HADRON PRODUCTION IN NUCLEAR COLLISIONS –
A NEW PARTON MODEL APPROACH*

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ABSTRACT

We consider a simple quark-parton model for inelastic hadron-nucleus
interactions where the wee partons of the projectile — uncorre-
lated in rapidity — interact with the wee partons of essentially inde-
pendent nucleons in the target. The ratio of multiplicities in the central
region is given by

\[ \frac{R_A}{<n>_{HA}} = \frac{\bar{\nu}}{<n}_{HN} = \frac{\bar{\nu}}{2} + \frac{\bar{\nu}}{\nu+1} \]

where \( \bar{\nu} = \frac{A\sigma_{HN}^{inel}}{\sigma_{HA}^{inel}} \) is the average number of inelastic collisions of
the projectile H in the nucleus. Including the effects of the leading par-
ticle regions, this prediction is in excellent agreement with experiment.

Predictions are also given for multiplicity distributions in hadron-nucleus
collisions and for multiplicities produced in nucleus-nucleus collisions.
The model, which is consistent with Glauber theory, predicts the ab-
sence of shadowing at large \( q^2 \) (independent of \( \omega \)) in electroproduction
or whenever the momentum transfer of a subprocess is large.

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Although the quark-parton model has been very successful in predicting the short distance behavior of hadronic interactions, the underlying mechanisms involved in the production of hadrons in ordinary high energy collisions have never been specified. In the case of particle production on nuclear targets, this fundamental uncertainty of the parton approach becomes amplified, and this has led to an extraordinary range of divergent predictions for even the most basic experimental parameters. In this letter we present a new approach to this problem based on a straightforward application of parton model concepts. The resulting picture for nuclear collisions is very simple and in good agreement with experiment. It is based upon (1) the assumption that each inelastically excited nucleon in the nuclear target produces hadrons independently of the others, and (2) a specific hadronic collision model based on wee parton interactions analogous to the Drell-Yan pair production process.

We begin with a simple parton model description of hadron-hadron interactions. Each hadron has a Fock-space decomposition in terms of multiparton states. An interaction occurs via a collision of a parton in the beam (B) with a parton in the target (A). The cross section takes the typical Drell-Yan form

\[ \sigma_{BA} = \sum_{a \in A} \int_0^1 dx_a \int_0^1 dx_b G_a/A(x_a)G_b/B(x_b) \tilde{\sigma}_{ab}(\hat{s}_{ab}) \tag{1} \]

where

\[ x_b = \frac{(k^o_b + k^z_b)}{(p^o_B + p^z_B)} \]

and

\[ x_a = \frac{(k^o_a - k^z_a)}{(p^o_A - p^z_A)} \]

are the light-cone fractions \((p^z_B > 0, p^z_A < 0)\) of the beam and target, respectively, and...
\[ s_{ab} = x_a x_b s + \frac{m_a m_b}{x_a x_b s} \]

is the collision energy squared of the subprocess. (For simplicity we do not display the transverse momentum dependence.) Expression (1) is Lorentz-invariant for boosts along the beam (z) direction. We presume that \( \hat{\sigma}_{ab} \) falls rapidly with increasing \( \hat{s}_{ab} \), as would be typical of quark-parton exchange\(^2\) or \( q\bar{q} \) annihilation processes,\(^6\) and that each distribution \( G(x) \) has the Feynman\(^2\) wee parton distribution \( xG(x) \rightarrow C \neq 0 \) at \( x \rightarrow 0 \). In this model \( \sigma_{BA}(s) \propto \log s \), and the location in rapidity of the parton-parton collision \( \hat{y} \) is distributed uniformly throughout the central region, where neither \( x_a \) nor \( x_b \) is forced into the finite \( x \), power-law damped regions of \( G(x) \). In inelastic collisions, the partons in the beam materialize as hadrons for \( \hat{y} < y < Y_B \), and those in the target materialize throughout the interval \( Y_A < y < \hat{y} \). Note that real hadron production from the beam partons cannot extend much below \( \hat{y} \) since this forces propagators off-shell where interactions are suppressed.

Turning to nuclear collisions, we shall assume that, aside from small binding corrections and Fermi motion effects, each nucleon in the nucleus independently develops its own parton distribution. Thus the partons of different nucleons interact with each other only minimally and do not shadow or coalesce with one another.\(^7\) In a high energy collision the various wee partons of the projectile can interact with the wee partons of different nucleons. The rapidity locations of the parton-parton collisions \( \hat{y}_i \) are uncorrelated and uniformly distributed in the central region. Each nucleon in the nucleus \( A \) participates in only one interaction, whereas the mean number of inelastic collisions of the beam hadron \( H \) is \( \bar{\nu} = \frac{\sigma_{HN}^{inel}}{\sigma_{HA}^{inel}}. \) On the average, then, the rapidity separation between parton collisions is \( \Delta y \approx Y_c / (\bar{\nu} + 1) \) where \( Y_c \) is the total length of
the central rapidity region. A typical multiparticle distribution for \( \bar{\nu} = 3 \) collisions is illustrated in Fig. 1. Since the collision rapidities are uncorrelated, each inelastically excited nucleon produces hadronic multiplicity on the average halfway across the central region. As the number of collisions increases, the range of the projectile hadron distribution extends further and further into the central region to the minimum \( y_1 \)—on the average over a rapidity length \( \Delta y = (\bar{\nu}/(\bar{\nu}+1))Y_c \). Thus we immediately obtain for the ratio of multiplicities in the central region

\[
\frac{\langle n \rangle_{HA}}{\langle n \rangle_{HN}} = \frac{\bar{\nu}}{2 + \frac{\bar{\nu}}{\bar{\nu} + 1}},
\]

where the only dependence on the projectile \( H \) is through the definition of \( \bar{\nu} \).

The distribution of particles averaged over events produced from the excitation of the nuclear partons is wedge-shaped. The ratio of distributions in the central region for hadron-nucleon to hadron-nucleus collisions is simply \( y_A = 0 \)

\[
R_A(y) = \frac{(dn/dy)_{HA}}{(dn/dy)_{HN}} = \bar{\nu} \left( 1 - \frac{y}{Y_c} \right) + \left[ 1 - \left( 1 - \frac{y}{Y_c} \right) \bar{\nu} \right].
\]

Although Eqs. (2) and (3) are derived assuming a uniform plateau height in the central region, corrections to this shape tend to cancel in the ratio.

Thus far we have ignored the effects of the fragmentation regions. Eq. (1) predicts that the fast (e.g., valence) partons interact only weakly and thus \( R_A(y) = 1 \) in the projectile fragmentation region, and \( R_A(y) = \bar{\nu} \) in the target fragmentation region. Let \( \langle n_{frag} \rangle_H \) and \( \langle n_{frag} \rangle_N \) be the average number of particles produced in the projectile and nucleon fragmentation regions (i.e., within \( \Delta y_{frag} \sim 2 \) units of the incident rapidity). Then, instead of Eq. (2), we obtain
\[ \frac{\langle n_{\text{tot}} \rangle_{\text{HA}}}{\langle n_{\text{tot}} \rangle_{\text{HN}}} = \left( \frac{\bar{v}}{2} + \frac{\bar{\nu}}{\nu+1} \right) \frac{\langle n_{\text{central}} \rangle}{\langle n_{\text{tot}} \rangle_{\text{HN}}} + \frac{\bar{\nu}}{2} \frac{\langle n_{\text{frag}} \rangle_{\text{H}}}{\langle n_{\text{tot}} \rangle_{\text{HN}}} + \frac{\bar{\nu}}{\nu+1} \frac{\langle n_{\text{frag}} \rangle_{\text{N}}}{\langle n_{\text{tot}} \rangle_{\text{HN}}} \]

where \( \langle n_{\text{tot}} \rangle_{\text{HN}} = \langle n_{\text{central}} \rangle + \langle n_{\text{frag}} \rangle_{\text{N}} + \langle n_{\text{frag}} \rangle_{\text{H}} \) is the total produced multiplicity for the H-N collision. In practice the fragmentation correction terms are small, of order \((\Delta y)_{\text{frag}} / Y_{\text{total}} \sim O(1/\log s)\) compared to \(\bar{\nu}/2\).

This result is compared with the data summary of Busza et al. in Fig. 2 for \(p_{\text{lab}} = 200\) GeV, taking \(\langle n_{\text{frag}} \rangle_{\text{H}} / \langle n_{\text{tot}} \rangle \sim \langle n_{\text{frag}} \rangle_{\text{N}} / \langle n_{\text{tot}} \rangle \sim 0.2\). It is in good agreement with the data for charged pion and proton collisions. In addition, the shapes of the observed multiplicity distributions are consistent with the predicted forms of Eq. (3) and Fig. 1. The slight energy dependence predicted in Eq. (4) is also consistent with the trend of the data.\(^{10}\)

We have analyzed the total nuclear cross section in this model and have found it to be consistent with the usual Glauber theory.\(^{11}\) In this picture the incident hadron, which is represented by its Fock-space parton distribution, can interact elastically (diffractively) via elastic parton interactions in the central region and can continue to propagate and interact as a coherent hadron through the nuclear medium.\(^{12}\) Thus one obtains the usual multiple-scattering Glauber series. Nonetheless, the multiplicity density \(dN/dy\) produced from the incident projectile parton distribution is not increased by the repeated collisions. Because of the Glauber series, the cross section of course does not factorize:

\[ \sigma_{\pi A} \sim \sigma_{pA} \quad \text{approach the geometric limit.} \]

The model proposed here is consistent with energy and momentum conservation. In the equal velocity frame, the central particles produced in the projectile direction have a typical total energy of order \(\bar{v}m_T, (m_T^2 - m^2 + \langle x_T^2 \rangle)\),
which can be compensated by a small loss of energy of the leading particles in
the projectile region, a correction of relative order $\frac{\bar{v}_{m_T}}{s}$.

One may also use this picture to predict the multiplicity distributions in
nucleus-nucleus collisions. For the central region one obtains

$$\frac{<n>_{A_1A_2}}{<n>_{NN}} = \frac{\bar{v}_{A_1/A_2}(\bar{v}_{A_2/N})}{\bar{v}_{A_1/A_2}(\bar{v}_{A_1/N+1}) + \bar{v}_{A_2/A_1}(\bar{v}_{A_2/N+1})},$$  \hspace{1cm} (5)

where

$$\bar{v}_{A_1/A_2} = \frac{A_1\sigma_{NA_2}}{\sigma_{A_1A_2}}$$

is the average number of inelastically excited nucleons in $A_1$ in collision with a
projectile $A_2$. Each such excited $A_1$ nucleon interacts inelastically with $\bar{v}_{A_2/N}$
nucleons in $A_2$ so that the average rapidity length of excited partons in $A_1$ is

$$\frac{\bar{v}_{A_2/N}}{\bar{v}_{A_2/N+1}} \frac{1}{y_{c}}.$$ 

Corresponding statements apply to $\bar{v}_{A_2/A_1}$ and $\bar{v}_{A_1/N}$. The above result pre-
dicts, for example, $<n>_{A_2/A_1} \sim 3.8$ for $A_2 > 100$, which is in agreement
with cosmic ray data for alpha-particle collisions.

Finally, we wish to point out the connection between our hypothesis of inde-
dependently interacting and materializing nuclear parton chains and deep inelastic
scattering measurements on nuclei. The latter directly probe the parton dis-
tributions within nuclei, and, according to our hypothesis, one should obtain

$$\nu W_2 A(x_{Bj}) \approx \Lambda \nu W_2 (x_{Bj})$$  \hspace{1cm} (6)

for all (including arbitrarily small) $x_{Bj} = -q^2/2M_N\nu \lesssim 1$ once $q^2$ is in the Bjorken
scaling region. For $x_{Bj} > 1$, Fermi motion corrections can be included and
computed using quark counting, but otherwise nuclear binding corrections to
(6) are considered negligible. Thus there is neither shadowing nor antishadowing\(^{16}\) of the partons of one nucleon by the partons of other nucleons. In general, we predict the absence of shadowing — independent of beam energy — for any reaction where the effective collision energy of the subprocess is large, e.g., for the Drell-Yan Process \(pA \rightarrow \ell^+\ell^-X\) at large \(Q^2\), as well as for large \(p_T\) hadronic reactions — ignoring multiple scattering effects.\(^{17}\) The absence of shadowing is also apparent in the ratio of distributions \(R_A(x) = (dn/dx)_{HA}/(dn/dx)_{HN}\) where \(x\) is the Feynman variable \(k_{c.m.}/k^\text{max}_{c.m.}\). At infinite energy \(R_A(x)\) reduces in our model to a step function \(R_A(x) = \bar{\theta}(-x) + \theta(x)\) since the central region is confined to \(x \rightarrow 0\). If we identify the nuclear parton distribution shape with the multiparticle distribution for \(x < 0\), this again corresponds to the absence of shadowing: \((d\sigma/dx)_{HA} = A(d\sigma/dx)_{HN}\)^{18}

In summary, we have found that the parton model can be consistent with both the strong absorption of nuclear cross sections and the relatively low multiplicity of hadron-nucleus collisions. Another problem which could be analyzed in this model is the propagation of virtual quark states and unstable resonances through the nuclear medium.\(^{19,20}\)

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References

1. For a review of the data and various theoretical models, see W. Busza, in Proc. VII Int. Colloquium on Multiparticle Reactions, Tutzing, Germany, 21-25 June 1976.


7. We emphasize that nuclear binding and Fermi–motion corrections are not related to the "A^{2/3}\" surface or nuclear size effects characteristic of shadowing.

8. This was first discussed by O. Kancheli, JETP Lett. 18, 274 (1973).


10. Recently, A. Capella and A. Krzywicki, Orsay preprint LPTHE 7712 (1977), proposed an extended multiscattering model in which higher order Glauber terms correspond to the interaction of n = 1, 2, ... independent constituent systems within the projectile, each of which shares the incident energy roughly equally, \( \hat{s} \sim s/n \). At present energies their model also agrees well with experiment. However, the energy dependence of this model
differs from the model discussed here; e.g., the nuclear chains all extend
to the projectile fragmentation region and \( \frac{\langle n \rangle_{HA}}{\langle n \rangle_{HN}} \rightarrow \beta \) at infinite en-
ergy. The effect of the multichains on the parton distribution of the pro-
jectile (\( vW_{2H} \)) must also be understood.

ten and G. Dunham (Interscience, New York, 1959). The generalized
Glauber theory resulting from this model includes inelastic diffractive states
which can be treated in the fashion of photon-hadron interactions, as in S. J.
Brodsky and J. Pumplin, Phys. Rev. 132, 1794 (1969), and V. Gribov,
JETP 30, 709 (1970). For a recent discussion and references for multi-
scattering models see J. Weis, CERN preprint TH.2197 (1976). We are
grateful for discussions with H. Miettinen and J. Weis on this subject.

12. Since the parton scattering is almost forward, the initial Fock–space state
is disturbed only minimally. It thus has considerable overlap and remains
nearly coherent with the incident projectile wave function. A complete dis-
cussion will be presented in S. J. Brodsky, J. F. Gunion, and J. Kühn, to
be published.

13. The experimental ratio is \( \sim 4 \). See M. F. Kaplon and D. M. Ritson, Phys.
Rev. 85, 932 (1952); Phys. Rev. 88, 386 (1952). We wish to thank D. Rit-
son for a discussion of these data. For other theoretical treatments see
and references therein.

14. This possibility was suggested, for instance, in S. J. Brodsky, F. Close,
and J. F. Gunion, Phys. Rev. D 6, 177 (1972). Note that this is contrary
to the result predicted by Zakharov and Nicolaev (Ref. 16). See also J. D.
Bjorken, Proc. SLAC Summer Institute, Stanford, California, 21–31 July
1975.


18. These results are also consistent with the cutting rules of V. A. Abramovski, V. N. Gribov, and O. V. Kancheli, Sov. J. Nucl. Phys. 13, 308 (1974).


20. Bjorken scaling of the virtual photoabsorption cross sections $\sigma_T(q^2, \nu) \propto (q^2)^{-1}$ at high $\nu$ and the scale-invariant behavior for $\sigma(e^+e^- \rightarrow \text{hadrons})$ at large $s$ implies, in a generalized vector dominance model, that the diffractive cross section for virtual meson states $\sigma(M^2, \nu)$ decreases at least as fast as $M^{-2}$. See J. D. Bjorken (Ref. 14), S. Brodsky and J. Pumplin (Ref. 11), and V. N. Gribov (Ref. 11). Since the high mass states interact weakly, this again implies the absence of shadowing in deep inelastic lepton scattering, Eq. (6).
Figure Captions

1. Idealized multiplicity distribution for an H-A collision with $\bar{\nu} = 3$ inelastic excitations. The $y_i$ are uniformly distributed in rapidity and can be produced in any sequence. The central and fragmentation ($s$-independent) regions are indicated.

2. The variation of $R_A = \langle n \rangle_{HA} / \langle n \rangle_{HN}$ with $\bar{\nu}$ for pion and proton beams. The data are for charged multiplicities from Ref. 1. The solid curve is the $s \to \infty$ prediction $R_A = \bar{\nu}/2 + \bar{\nu}/(\bar{\nu}+1)$. The dashed curve is the line $R_A = \bar{\nu}/2 + 1/2$ corresponding to no central region. The prediction of the model, Eq. (4), for $E_{\text{lab}} = 200$ GeV (taking $\langle n_{\text{frag}} \rangle_{H, N} / \langle n_{\text{tot}} \rangle = .2$) is the dashed-dotted curve, $R_A = \bar{\nu}/2 + \bar{\nu}/(\bar{\nu}+1) - .2(\bar{\nu} - 1)/(\bar{\nu}+1)$. 