ORIGINS OF NEUTRINO EVENTS WITH THREE MUONS*

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ABSTRACT

The experimental observation of events in neutrino scattering with three outgoing muons presents a potential challenge to $SU_2 \times U_1$ gauge models, depending on the source of such events. In this paper a comprehensive study is reported of four possible modes of trimuon production (hadronic, mixed, leptonic and trident modes). Background sources are expected to be significant and will have to be separated from other sources. Attention is also given to two trimuon events with extraordinarily energetic muons.

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I. Introduction

There has been considerable attention recently to the successes of the "standard" model of the Weinberg-Salam SU$_2 \times$ U$_1$ gauge theory of weak and electromagnetic interactions. The data for virtually all charged-current and neutral-current phenomena are consistent with the predictions of this model. There is some debate over whether the apparent lack of parity-violation in certain atomic transitions constitutes a problem; however, even this could be handled with SU$_2 \times$ U$_1$ with modification of the standard model.

SU$_2 \times$ U$_1$ models have only four gauge bosons, W$^\pm$, Z$^0$ and $\gamma$. No data up to this time give any substantial motivation for considering other gauge bosons, i.e. for higher groups than SU$_2 \times$ U$_1$. However, the observation in neutrino scattering of events with three outgoing muons ($\mu^- \mu^- \mu^+$) raises the possibility that these events reflect a new phenomena which might conceivably require new bosons.

It is important, therefore, to determine whether these "trimuon" events are the result of some background process or are evidence of the production of new, heavy particles. Consideration of the characteristics of and rates for these events can provide the information for determination of their source. One can then see if SU$_2 \times$ U$_1$ models are able to account for these trimuon events. It is possible that while being consistent with SU$_2 \times$ U$_1$ models, these events require the presence of new leptons and/or quarks and indicate the structure of the model. It is also possible that the production of any new, heavy particles requires additional gauge bosons and, therefore, a larger group.

Various backgrounds have been discussed by the experimentalists and others; there is debate over whether they are sufficient to account for the observed rate. Here consideration is given to the possibility that in ordinary deep-inelastic scattering a photon is occasionally radiated from an off-shell muon or quark and is
converted into a muon pair. In this paper this process will be called "trident" production (but should not be confused with pair production in the Coulomb field of a nucleus). Some have speculated that this type of background trimuon process could be quite large.

A variety of interesting sources for trimuon events have been considered by the present authors \textsuperscript{10} and others. \textsuperscript{11-13} Here further details are given on three modes of trimuon production with new data added to the analysis. The three possible modes considered (in addition to tridents) are the "hadronic" mode (with the two "additional" muons coming from the hadronic vertex), the "mixed" mode (with one additional muon from each vertex) and the "leptonic" mode (from the leptonic vertex). Each of these modes encompasses several possible sources of trimuon production. For each mode we will focus on one particular source since our studies indicated that the differences among characteristics of particular sources within a given mode are relatively small when compared with differences among modes.

The hadronic mode \textsuperscript{10,11} includes the production of a heavy quark and its decay via another heavy quark or lepton. Also within this mode are various background sources (not studied explicitly here) such as the associated production and decay of pairs of charmed particles, vector meson production and decay to $\mu^+ \mu^-$, ordinary dimuon production with a coincidental $\pi$ or K decay, etc. The rates for most of these processes should be relatively low, and in any case their characteristics should be qualitatively similar to other hadronic sources. The hadronic source of trimuon events considered here is the production of a quark $t$ (charge $\frac{2}{3}$) and its decay to hadrons, a muon and a heavy neutral lepton which then decays to a muon plus other particles, see Fig. 1a. This process, related processes and, of course, background processes are certainly possible in SU$_2 \times$ U$_1$ models.
The "mixed" mode\textsuperscript{10,11} considered here is the simultaneous production of a lepton $M^0$ (which decays to $\mu^- \mu^+ \nu$) and a quark $b$ (charge $-\frac{1}{3}$; which decays to $\mu^- \bar{\nu} X$), see Fig. 1b. This is an off-diagonal neutral-current coupling. Were the boson having these couplings to be the usual $Z^0$, it would be very difficult to understand how it could have bottom-changing coupling (of such magnitude) and yet have no strangeness-changing-neutral-currents. It is safe to say that all sources within the mixed mode require the exchange of a new boson (call it $U^0$) which may be heavier than the $Z^0$. The presence of such a gauge boson, of course, implies a larger group than $SU_2 \times U_1$; so that if experiment indicates a mixed mode source of trimuons, it would be a clear indication that the gauge group $SU_2 \times U_1$ was inadequate (although perhaps a good subgroup).

The "leptonic" mode\textsuperscript{10,13} of trimuon production involves the production of a charged heavy lepton which decays into a neutral heavy lepton, a muon and a neutrino. The neutral lepton then decays into two muons and a neutrino. It is possible for $M^-$ to decay to either $M^0$ or $M^0$ (depending on whether $M^-$ couples to $M^0$ or to $\nu$ in its decay). Here the case involving $\bar{M}^0$ is considered. If experiment indicates a leptonic source of trimuons, the question of the viability of $SU_2 \times U_1$ may rest in part on the observed rate. Within $SU_2 \times U_1$ (with W exchange) one can obtain $M^-$ production by allowing some mixing between $\mu^-$ and $M^-$; however this mixing is limited so that the trimuon production rate is limited. For a sufficiently high rate $SU_2 \times U_1$ would therefore be ruled out.

In general the problem of rate is a difficult one, since it involves many assumptions about the branching ratios of heavy quarks and leptons, the mixing angles among quarks and among leptons, the masses of any new gauge bosons (in production or decays), the masses of heavy quarks and leptons, etc. Various experiments use different neutrino fluxes (spectra of incoming neutrino energy), so that different rates are expected. In general we leave the subject of rates to those with
specific models in mind although some useful information is given in Sec. VI. Instead the expected distributions of a variety of variables will be discussed. These distributions can distinguish the modes, and are not so sensitive to model-dependent assumptions. However, for "trident" production the calculation does not involve such assumptions, so that both rates and distributions will be given.

Almost all present data is from neutrino rather than antineutrino beams. The antineutrino fluxes are smaller than neutrino fluxes at all energies, and are much more peaked toward lower energies. Since trimuon production appears to be a high energy process, much lower trimuon rates are expected in antineutrino scattering (perhaps an order of magnitude lower).

Nevertheless, antineutrino trimuon production does provide a valuable test for distinguishing the mixed mode from virtually all other modes. In neutrino scattering all modes discussed here give dominantly $\mu^- \mu^- \mu^+$ rather than $\mu^+ \mu^- \mu^-$. In antineutrino scattering most modes clearly give $\mu^+ \mu^+ \mu^-$; however, the mixed mode gives dominantly $\mu^+ \mu^- \mu^-$ since $M^0 \rightarrow M^0$ but $b \rightarrow b$. This simple test (after accounting for any background) could indicate the mixed mode and thereby provide a serious challenge to $SU_2 \times U_1$.

At the time of this writing sixteen neutrino trimuon events have been reported: There are two from the Cal Tech-Fermilab (CF) collaboration\(^7\), twelve from the Fermilab-Harvard-Pennsylvania-Rutgers-Wisconsin (FHPRW) collaboration\(^8\) and two from the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration\(^9\). In this paper most attention will be devoted to the FHPRW data which is the most extensive data at present. While FHPRW report 12 events, two events contain extremely slow muons and such events are very likely to be from background sources. To minimize background contamination we require for both theory and experiment that all muons have $E_\mu \geq 4$ GeV. This requirement eliminates those two events from consideration. Another event is eliminated because it was not fully measured.
Of the remaining nine events, two appear to belong to an entirely separate class from the other seven (there is no continuum between the two classes). The average total muon energy for the two "super" events is 245 GeV compared to 83 GeV for the other seven. For the fast $\mu^-$, slow $\mu^-$ and $\mu^+$ the average energies for the "super" events are 127, 53 and 65 GeV compared to 55, 8 and 20 GeV for the other seven. However, other characteristics of these two events (such as angular distributions) are not unusual. As discussed in Sec. VI no mode considered here can account for the two "super" events and therefore they are considered separately.

The theoretical calculations reported here are in general the result of Monte Carlo calculations. However, in all cases these results were checked against analytic results. In doing the calculations the polarizations of particles were always kept when appropriate. A large variety of masses and helicities (for each coupling) were compared. Since experimentalists can only distinguish a fast $\mu^-$ from a slow $\mu^-$ and cannot tell the origin of either $\mu^-$ (which vertex), the Monte Carlo calculations identify the slow $\mu^-$ as such, irrespective of its origin (although, of course, the matrix element is calculated with muons identified properly). Distributions and rates have been calculated with the FHPRW and the CERN wide-band neutrino fluxes folded in. Except where stated otherwise, the results are quite similar for the two cases.

In Secs. II - V the trident, hadronic, mixed and leptonic modes of trimuon production are discussed. In Sec. VI the results are summarized and conclusions given.

II. The Trident Mode of Trimuon Production

During a conventional deep inelastic charged current interaction it is possible that an off-shell muon or quark radiates a timelike photon which converts to a $\mu^+\mu^-$ pair. At high neutrino energies, the production of three high energy
muons via this "trident" mechanism is expected to occur at order $\alpha^2$ relative to single muon production. Trident production is thus potentially an important source of trimuons. In this section we describe our calculation of the rate and the distributions of trimuons due to tridents.

Trident production was calculated in the quark-parton model from the Feynman diagrams of Fig. 2. Although the effect of radiation from the spectator quarks on the production of a high energy $\mu^+\mu^-$ pair can be neglected it is essential that radiation off the interacting $d$ and $u$ quarks not be neglected. Radiation off the leading muon alone is not gauge invariant. This is born out by our calculation. In Feynman Gauge there are large cancellations between the square of Fig. 2a) and its interference with Fig. 2b) and 2c), even when only high energy $\mu^+\mu^-$ pairs are radiated.

In evaluating the diagrams of Fig. 2 the effect of Fermi-Dirac statistics which is known to be small, is neglected. Radiation off the exchanged $W$-boson is neglected since $-q^2 << m_W^2$. The use of the quark-parton model has been tested in a similar situation in the emission of a real photon during an $e^\pm p$ deep inelastic collision. It was found in quite good agreement with experiment.

From Fig. 2 the differential cross-section $d\sigma/3\mu$ for trident production is given by

$$d\sigma_{3\mu} = \frac{1}{4ME} \sum_{if} \prod_{i=1}^{3} \frac{d^3k_i}{2E_i} \frac{d^3p_x}{2E_x} \frac{dM_x^2}{2M\nu X} \times \delta^4(p + q - p_x) \quad (2.1)$$

Here $k_1$, $k_2$, $k_3$; $E_1$, $E_2$, $E_3$ are the muon momenta and energies, $k$, $E$ is the beam momentum and energy, $p_x$, $M_x$ is the momentum and mass of the outgoing hadrons, $d(x) = 2xF_2(x) = 2x(\nu W_2(x))$, with $\nu = E_1 - E_2 - E_3$, and $x = -q^2/2M$ with $q = k - (k_1 - k_2 - k_3)$, and $M$ is the proton mass.
Several distributions were obtained from Eq. 2.1 using a phase space Monte Carlo program. Our results were found to be insensitive to the details of the choice of $F_2(x)$. Events were generated with FHPRW and CERN (wide band beam) fluxes.

The total rate for trimuon production and the ratio of trimuons to single muons was also evaluated. From Eq. 2.1 the total rate is given by

$$
\sigma_{3\mu} = \frac{1}{4ME} \left[ \sum_{i=1}^{2} \left| a_i \right|^2 \int \frac{d^3k_i}{2E_i} \frac{F_2(x)}{4M^2} \theta(p_i^0 - M) \theta(p_i^2 - M^2) \right]
$$

The integral in 2.2 was done by Monte Carlo techniques and numerically on an adaptive integration program ("Vegas"16). The ratio of tridents to single muon events (for tridents with muon energies greater than 4 GeV and for neutrino energies of the FHPRW flux between 100 and 300 GeV) was found to be:

$$
\frac{\sigma_{3\mu}}{\sigma_{\mu}} \approx 2 \times 10^{-5}
$$

This is to be compared with the rate of $10^{-4}$ quoted by the FHPRW group.8

It is important to note that the rate from Fig. 2a) alone is more than five times as large as the correct result. Gauge invariance is thus essential in obtaining a meaningful estimate for trident production.

For the CERN beam, with the same cuts and energy range, the ratio of tridents to single muons is about two thirds as large. For tridents, about half the events occur at energies below 100 GeV. Whereas for the other modes most trimuon events occur above 100 GeV.

Besides providing trimuons, the trident mechanism also provides a source for $\mu^+ e^- e^-$ events which could be observed in bubble chamber experiments. This rate was computed for both the FHPRW flux and the CERN flux above 30 GeV.
with a cut of 140 MeV on the invariant mass of the $e^+e^-$ pair, and with muon and $e^\pm$ energy cuts of 4 GeV and 800 MeV respectively. For the FHPRW flux $\sigma_{\mu e^+e^-}/\sigma_{\mu} \approx 2.5 \times 10^{-5}$.

III. The Hadronic Mode of Trimuon Production

One possible mode$^{10,11}$ of trimuon production involves the ordinary production of a muon at the leptonic vertex along with two muons from the hadronic vertex. The hadronic muons could come from several sources. Among these are several "background" sources. In events in which a single charmed quark is produced and then decays into a muon, a third muon might result from the decay of a pion or kaon (but this muon need not always have a minus charge). It may be possible to produce a charm-anticharm pair indirectly (not a direct result of the weak interaction) and have both charmed quarks decay into muons. However, the observed positive muons are more energetic than naively expected from this process. Particles such as $\rho, \omega, \phi$, and $\psi$ may be produced at the hadronic vertex and decay to $\mu^+\mu^-$. It is possible to give qualitative consideration to such sources, since their distributions are expected to be somewhat similar to the "non-background" hadronic sources which were studied intensively.

There were two types of hadronic sources studied. Both involve the production of a heavy quark which decays to a muon, a heavy particle, etc. The heavy particle can be either a quark or a lepton which decays into a muon. The case reported in detail here (Fig. 1a) is the production of a quark $t$ (charge $\frac{2}{3}$) which decays into a quark $d$, a positive muon and an $M^0$, the $M^0$ then decays into a negative muon plus other particles. Such processes can result in models with the couplings of $t$ to $d$ quarks and $M^0$ to muons. For our calculations, a model$^{17}$ such as $SU_2 \times SU_2 \times U_1$ was imagined in which $t$ is produced by a
left-handed coupling to d quarks (with a small mixing angle $\theta$) via the standard W boson and decays via a "right-handed" boson $W'$. However, the results are virtually identical in SU$_2 \times U_1$ models. In addition, several variations were studied (with decays to both quarks and leptons).

Any production mechanism involving the hadronic vertex requires an assumption about the conversion of the heavy quark into a hadron (which then decays). A fragmentation function describes what fraction $z$ of the quark's momentum is kept by the resulting hadron. While an approximate form of this function is known for light quarks, Bjorken has suggested that the form may be quite different for heavy quarks. We have tested the sensitivity of our results to the form of the fragmentation function by trying forms ranging from peaking at small $z$ to peaking at large $z$ (and also flat forms). Obviously the energy of hadronic muons is sensitive to this form; however, most of the distributions covered here are quite insensitive to the form. Only the extreme case with $f(z) = 5(1-z)$ (as opposed to smooth peaking at $z=1$) gives somewhat different results from other forms. As a compromise, the fragmentation function used in the results shown here, rises linearly from zero until $z=0.5$ and then falls linearly.

The mass of the t quark was chosen to be 5 GeV (assuming T (9.5) to be associated with the t quark). If a higher mass is required, invariant mass distributions would be shifted higher (away from present data). For $M^0$, a mass of 2 GeV was chosen; there are no experimental or theoretical arguments against neutral leptons of such mass.

IV. The Mixed Mode of Trimuon Production

Another possible mode of trimuon production involves the simultaneous production (see Fig. 1b) of a neutral lepton $M^0$ and a heavy quark $b$ (of charge $-\frac{1}{3}$). The $M^0$ decays into $\mu^- \mu^+ \nu$ and the $b$ quark decays into $\mu^- \bar{\nu}$ plus other particles.
The $M^0$ and $b$ production involves an off-diagonal neutral-current. It appears impossible to accomplish this process with the ordinary $Z^0$ boson; were $b$-changing-neutral-currents ($d$ to $b$) to be so large it would be quite difficult to suppress strangeness-changing-neutral-currents. As a result, we assume this process must occur via a gauge boson $U^0$ and that a group larger than $SU_2 \times U_1$ would be required. There are $SU_3 \times U_1$, $SU_6$, and $SU_3 \times SU_3$ models in which trimuon production via simultaneous $M^0$ and $b$ production can occur.

Here primary consideration is given to an $SU_3 \times U_1$ model with the couplings:

$$
\begin{pmatrix}
  u \\
  d \\
  b
\end{pmatrix}_L
\begin{pmatrix}
  c \\
  s \\
  g
\end{pmatrix}_L
\begin{pmatrix}
  u \\
  g \\
  b
\end{pmatrix}_R
\begin{pmatrix}
  c \\
  s \\
  d
\end{pmatrix}_R
$$

In each triplet the top two fermions are coupled by $W^\pm$, the bottom two fermions by $U^0$ and the top and bottom fermions by $V^\pm$. The Higgs structure is arranged to make $U^0$ and $V^\pm$ three or four times heavier than $W^\pm$. In Ref. 10 the $b$ quark was required to have a mass of about 5 GeV if the mixed mode was to explain trimuons. Since then, the meson $T$ has been discovered with a mass of 9.5 GeV presumably indicating the presence of such a 5 GeV quark (although it obviously is not proof of the mixed mode).

The $b$ quark in this model cannot be produced in neutrino and antineutrino scattering (without $M^0$ leptons) except via small mixing angles. The quark $g$ is assumed to have a mass of say 15 GeV, and can be produced only when higher energy beams exist. The neutral-current phenomenology is consistent with present experiment. It should be emphasized that the details of these example...
models are not relevant to this paper since it is only the mixed mode being tested here, and there are other models which contain this mixed mode of trimuon production.

In the above model the exchange of the $U_0$ boson couples $d$ to $b$ (left-handed) and $\nu_\mu$ to $M^0$ (left-handed). The decay of $M^0$ to $\mu^-$ is right-handed as is the decay of the $b$ quark. The $b$ quark fragmentation to hadrons involves the same assumptions discussed in Sec. III.

The test fits to the limited data required $M_b \approx 5$ GeV and $M_{M^0} \approx 3$ GeV but there is flexibility so that these masses could be larger or smaller.

V. The Leptonic Mode of Trimuon Production

Trimuon events could also occur in the production and decay (see Fig. 1c) of a charged heavy lepton $M^-$. The decay of such a charged lepton can proceed (at least) two distinct ways: one involving a neutral lepton and one a neutral anti-lepton. The $M^-$ may be produced via a small mixing angle (between $M^-$ and $\mu^-\bar{\nu}$). If $M^-$ couples to $M^0$, and $M^0$ is lighter than $M^-$, then $M^-$ can decay into $M^0$ and $\mu^-\bar{\nu}$. Then $M^0$ can decay into $\mu^-\mu^+\nu$. The case receiving primary attention here involves the $M^-$ decay into $\nu_{M^-}$ and $\mu^- M^0$. The $M^0$ then decays into $\mu^+\mu^-\bar{\nu}$. We assume the couplings $(\nu_{M^-} M^-)_{L}$ and $(M^0 \mu^-)_{R}$ and except for possible rate problems, this process is conceivable in $SU_2 \times U_1$ models (and certainly is possible in larger groups such as $SU_2 \times SU_2 \times U_1$, $SU_3 \times U_1$, etc.). In addition, many variations were studied (with $M^0$ and $M^0$ and with left- and right-handed couplings), but the differences were smaller than between modes.

While good fits are obtained with $M_{M^-} \approx 8$ GeV and $M_{M^0} \approx 3$ GeV, more data is needed before these masses can be determined accurately.
VI. Conclusions and Summary

Many different distributions were examined in our study. Here a number of
distributions which were interesting and/or useful will be shown and discussed.
For each case, four theoretical curves (for hadronic, mixed, leptonic and tri-
dent modes) are shown; however, these curves are not normalized to each other or
to the data since rates usually involve further model-dependent assumptions (see
Sec. I). The experimental neutrino flux of FHPRW is folded in (but the results for
the CERN flux (wide band beam) are very similar). The muons were required to
have $E_\mu > 4$ GeV. The notation which follows has the muons labelled as: $\mu_1 =$
fast $\mu^-$, $\mu_2 =$ slow $\mu^-$ and $\mu_3 = \mu^+$. With so few trimuon events and the possibility
of large background contamination, no strong conclusions can be reached now, but
in the near future, considerably more data are expected.

The energy of the positive muon can be studied with the energy asymmetry
$E_{13} = (E_1 - E_3)/(E_1 + E_3)$. For both the trident and hadronic modes, one ex-
pects the positive muon to be relatively slow so that $E_{13}$ would be peaked near
one (not at one, since experiment requires $E_1 > 4$ GeV). Fig. 3 indicates just
such a result but does not distinguish the mixed and leptonic modes from each
other. The data gives a slight preference to the mixed and leptonic modes. For
the hadronic mode, this distribution is mildly sensitive to the assumed fragmen-
tation function.

The energy of the slow negative muon can be studied with the energy asymmetry
$E_{12} = (E_1 - E_2)/(E_1 + E_2)$. The hadronic mode should give slower $\mu_2$ (higher $E_{12}$)
than other modes since it comes from the decay of the (lighter) particle produced
in the decay of the $t$ quark. The mixed mode should give faster $\mu_2$ since this
muon comes directly from the initial decay of the $b$ quark. For the leptonic
mode, the $\mu_2$ should also be faster even though it comes from a secondary decay,
simply because these are direct decays with no fragmentation of quarks into hadrons. The data in Fig. 4 is consistent with all modes although slightly favoring the hadronic mode.

Of special interest to the possibility of the trident mode is the invariant mass \( M_{23} \) of the slow muon pair; for this mode the muon pair is produced from the photon and \( M_{23} \) should be peaked at very small values. Of course, on rare occasions the slow \( \mu^- \) is not from the photon, so that large \( M_{23} \) result. For the hadronic mode, relatively small \( M_{23} \) are expected since both \( \mu_2 \) and \( \mu_3 \) are from the hadronic vertex and are relatively slow. Fig. 5 reflects these expectations. The limited data is consistent with all modes, especially with the trident mode.

Another invariant mass of interest is the total invariant mass \( M_{123} \). However, this variable is sensitive to some model-dependent assumptions and to the heavy particle mass assumptions. As seen in Fig. 6, it does not distinguish most of modes except for tridents which should have lower \( M_{123} \) than other modes.

Aspects of the transverse angle distributions are expected to be interesting. Two such angles studies are \( (\theta_{23} - \theta_1) \) and \( (\theta_{13} - \theta_2) \). \( \theta_{ij} \) is the direction of \( p_{\mu_i} + p_{\mu_j} \) and \( \theta_k \) is the direction of \( p_{\mu_k} \). For the hadronic mode, one expects that the slow muon pair \( (\mu_2 - \mu_3) \) which comes from the hadronic vertex will have transverse momentum directed away from the fast muon which comes from the leptonic vertex (the exchanged W boson pushes \( \mu_2 - \mu_3 \) away from \( \mu_1 \)). In Fig. 7 one sees that the hadronic mode does peak at \( (\theta_{23} - \theta_1) = 180^\circ \) although the data does not. This angle does not distinguish the leptonic and mixed modes and we hoped the angle \( (\theta_{13} - \theta_2) \) would distinguish them since for the mixed mode \( \mu_1 \) and \( \mu_3 \) come from the opposite vertex from \( \mu_2 \). However, Fig. 8 does not reflect our expectations. The leptonic mode also has some tendency to peak at 180°. The hadronic
mode peaks even more sharply at $180^\circ$ since $\theta_{13}$ is dominated by $p_{\mu 1}$ ($\mu_1$ is produced directly without any decay). The data does not follow the hadronic mode curve very well.

If one assumes that the hadronic mode is correct, then one can determine the momentum $p_W$ of the W boson; it is the momentum of the incoming neutrino minus the momentum of the muon from the leptonic vertex (which is, under this assumption, the only particle from that vertex). Of course, sometimes there are outgoing neutrinos whose presence can confuse the determination of the incoming neutrino momentum, and sometimes the leading muon is not from the leptonic vertex; however, one can also "miscalculate" $p_W$ theoretically, so in either case there is no problem. To know $p_W$ it is necessary to measure the hadron energy which was possible for only two of the present (non-super) FHPW events, so that at this moment this data is very limited (but much more data is expected soon). For the other modes, the theoretical calculations are not calculations of the momentum of the exchanged boson, but of some meaningless but well-determined momentum (which we can still call $p_W$).

Given this momentum $p_W$, one expects for the hadronic mode that the angle $\theta_{W13}$ of the positive muon ($\mu_3$) in the plane perpendicular to $p_W$ (with $p_{\mu 1}$ as reference) is randomly distributed, whereas for other modes it would not be random. Such a result appears in Fig. 9 with the limited data shown.

However, Fig. 9 does not distinguish the mixed and leptonic modes (nor do most distributions). One possible means of separating the mixed mode would be to examine the angle $\theta_{U2}$ of the slow negative muon ($\mu_2$) in the plane perpendicular to the momentum $p_U$ of the exchanged boson, $U^0$. Unfortunately $p_U$ cannot be determined since a neutrino is involved in the decay at the leptonic vertex. However, some residual effect may remain if one defines $P_U'$ as the incoming neutrino
momentum minus $p_{\mu_1}$ and $p_{\mu_3}$. The hadronic energy must be measured, and the "limitations" mentioned for $\theta_{W13}$ apply for $\theta_{U2}$ also. In Fig. 10 one sees that the angle $\theta_{U2}$ is relatively randomly distributed for the mixed mode, and it may be possible to distinguish the mixed and leptonic modes when more data is available.

Finally, it is interesting to examine the rate at which trimuons are produced as a function of incoming neutrino energy. Here we use the true energy not the observed energy which is missing the energy of the outgoing neutrinos. This is shown in Fig. 11 with no flux included, thus displaying the energy dependence of trimuon production independent of any particular flux (with arbitrary normalization). Remember, for comparison, that the single muon rate rises linearly with energy. For hadronic, mixed and leptonic modes, the shape of the results is sensitive to the produced masses; however, if these masses are changed by 20 or 30%, the effects on the scale of Fig. 11 are relatively small. For these modes the suppression due to phase space at present energies is approximately 20%. The details of rates are left to those who have in mind specific models with specific masses, couplings, branching ratios, etc. (which are essential for determining rates). The trident rate, of course, can be calculated without such assumptions (see Sec. II).

In comparing rates from different experiments, one should be careful to account for two matters. First, the energy range considered is important. For tridents, the ratio $\sigma_{3\mu}/\sigma_{\mu}$ for $E > 30$ GeV is half that for $E > 100$ GeV (for both FHPRW and CERN fluxes). For other modes, the difference is even greater. Second, the CERN (wide band beam) flux is shaped quite differently from the FHPRW flux (the latter is the same as the new CF flux). As a result, the rates expected with the FHPRW flux are about 50% larger than with the CERN flux (for either energy range).

As discussed in Sec. 1, there appear to be two distinct types of

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As discussed in Sec. I, there appear to be two distinct classes of trimuon events: The "ordinary" events (shown with cross-hatching in the figures) and the two "super" events (shown separately in the figures). For each mode we investigated the possibility of producing "super" events. Our procedure, for each observed event, was to find what fraction of calculated trimuon events had

\[ E_{\mu_1} > E_a, \quad E_{\mu_2} > E_b, \quad \text{and} \quad E_{\mu_3} > E_c, \]

where \( E_a, E_b, \) and \( E_c \) were the observed muon momenta reduced by one standard deviation. For both "super" events, no mode discussed here had any possibility of producing such energetic muons (at the same time). For the hadronic, mixed and leptonic modes, less than one trimuon event in \( 10^4 \) trimuon events would have such large energies. In most cases, one expects the outgoing neutrinos to take a significant fraction of the energy, but for these events the observed energy is so high that there is almost no remaining energy available. For the trident mode, there are no outgoing neutrinos, and this improves the ratio to one in \( 10^3 \) trimuon events which is still inadequate (especially since the calculated trident rate is also low). One might assume that vector meson production (and decay to \( \mu^+\mu^- \)) results in a similar ratio (one in \( 10^3 \)).

From the FHPRW data, the observed rate of "super" events is more than one in 10.

The most difficult problem in analyzing trimuon production in neutrino scattering will be the certain presence of "background" trimuon events. Our calculated rate for trident production is about 20% of the observed rate. It is possible that vector meson production and decay to \( \mu^+\mu^- \) may lead to a trimuon rate also on the order of 20%. It is reasonable to expect that perhaps half of (and conceivably all of) the observed trimuon events come from conventional sources (involving no heavy particles).

The separation of these background events may be difficult, and the identification of the source of any non-background trimuons may be even more difficult. To lower the fraction of background events, one could examine only those events
with total energy above 100 GeV. However, if there are trimuons from interesting sources, they are likely to have two outgoing neutrinos so that the measured total energy will be lower than the true energy. Our calculations show that this leads to very similar $E_{\text{visible}}$ distributions for "background" and "non-background" sources. Perhaps when there is more data, if one divides the trimuon rate at each energy by the flux and by the single muon rate, one will be able to observe the fall off of the trimuon rate at high energies due to the loss of energy to outgoing neutrinos. Of course, if the neutrino fluxes can be chosen to eliminate low energy neutrinos, this could raise the fraction of "non-background" trimuon events.

Alternatively, one can apply various cuts such as $E_\mu > 8$ GeV or $(\theta_{23} - \theta_1) < 120^\circ$ both to observed events and to calculated distributions. Unfortunately this reduces the amount of data to be analyzed, but it may be necessary to raise the fraction of "non-background" events. These will be matters of judgement as more data becomes available. The problem will be more severe if any "interesting" trimuons are from heavy quark cascades (a hadronic mode) since their characteristics will be more similar to those of backgrounds than would be other sources.

For the mixed mode, antineutrino scattering provides an important test since only that mode predicts $\mu^+\mu^-\mu^-$ events. In general the energy and mass distributions are more subject to details of produced masses, fragmentation functions, etc. Of course, $M_{23}$ is an important test of the presence of the trident mode. We believe that the angular distributions, such as discussed here, are the superior means of distinguishing the various possible sources of neutrino-induced events with three muons.

Note added - Upon completion of this work, we received manuscripts from R. B. D. and from J. R. B. and R. A. B. We understand that results similar to the present one are reported there.
Note added - Upon completion of this work, we received manuscripts from V. Barber, T. Gottschalk and R. J. N. Phillips and from J. Smith and J. A. M. Vermaseren who obtain results similar to the trident results reported here. We thank J. Smith for discussing his work with us before publication.

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REFERENCES

5. Addition of a coupling (N eV ) R makes the electron couplings vector; see M. Barnett, Phys. Rev. D13, 671 (1976) and references therein.


FIGURE CAPTIONS

1. Three modes of trimuon production: a) the hadronic mode, b) the mixed mode and c) the leptonic mode.

2. The trident mode of trimuon production with a bremsstrahlung photon from an off-shell a) muon, b) incoming d quark, c) outgoing u quark.

3. The event rate versus the energy asymmetry between the fast negative muon ($\mu_1$) and the positive muon ($\mu_3$). The unnormalized theoretical curves are for the hadronic mode (dotted), the mixed mode (dashed), the leptonic mode (solid) and the trident mode (dashed-dotted). The FHPRW flux is folded in and $E_\mu > 4$ GeV required for all muons. The data is shown with shaded squares for each observed event except for the two "super" events which are marked with $S_1$ and $S_2$. The data is from Ref. 8.

4. The event rate versus the energy asymmetry between the fast ($\mu_1$) and slow ($\mu_2$) negative muons. The curves and data are as described for Fig. 3.

5. The invariant mass squared of the slow negative muon ($\mu_2$) and the positive muon ($\mu_3$) which is $(p_{\mu_2} + p_{\mu_3})^2$. The curves and data are as described for Fig. 3.

6. The event rate versus the total invariant mass squared of all three muons which is $(p_{\mu_1} + p_{\mu_2} + p_{\mu_3})^2$. The curves and data are as described for Fig. 3.

7. The event rate versus the angle in the plane perpendicular to the incoming neutrino between $(\vec{p}_{\mu_2} + \vec{p}_{\mu_3})$ and $\vec{p}_{\mu_1}$. The curves and data are as described for Fig. 3.

8. The event rate versus the angle in the plane perpendicular to the incoming neutrino between $(\vec{p}_{\mu_1} + \vec{p}_{\mu_3})$ and $\vec{p}_{\mu_2}$. The curves and data are as described for Fig. 3.
9. The event rate versus the angle in the plane perpendicular to $p_W$ between $\vec{p}_{\mu 3}$ and $\vec{p}_{\mu 1}$ where $\vec{p}_W$ is $(\vec{k} - \vec{p}_{\mu 1})$ and $\vec{k}$ is the calculated incoming neutrino momentum. The curves and data are as described for Fig. 3.

10. The event rate versus the angle in the plane perpendicular to $p_U$ between $\vec{p}_{\mu 2}$ and $\vec{k}$ where $\vec{p}_U$ is $(\vec{k} - \vec{p}_{\mu 1} - \vec{p}_{\mu 3})$ and $\vec{k}$ is the calculated incoming neutrino momentum. The curves and data are as described for Fig. 3.

11. The cross-section versus real total energy with no flux folded in (with $E_\mu > 4$ GeV). The unnormalized curves correspond to the modes as described for Fig. 3.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9
EVENTS (arbitrary units)

Fig. 10