QCD PREDICTIONS FOR THE ASSOCIATED PRODUCTION
OF CHARM BY NEUTRINOS

H. Goldberg
Stanford Linear Accelerator Center†
Stanford University, Stanford, California 94305

and

Department of Physics
Northeastern University, Boston Massachusetts 02115††

ABSTRACT

Cross sections for the inclusive production of charm-anticharm
pairs in the hadron showers of neutrino scattering are calculated
within the framework of QCD. A branching ratio of less than $10^{-3}$,
in insufficient to account for the like-sign dimuons at FNAL and CERN,
and trimuons at FNAL, is obtained for $\alpha_s = 0.4$ at values of $x$ between
0.05 and 0.3, and $\nu \sim 50 - 75$ GeV.

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†Supported by the Energy Research and Development Administration.
††Permanent address.
Trimuons\textsuperscript{1,2} and like-sign dimuons\textsuperscript{3,4,5} have recently been observed in high energy neutrino experiments at FNAL and CERN. It has been proposed\textsuperscript{6} that the production and subsequent decay of new heavy leptons ($m \sim 10$ GeV) are responsible for these events. However, at least in the case of the like-sign dimuons, the events may be accounted for through the associated production of charm-anticharm pairs in 0.5-1\% of the hadron showers.\textsuperscript{4,7} Hence the heavy lepton interpretation of the multimuon events must be measured against at least this alternative.

In a recent letter, F. Bletzacker, H. T. Nieh and A. Soni\textsuperscript{8} have presented a phenomenological model of $c\bar{c}$ pair production in the diffractive (small $x$) region in order to account for the kinematic distributions of the multimuons. Since the publication of this work, however, a "large" sample of 47 $\mu^+\mu^-$ events have been reported\textsuperscript{5} in a $\nu$ Fe experiment at CERN. The background from $\pi^-$ and $K^-$ decay is estimated to contribute 30\pm7 events, so that it is possible that there are 17\pm7 $\mu^+\mu^-$ events of direct origin. The $\langle x_{\text{vis}} \rangle$ of all the events is 0.28, so that if there are events of direct origin, it is likely that they are not in the diffractive region. (The 7 events reported in Ref. 3 also have $\langle x_{\text{vis}} \rangle \approx 0.2$.) In addition, the work of Ref. 8 does not provide a theoretical basis for the overall normalization of the cross section for the associated production of charmed hadrons.

Thus, it is of considerable practical interest to provide a theoretical model for inclusive charm-anticharm production in the nondiffractive ("normal $x$") region of $\nu$-nucleus scattering. Such a model, based on the standard SU(3) color gauge theory of the strong interactions ("QCD") is presented in this paper. It will be seen that the estimates obtained, for average values\textsuperscript{9} of the QCD coupling constant $\alpha_s(m) \approx 0.4$ and the charmed quark mass $m_c(m) \approx 1.6$ GeV (for
3 GeV \leq \mu \leq 13 \text{ GeV}) are very strong functions of \nu, the total hadron energy, and weak functions of x. In the range of the CERN experiments \( \langle x \rangle \sim 0.3, \langle \nu \rangle \sim 70 \text{ GeV} \), the branching ratio is predicted to be about \( 0.6 \times 10^{-3} \), a factor of 10 too small to account for the data.

The model is essentially described by the graphs in Fig. 1a. A parton in a nucleon is struck by a W, and emits (or pre-emits) a timelike color gluon, which subsequently "decays" into a \( c\bar{c} \) pair. Other contributions, such as the emission of the \( c\bar{c} \) pair by one of the spectator partons (depicted in Fig. 1b), are expected to be small because they involve large momentum transfers along more than one gluon line, resulting in extra factors of \( \alpha_s^2 \).

All dressings of the quarks in the final state are assumed (as usual) to proceed with unit probability, and all momenta are integrated over. The arithmetic is straightforward but tedious, and consists primarily of squaring the amplitude, integrating over phase space and subsequently identifying the contributions of a single parton to the structure functions \( W_1, W_2, W_3 \). These are then convoluted with parton distribution functions \( \frac{1}{\hat{q}} \) to obtain the contribution of inclusive associated charm production to neutrino-nucleon scattering,

\[
\frac{d\sigma^c\bar{c}}{dxdy} = \frac{(G^2M_p)}{\pi} \left( \frac{1}{2} y^2 xF_{1c\bar{c}} + (1-y)F_{2c\bar{c}} + y \left( 1 - \frac{1}{2} y \right) xF_{3c\bar{c}} \right) \ldots (1)
\]

where \( F_{1c\bar{c}} = 2M_N W_{1c\bar{c}}, F_{2c\bar{c}} = \nu W_{2c\bar{c}}, F_{3c\bar{c}} = \nu W_{3c\bar{c}} \), \( y = \nu/E _\nu \), with \( W_1, W_2, W_3 \) defined in the standard manner. The nucleon structure functions \( W_i \) are given in terms of parton structure functions \( w_i \) by

\[
W_{1,2} = \int (d\eta/\eta) \, w_{1,2}(q^2, \nu, \eta) \, (q(\eta) + \bar{q}(\eta)) \ldots (2)
\]

\[
W_3 = \int (d\eta/\eta) \, w_3(q^2, \nu, \eta) \, (q(\eta) - \bar{q}(\eta)) \ldots
\]
where \( q(n) = \frac{1}{2} (u(\eta) + d(\eta)) \), \( \bar{q}(n) = \frac{1}{2} (\bar{u}(\eta) + \bar{d}(\eta)) \) and \( u, d (\bar{u}, \bar{d}) \) are the up and down quark (antiquark) densities in the proton, as a function of the longitudinal momentum fraction \( \eta \). (Note that \( \eta \neq x \).)

In order to simplify the arithmetic to some extent, all the light quark masses have been set equal to zero. In that case, most of the phase space calculation can be performed analytically. There remains a two-fold numerical integration: over \( \tau \), the (c.m. energy)\(^2\) of the \( cc \) pair, and over \( \eta \). For fixed \( x = -q^2/2M_N \nu, \nu \) and \( \eta \), the range of \( \tau \) is

\[
4m_c^2 \leq \tau \leq 2M_N \nu(\eta-x)
\]

whereas \( \eta \) ranges over the values \( \eta_{\text{min}} = x + 4m_c^2/2M_N \nu \) to 1. From this we note the important fact that for moderate \( \nu \) (\( \nu < 100 \text{ GeV} \)), \( \eta_{\text{min}} > 0.05 \) even for \( x \) close to zero. Hence we partons play no role in the calculation, and the model is expected to give meaningful results even for very small \( x \) \((x < 0.1)\).

From Eq. (1), the theoretical branching ratio to charm-anticharm pairs in neutrino scattering is

\[
\frac{B_{\nu N}^{cc}}{\nu N} = \frac{\frac{1}{2} y^2 xF_{1}^{cc} + (1-y)F_{2}^{cc} - y \left(1 - \frac{1}{2} y\right) xF_{3}^{cc}}{x(u+d) + (1-y)^2 x(\bar{u}+\bar{d})} \ldots \tag{3}
\]

where, for consistency, we have made use of the parton model relations\(^{11}\)

\[
xF_{1}^{\nu N} = F_{2}^{\nu N} = x(u+d + \bar{u}+\bar{d}), \quad -xF_{3} = x(u+d - \bar{u}-\bar{d}).
\]

In Fig. 2 are plotted some sample results for \( y=0.5 \). The branching ratio is slowly varying in \( x \), but very rapidly varying in \( \nu \), so that a comparison with experiment would require a fairly accurate knowledge of \( \langle \nu \rangle \) for the events in question. For the 17 \pm 7 possible events of Ref. 4, we may estimate \( \langle x \rangle \sim 0.3, \langle \nu \rangle \sim 70 \text{ GeV} \) (the latter being obtained from the stated values of \( \langle y \rangle, \langle E_{\mu_1} \rangle \)).
and \( \langle E_\mu \rangle \), and hence (from Fig. 2) propose a theoretical branching ratio of about \( 0.6 \times 10^{-3} \) for associated production of charmed-anticharmed pairs in hadron showers of neutrino collisions. Folded with a branching of 0.15 of \( c \rightarrow \mu + \ldots \), and a detection efficiency\(^5\) of 0.3, we are led to a branching ratio of \( 2.7 \times 10^{-5} \) for \( \mu^- \mu^- \) pairs. This is a factor of 10 too small to account for the experimental results. The corresponding trimuon branching from \( cc \) pairs is then predicted to be \( 3 \times 10^{-6} \), also a factor of 10 smaller than experiment.\(^1,2\)

For completeness I have plotted in Fig. 3 the branching into charm-anticharm pairs for \( \bar{p}N \) experiments. The relevant formula (corresponding to Eq. (3), is

\[
B_{CC}^{\bar{p}N} = \frac{1}{2} y^2 x F_{1} c \bar{c} + (1-y) F_{2} c \bar{c} + y \left(1 - \frac{1}{2} y\right) x F_{3} c \bar{c} \]

\[ \frac{1}{x(u+d)(1-y)^2 + x(u+d)} \ldots \]  \( (4)\)

Except for being slightly larger, the behavior of this branching ratio is similar to that in the case of \( \nu N \). The branching into multimuons can be found as before.

Curves for smaller values of \( x \) (\( x = 0.01, 0.05 \)) have also been calculated, but not plotted. They differ by less than 30% from the \( x = 0.1 \) curves in Figs. 2-3.

To conclude, I have calculated within the framework of QCD the cross section for the inclusive production of charm-anticharm pairs in neutrino-nucleus scattering. I have found values of the order of \( 10^{-3} \) at presently available energies. This is insufficient to account for the like-sign dimuon events observed at FNAL and CERN, and trimuons at FNAL. If these events persist
at the rates quoted, we would tend, in the light of this calculation, to consider
more seriously the heavy lepton alternative, or a recalculation of $\pi/K \rightarrow \mu$
background.

There remains the question: is the mechanism proposed in this paper the
dominant one for associated charm production in neutrino interactions? We
would expect this to be so in the scaling region, $Q^2 \gtrsim 1$ GeV$^2$. For $<v> \sim 50$ GeV,
this means $x \gtrsim 0.01$. Hence we would not expect a virtual hadronic diffractive
mechanism to play a significant role in the like-sign dimuon production at
$x \sim 0.2-0.3$ discussed in this paper. 12

Calculational details are deferred to a later publication.

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12. The case of $\mu N$ scattering is considerably different: the presence of a substantial $\psi$ component in the virtual photon (F. E. Close, D. Scott, D. Sivers, Nucl. Phys. B117, 134 (1976)) introduces a possibly dominant $c\bar{c}$ production mechanism into electroproduction which is not present in $\nu N$ and $\bar{\nu} N$ interactions. [Also, D. P. Roy, Phys. Lett. 68B, 76 (1977).]

There are also additional parton diagrams in this case, due to the presence of gluons in the proton. These give contributions of order $\alpha_s$. 
FIGURE CAPTIONS

1. (a) The principal diagrams contributing to $c\bar{c}$ production in the nondiffractive region. $\gamma_s$ is a QCD gluon. (b) Diagrams suppressed by order $\alpha_s$ (see text).

2. Branching ratio $R_{\nu N}^{c\bar{c}} = \left( \frac{d\sigma_{\nu N \rightarrow \mu^- c\bar{c}X}}{dxdy} / \frac{d\sigma_{\nu N \rightarrow \mu^- X}}{dxdy} \right)$, evaluated at $y=0.5$, vs. total hadron energy $\nu$.

3. Branching ratio $R_{\nu N}^{c\bar{c}} = \left( \frac{d\sigma_{\nu N \rightarrow \mu^+ c\bar{c}X}}{dxdy} / \frac{d\sigma_{\nu N \rightarrow \mu^+ X}}{dxdy} \right)$, evaluated at $y=0.5$, vs. total hadron energy $\nu$. 
Fig. 1

(a)

\[ W^+ \]

\[ d \]

\[ u \]

\[ \gamma_s \]

\[ c \]

\[ \bar{c} \]

\[ + \]

\[ W^+ \]

\[ d \]

\[ \gamma_s \]

\[ c \]

\[ \bar{c} \]

(b)

\[ W^+ \]

\[ d \]

\[ \gamma_s \]

\[ \gamma_s \]

\[ \bar{c} \]

\[ u \]

\[ c \]

\[ + \cdots \]

\[ u \]

\[ u \]

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Fig. 2

$B_{\nu N}^{c\bar{c}}$

$\nu$ (GeV)

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$x = 0.1$

$x = 0.3$
Fig. 3