xF_1\)) has been assumed in the analysis of neutrino experiments. (71) As indicated earlier, the parton model predicts \(R = a(x)/\nu\) for general spin 1/2 constituents. The Callan-Gross relation is specifically for unbound constituents (i.e., \(a(x) = Q^2/\nu = 2Mx\)). We note that as \(\nu \rightarrow \infty\), \(R \rightarrow 0\) in either case and the relation \(F_2 = xF_1\) is satisfied. Here we present the deviation

\[
K = \frac{F_2}{xF_1} - 1 = \left(\frac{\nu}{Q}\right)^2 \left[\frac{1 + \frac{R}{1 + \frac{\nu}{Q}}}{}\right] - 1 \quad \text{(VI.4)}
\]
for the $Q^2, \nu$ range of this experiment. Figure (34) shows $K$ averaged over $Q^2$ versus $x$ for the proton and deuteron, and Figure (35) shows $K$ averaged over $x$ versus $Q^2$. Significant deviations from Callan-Gross are seen at low $x$ and low $Q^2$. These deviations are expected and may come from binding effects of spin 1/2 constituents, low and high $Q^2$ non-scaling effects, or spin 0 constituents.

VI.D. Separation of the Structure Functions

At each kinematic point of the $x - Q^2$ array, the quantities $2M\nu_1$ and $\nu W_2$ were derived from $\sigma_L$ and $\sigma_T$ for both proton and deuteron according to equation (I.3). The separated values of $F_1(x,Q^2) = 2M\nu_1(x,Q^2)$ and $F_2(x,Q^2) = \nu W_2(x,Q^2)$ are reported in Table (XV), along with the random errors and relative systematic uncertainties in these quantities. Plots of $F_1(x,Q^2)$ and $F_2(x,Q^2)$ versus $Q^2$ for selected fixed values of $x$ are presented in Figures (36) and (37) for both the proton and deuteron. The random errors in $F_1$ and $F_2$ were computed from the error matrix of the least-square fit to $\Sigma$, and therefore include a contribution from the random error in $R$ at each point. As most of our cross section data were measured at values of $\epsilon$ between 0.6 and 0.9, this contribution is, in general, much larger for $F_1$ (corresponding to $\epsilon = 0$) than for $F_2$ (corresponding to $\epsilon = 1$). The relative uncertainties, which arise from the normalization uncertainties
Fig. 34. Values of $K$, averaged over $Q^2$, plotted against $x$ for the proton and deuteron. $K$ is defined in equation VI.4.
Fig. 35. Values of $K$, averaged over $x$, plotted against $Q^2$ for the proton and deuteron. $K$ is defined in equation VI.4.
Table XV. Separated values of 2MW$_1$ and vW$_2$ and their random errors and relative systematic uncertainties.
Fig. 36. Separated values of $2M_1 = F_1(x, Q^2)$ for the proton and deuteron plotted against $Q^2$ for fixed values of $x$. The errors shown are purely random.
Fig. 37. Separated values of $y W_2 = F_2(x, Q^2)$ for the proton and deuteron plotted against $Q^2$ for fixed values of $x$. The errors shown are purely random.
and from the cross section uncertainties listed in Table (VII), are those which can affect the $Q^2$-dependence of $F_1$ and $F_2$. They were estimated in a manner similar to that used to estimate the uncertainties in $R$, and were added in quadrature to produce the numbers listed under $\Delta$ in Table (XV). The relative uncertainty arising from the uncertainty in the radiative corrections ranged from 2% to 10% in $F_1$ and from 1.5% to 2% in $F_2$. Overall normalization uncertainties in $F_1$ and $F_2$, arising from the cross section uncertainties of Table (VII), are estimated to be 3.4% for the proton structure functions and 3.6% for the deuteron.