WEAK INTERACTION EXPERIMENT AND THEORY*

R. Michael Barnett
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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I. INTRODUCTION

Given the mood of many in this field, I had considered dropping the word "interaction" from the title. There has been a widely popular disrespect for both experiment and theory during the last couple of years. It is, however, an attitude which I do not share. There have been a series of extremely difficult experiments completed recently with surprisingly clear and important results; these various experiments have shown a remarkable consistency among themselves. Equally impressive have been the successes of the parton model and of the general framework of the Weinberg-Salam gauge theory of the weak and electromagnetic interactions. The total and differential cross sections found in neutrino scattering were close to those predicted. Most predicted neutral currents exist at approximately the expected magnitudes (and in the simplest model, with the same value of $\sin^2\theta_W$). The proposed explanation for the lack of strangeness-changing-neutral-currents (charm) appears to be correct.

While there have been great successes, there remain further problems to be addressed. Are there more than four quarks? If so, why does the GIM mechanism work? Are there any charm-changing-neutral-currents? How is CP violation to be understood? Is asymptotic freedom relevant to the anti-neutrino anomalies? Do quarks and leptons have right-handed currents? What are the weak interactions of the heavy lepton? Do neutrinos have finite mass? And, of course, there are deeper questions which must eventually be considered.

In this limited review, I will first discuss three recent theoretical developments relevant to quark-lepton models of the weak interactions (Secs. II-IV). The implications of recent experimental results on these models is considered next (Secs. V-IX). Finally, a few aspects of the Pati-Salam gauge model with integer, nonconfined quarks, leptons, and gluons are examined (Secs. X-XIII).
II. NATURALNESS OF THE GIM MECHANISM

The absence of strangeness-changing neutral-currents (SCNC) in weak-interactions is observed to be of very high order. Glashow, Iliopoulos, and Maiani (GIM) proposed in 1970 a mechanism which assures the absence of SCNC. The original model which incorporates this mechanism, the Weinberg-Salam-GIM (W-S-GIM) model, automatically cancels the SCNC. No choice of parameters, such as mixing angles, can avoid the cancellation. However, some other proposed quark models, which also contain the GIM mechanism, obtain this cancellation of SCNC only by specifying that certain mixing angles take specific values.

In a recent paper Glashow and Weinberg argue that, since SCNC are so completely negligible, it is highly unlikely that this absence is the result of an accident – an accident by which the parameters of the theory happen to take the precise values needed. If parameters are chosen to avoid certain mixings leading to SCNC, one could still find weak, radiative effects which lead to SCNC of order $\alpha G$ (which are not observed).

Glashow and Weinberg (GW) show that in SU(2) x U(1) models the parameter-independent incorporation of GIM (which they call "natural" GIM) is assured under the following condition. If we define $\tau$ as the weak isospin (from SU(2)_{weak}), then one must require that all quarks of charge $-\frac{1}{3}$ have the same values of $\tau_2^L$, $\tau_2^R$, $\tau_3^L$, and of $\tau_3^R$, where L and R refer to left-handed and right-handed couplings.

In Table I three examples of models are shown. In the W-S-GIM model, all charge quarks (the d and s) are in the bottom of left-handed doublets and are in right-handed singlets; therefore the GW condition is met, and GIM is "natural". In the vector model, all $-\frac{1}{3}$ charge quarks (d, s, and b) are in the bottom
TABLE I

A Small Sampling of SU(2) \times U(1) Models

(Many others are discussed in Refs. 4 and 5.)

<table>
<thead>
<tr>
<th>W-S-GIM Model$^2$</th>
<th>Vector Model$^6-9$</th>
<th>CHYM Model$^{10-13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u)(\begin{pmatrix} c \ s \end{pmatrix} ), (d)(\begin{pmatrix} c \ s \end{pmatrix} ), u_R c_R d_R s_R</td>
<td>(u)(\begin{pmatrix} c \ s \end{pmatrix} ), (d)(\begin{pmatrix} c \ s \end{pmatrix} ), (t)(\begin{pmatrix} u \ b \end{pmatrix} ), (c)(\begin{pmatrix} u \ b \end{pmatrix} ), (d)(\begin{pmatrix} t \ d \end{pmatrix} ), (t)(\begin{pmatrix} t \end{pmatrix} )</td>
<td>(u)(\begin{pmatrix} c \ s \end{pmatrix} ), (d)(\begin{pmatrix} c \ s \end{pmatrix} ), b_L g_L (u)(\begin{pmatrix} c \ s \end{pmatrix} ), (d)(\begin{pmatrix} c \ s \end{pmatrix} ), d_R s_R</td>
</tr>
<tr>
<td>(\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} N_e \ N_\mu \end{pmatrix} ), (\begin{pmatrix} N_E \end{pmatrix} ), (\begin{pmatrix} N_\mu \end{pmatrix} ), (\begin{pmatrix} N_E \end{pmatrix} )</td>
<td>(\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} N_e \end{pmatrix} ), (\begin{pmatrix} N_\mu \end{pmatrix} ), (\begin{pmatrix} N_E \end{pmatrix} )</td>
<td>(\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), (\begin{pmatrix} \nu_e \ \mu \end{pmatrix} ), singlets, etc., (\begin{pmatrix} N_E \end{pmatrix} ), (\begin{pmatrix} N_\mu \end{pmatrix} ), (\begin{pmatrix} N_E \end{pmatrix} ) + singlets, etc.</td>
</tr>
</tbody>
</table>

*possible in E_7
of doublets left-handed and right-handed, thereby obtaining GIM naturally.

However, the CHYM model does not meet the GW condition. In this model for d and s
\[ \tau_{3L} = -\frac{1}{2}, \quad \tau_{3R} = 0, \quad (2.1) \]
whereas for b and g
\[ \tau_{3L} = 0, \quad \tau_{3R} = -\frac{1}{2}. \quad (2.2) \]

As written, with absolutely no mixing between the light quarks (d and s) and the heavy quarks (b and g), the CHYM model does not have SCNC. The question is how can one assure that the mixing is zero, that there are no weak radiative effects leading to mixing. While there is no strong argument now, Gürsen, Ramond, Sikivie, and I believe that, in some future theory, the lack of mixing may be connected with the great splitting of quark masses (between d and b).

While one can imagine a rationale for imposing zero mixing, it is difficult to defend those models which require a particular, nonzero mixing between \(-\frac{1}{3}\) charge quarks in different representations of SU(2)\(_{\text{weak}}\).

Whatever one's viewpoint, the experimental fact that SCNC are extremely small is a vital and relevant observation for those considering theories of the weak interactions.

Of course, one need not stop with \(-\frac{1}{3}\) charge quarks. When \(\frac{2}{3}\) charge quarks in a given model meet the GW condition, there will be no charm-changing neutral-currents, CCNC (or other off-diagonal currents among 2/3 charge quarks). If no CCNC are observed, then Glashow and Weinberg\(^3\) argue that their condition must apply here also. If the GW condition is not met then CCNC should not be particularly small and in any case \(D^0 - \bar{D}^0\) mixing\(^3,14\) would be large. Then at SPEAR in addition to
\[ e^+e^- \rightarrow D^0\bar{D}^0 \quad (2.3) \]
one would see

\[ e^+ e^- \rightarrow D^0 D^0 \text{ or } \bar{D}^0 \bar{D}^0. \]  \hspace{1cm} (2.4)

There is certainly no difficulty in finding models which have no SCNC but do have CCNC.\(^3\),\(^4\) The absence of CCNC (if observed) will be an important constraint on model building.

III. WHY IS CP VIOLATION SO SMALL?

For several years, people have wondered why and how CP violation occurs, and various models incorporating it have been proposed.\(^15\) A recent paper by Weinberg\(^16\) maintains that the real mystery is not why CP is violated but why the violation is so small. The observed violation of CP is not weak but "milli-weak".

Models with right-handed currents\(^15\) have been proposed to obtain CP violation. It has also been shown that some models with more than four quarks\(^15\) inherently have CP violation even if there are only left-handed currents. However, there are parameters in the Lagrangian which must take particular values in order to insure that CP is approximately conserved, and this, Weinberg feels, is "unnatural".

The alternative\(^16\) is to insist that there are only four quarks and only left-handed currents; then CP is automatically and completely conserved in the quark sector. The violation is proposed to occur only in Higgs exchange where CP conservation is to be violated strongly. Particular models have been proposed to accomplish this.\(^16\)

While this proposal is reasonable, the discovery of more quarks might suggest the need to find another explanation of CP violation (such as in Ref. 15).
IV. ASYMPTOTIC FREEDOM CORRECTIONS TO NEUTRINO SCATTERING

Several months ago Altarelli, Parisi, and Petronzio\textsuperscript{17} (APP) showed that asymptotic freedom leads to large corrections to parton model calculations of deep-inelastic, charged-current neutrino scattering. The corrections they found gave an energy dependence of the kind seen in the observed antineutrino anomalies,\textsuperscript{18-21} i.e., an increase with energy of \(\langle y\rangle\) where

\[ y \equiv \frac{(E^{-}_\nu - E^{+}_\mu)}{E^{-}_\nu} \quad (4.1) \]

and of

\[ R_0 \equiv \frac{\sigma(\bar{\nu}N \to \mu^+X)}{\sigma(\nu N \to \mu^-X)}. \quad (4.2) \]

Georgi, Politzer, and I\textsuperscript{22} agreed with the qualitative conclusions of APP, but we were skeptical about the asymptotic freedom corrections being of sufficient magnitude to explain the reported anomalies (assuming the W S GIM model). As a result we decided to do a detailed but conservative calculation. By "conservative" we meant that whenever approximations needed to be made, they would be chosen to oppose our bias, i.e., to increase the size of the corrections with energy. Our eventual conclusions\textsuperscript{22} (which are shared by others\textsuperscript{23}) were that asymptotic freedom corrections are probably not of sufficient magnitude to provide a solution to the problem of antineutrino anomalies (see also Sec. V).

When the effects of asymptotic freedom are included in parton model calculations of deep-inelastic scattering, the relative amounts of u, d, s, \(\bar{u}\), \(\bar{d}\), \(\bar{s}\), c, \(\bar{c}\), and gluons change as a function of \(Q^2\), i.e.,

\[ u(x) \rightarrow u(x, Q^2) \quad (4.3) \]

where \(u\) is the u quark distribution function. As \(Q^2 \rightarrow \infty\), the valence quark functions decrease while sea quarks increase, all approaching the same value at infinity.
In addition, asymptotic freedom results in the $x$-dependence of quark distribution functions changing with $Q^2$. The mean $x$ of the structure functions decreases with increasing $x$.

The first of our "conservative" approximations is to ignore the changing $x$-dependence. It is conservative because we then overestimate the increase in average $Q^2$ since $Q^2 = 2mExy$. Furthermore, keeping $<x>$ larger diminishes the impact of $\xi$ scaling in delaying the contributions from charm production. This approximation corresponds to the factorization

$$u(x,Q^2) = u(x)U(Q^2) \quad (4.4)$$

Other approximations (which can be shown to be conservative) are: (2) Ignoring $m^2_p/Q^2$ terms ($m_p =$ proton mass) in $\xi$, the scaling variable; (3) Ignoring $m^2_q/Q^2$ terms ($m_q =$ quark masses) when appropriate; (4) Choosing $\Lambda = 500$ MeV (twice the value used by APP) in

$$\alpha_s(Q^2) = \frac{g^2}{4\pi} - \frac{12\pi}{25} \left( \log \frac{Q^2}{\Lambda^2} \right)^{-1} \quad (4.5)$$

which corresponds to choosing $\alpha_s(Q^2=1) = 1.1$ or $\alpha_s(Q^2=9) = 0.42$; (5) Assuming $s = \bar{u} = \bar{d}$.

Given the above assumptions one can calculate valence ($u$ and $d$) and sea ($s$, $\bar{u}$, and $\bar{d}$) functions (see Eq. (4.4)) as follows:

$$U(Q^2) = \frac{1}{4} \left[ \frac{3}{14} + \left( U_0 + 2S_0 + C_0 - \frac{3}{14} \right) \right] L^{-56/75} + \left( 3U_0 - 2S_0 - C_0 \right) L^{-32/75} \quad (4.6)$$

$$S(Q^2) = \frac{1}{4} \left[ \frac{3}{14} + \left( U_0 + 2S_0 + C_0 - \frac{3}{14} \right) \right] L^{-56/75} + \left( 2S_0 - U_0 - C_0 \right) L^{-32/75} \quad (4.7)$$

where $U_0$ is $U(Q^2=Q^2_0)$ (etc.), $Q^2_0 = 4$ GeV$^2$, and

$$L = \frac{\log Q^2/\Lambda^2}{\log Q^2_0/\Lambda^2} \quad (4.8)$$
The powers of $L$ change very slightly if there are additional flavors of quarks.

While we found that asymptotic freedom corrections were not the full answer to the problems of antineutrino anomalies, these corrections are large enough that they should not be ignored in studying neutrino scattering.

V. DEEP-INELASTIC, CHARGED-CURRENT $\nu$ SCATTERING

In examining the published results from charged-current neutrino scattering

$$\nu + N \rightarrow \mu + X$$

it is crucial to understand that all of these data have experimental cuts that significantly affect the nature of the results. Those doing theoretical calculations can include these cuts if they are given clearly and explicitly by experimentalists (the figures shown below do include the cuts).

As an example, when HPWF showed their results for $R_C$ (Ref. 18) and $\langle y \rangle_{\bar{\nu}}$ (Ref. 19), their data contained the following cuts (see Eqs. (4.1) and (4.2)):

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$\langle y \rangle$ & $R_C$ \\
\hline
$E_{\mu} > 4$ GeV & $E_{\mu} > 4$ GeV \\
$\theta_{\mu} < 0.225$ radians & $\theta_{\mu} < 0.225$ radians \\
x < 0.6 & but see below \\
$Q^2 > 1.0$ or $W^2 > 2.56$ GeV$^2$ & \\
\hline
\end{tabular}
\caption{Table II}
\end{table}

The reported results of HPWF for $R_C$ include a "correction" found by assuming there is a 6% sea at all energies (using only u, d, $\bar{u}$, and $\bar{d}$ quarks). One can then calculate what fraction of the cross sections the two above cuts would cause to be missed, and "correct" the observed $\nu$ and $\bar{\nu}$ cross sections. While the assumption of this procedure is somewhat inconsistent with the observed data, the resulting correction is not that far from an ideal correction. In any case, the
theoretical calculations can easily follow the same procedure.

The first two cuts for \(<y>\) eliminate very high \(y\) and thereby lower \(<y>\); the fourth cut eliminates very low \(y\) and thereby raises \(<y>\). This set of cuts (which are energy-dependent) tends to cancel in \(<y>\).

But the point I wish to emphasize is that these systematic "errors" (cuts) are not relevant to the problem of analyzing these experiments in the sense that they can be totally compensated for.

Bubble chamber experiments are not free of cuts and biases either. Not all energy can be detected. Some neutral particles will always be missed. Present experiments vary in their ability to detect converting gammas. In some cases a given fraction of gammas are counted. This of course affects calculations of \(y\), and theorists should be certain about what is meant by "\(E\)". It should also be understood that the spectra of incoming neutrino energies are often extremely different in different experiments. Other cuts which are used in some bubble chamber experiments include cuts on the energy transfer and cuts such as

\[ E_\mu > 4 \text{ GeV}. \]

The present status for experiment and theory\(^4,24\) of \(R_c\) and \(<y>\) is shown in Fig. 1 (round points from Ref. 18 and square points, Ref. 20) and in Fig. 2 (data from Ref. 19), where experimental cuts are always included. Bubble chamber data\(^21\) for \(<y>\) are not shown here since the cuts are different; as their statistics become comparable to those of counter experiments, they too should be considered. The dotted curves show calculations in the W-S-GIM model.

Fig. 1
without asymptotic freedom corrections. The solid curves show the corrected W-S-GIM model. For comparison, the asymptotic freedom corrected predictions of models (see Table I) with a \((u,b)_R\) coupling are shown with a dashed curve (mass \((b) = 5\) GeV).

While it appears that the solid curves do not miss most error bars by much, those curves do not really show the general trend of the data; furthermore, it will be recalled from Sec. IV that the rise of these curves has been exaggerated by the choice of "conservative" approximations. Given the statistical errors in present data, there is need for more data before strong conclusions can be drawn concerning the viability of the W-S-GIM model or about the need for right-handed currents.

It can also be argued from data not shown here that there is no possibility of a coupling \((c,d)_R\) of strength comparable to \((u,d)_L\).

VI. DILEPTONS IN NEUTRINO SCATTERING

If one wishes to pursue the possibility of the existence of a right-handed coupling \((u,b)_R\), then there are vital tests to be passed in the area of dilepton production in charged-current neutrino scattering. Here the b quark is assumed to have charge \(-\frac{1}{3}\) and mass 5 GeV.

The expected decays for particles with a b quark are:

\[
\begin{align*}
\bar{b} \to u &+ \\
&\begin{cases}
\mu \bar{\nu} \\
e \bar{\nu} \\
(\bar{u}d \text{ (in 3 colors)}) \\
(\bar{c}s \text{ (in 3 colors)})
\end{cases}
\end{align*}
\]
The simplest assumption is that each decay mode has equal probability so that
the $\mu \bar{\nu}$ (or $e \bar{\nu}$) mode occurs about 12% (compared to charm to $\mu \bar{\nu}$ of 20%). However, Cahn and Ellis\textsuperscript{25} emphasize that the fourth decay mode ($\bar{c}s$) also leads to
$\mu \bar{\nu}$ (20% naively) so that the total b branching ratio to $\mu$ is about the same as
that for charm. Of course, the $\bar{c}s$ mode may be suppressed by phase space or
dynamical factors, but the b branching ratio to $\mu$ is clearly not sensitive to that.
In fact there is no obvious reason why $b \rightarrow \mu$ should be significantly less than
$c \rightarrow \mu$.

Since $b \rightarrow \mu$ is presumably not suppressed and since the rise of $R_c$ is easily
attributed to b production from u valence quarks, Cahn and Ellis\textsuperscript{25} conclude
that the rate of dilepton production in antineutrino scattering (see Fig. 3) should
rise sharply with energy and easily surpass the rate for neutrinos. The
average $x$ is not expected to be par-
ticularly small except at lower ener-
gies where sea production dominates,
and, at these low energies, the rate
should be less than that for neutrinos. Very few K's are expected in $\bar{\nu}$ dilepton
events unless $\bar{c}s$ ($\rightarrow \bar{s}s\mu\bar{\nu}$) is a significant decay mode of b or unless E is quite
low.

If the results for neutrinos reported at this meeting by C. Baltay indicating
only one K per $\nu \rightarrow \mu^- e^+$ event are correct, it is quite suggestive that Cabibbo-
suppressed production of charm dominates sea production; and one might con-
clude that the strange sea is not large.

While $D^0 - \bar{D}^0$ mixing may or may not be possible, the discussion of Sec.
II indicates that it is highly unlikely that there is ($3b$) - ($3\bar{b}$) mixing.
The study of $p_L$ distributions in antineutrino dilepton events, in which an indication of a high mass is sought, would be confused if $b \rightarrow \bar{c} \rightarrow \mu$ is a significant fraction of $b \rightarrow \mu$. However, the study of $W$ distributions should not be affected.

It is important to realize that $(u, b)_R$ is not the only possibility if right-handed currents are needed. The coupling $(d, a)_R$ is also possible although the "a" quark must have a charge of $-4/3$ (and a mass of about 5 GeV). Since most experiments so far have been done on isoscalar targets, it would not be easy to distinguish these two couplings. However, bubble chamber experiments with high statistics might see evidence of a difference by comparing results with hydrogen targets to those with isoscalar targets. Certainly PETRA and PEP will have no problem noticing the presence of a $-4/3$ charge quark: the relative area under an $a \bar{a}$ resonance would be four times as much as that under the $\psi$, and $R(e^+e^-)$ would increase by 5.3.

With reports in neutrino scattering experiments of same-sign dilepton events and of trimuon events likely to increase greatly in the near future, it is necessary to consider their possible sources. The most conservative guess for both is associated production, which is to say charm pairs are produced incidentally to the weak interaction. This hypothesis is best tested by comparison with dilepton and trilepton production in $\mu p$ scattering. Associated production requires fairly large $\gamma$ since two charmed particles are produced. A more interesting source could be charm-changing neutral-currents which would also lead to large $D^0 - \bar{D}^0$ mixing. This possibility could soon be eliminated by looking for $D^0\bar{D}^0$ pairs at SPEAR. If, however, both above hypotheses hold, then one might see $\mu^-\mu^-\mu^-$ events for which one expects little background.
VII. NEUTRAL CURRENTS IN NEUTRINO SCATTERING

The most recent results (Cal Tech - Fermilab, HPWF, Gargamelle) and the predictions of the models of Table I for neutral-current scattering are shown in Fig. 4, where

\[ R_\nu = \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu X)} \]  

that is, the ratio of neutral to charged current cross sections. \( R_\nu \) is similarly defined. The theoretical values depend on \( \sin^2 \theta_W \) so that each model has a line with tenth values of \( \sin^2 \theta_W \) marked. These theoretical calculations are somewhat naive in that they neglect sea contributions and asymptotic freedom corrections in the neutral currents, but these should cause only small changes.

There are two important differences between the experimental and theoretical points. First, not all regions of \( \gamma \) are seen in present experiments. However, when comparing data with particular models, one can make model-dependent extrapolations. This has been done only with the HPWF data shown; the two points represent the extremal results of extrapolation for all "reasonable" models with V and A. Second, since the data points shown are from different energy regions where the charged currents (the denominator of Eq. (7.1)) are different, I took only the naive charged-current cross sections off valence quarks. The simplest
correction is to multiply $R_p$ for each data point by $(R_c/0.33)$ (see Eq. (4.2)).

For low (high) energies this factor is about 1.1 (1.3-1.4).

In Fig. 4 I have assumed $m(Z^0)/m(W^\pm)$ to be the same as required in the W-S-GIM model. However, in other models that ratio could vary; then theoretical points in Fig. 4 would move toward or away from the origin.

While the W-S-GIM and CHYM models are consistent with the data, this is the one (and only) really convincing experimental result which sheds doubt upon the Vector model.

However, with better statistics the $\nu p$ elastic scattering experiments reported by L. Sulak and collaborators and by W. Lee and collaborators may soon be able to give important evidence on the validity of the Vector model, of other $V,A$ models and of $P,T,S$ models. Fig. 5 (compare with Fig. 4) shows the expectations of the models of Table I for elastic $\nu p$ scattering, where $R_p$ and $R_\nu$ are defined equivalently to Eq. (7.1). The data point is that of Ref. 33. W. Lee et al. report a value for $R_p = 0.23 \pm 0.08$. These experiments have comparable systematic errors also.

With the recent observation by Reines et al. of $\bar{\nu}_e e$ elastic scattering, one can really narrow down the allowed values of $g_A$ and $g_V$ (found from the neutral-current couplings of models). At this conference, we have heard that corrections to Aachen-Padua data have brought it into reasonable consistency with
Gargamelle data\textsuperscript{38} for $\nu_\mu e$ and $\bar{\nu}_\mu e$ elastic scattering. In Fig. 6 I have shown, in the shaded area, the overlap of the allowed regions of $g_A$ and $g_V$ from the three types of experiments. A given cross section measured can only determine a set of possible values for $g_A$ and $g_V$, and I have shown the 90\% confidence upper and lower (if any) limits which are determined. The data for $\nu_\mu e$ and $\bar{\nu}_\mu e$ are from Ref. 38 and for $\bar{\nu}_e e$ from Ref. 36. The lines with dots represent the model predictions,\textsuperscript{4} with dots indicating tenth values of $\sin^2 \theta_W$. The lowest line applies to the W-S-GIM model and to the CHYM model with $(N_E, E)_R$ coupling; the middle line to the Vector model and to the CHYM model with $(N_e, e)_R$ coupling; the upper line to the CHYM model with $(E^+, N_E, e^-)_R$ coupling.

From Fig. 6 one concludes that the W-S-GIM and Vector models contain allowed regions, and the CHYM model has allowed regions with coupling $(N_E, E)_R$ and $(N_e, e)_R$ but not with coupling $(E^+, N_E, e^-)_R$.

VIII. WEAK PARITY-VIOLATION IN ATOMIC PHYSICS

Several years ago the Bouchiats\textsuperscript{39} and Khriplovich\textsuperscript{40} suggested that a unique test of weak parity-violation might be found by searching for forbidden atomic transitions in heavy atoms. Sandars,\textsuperscript{41} Fortson,\textsuperscript{42} Henley and Wilets,\textsuperscript{43} and they have pursued such experiments and calculations extensively during the last year or so. If there is a parity-violating neutral-current in the weak interactions, then one could excite, with a tunable laser, states of the wrong parity because of mixing with right parity states. This would result in a rotation of the
polarization of the laser light being detected.

The theoretical calculation of this rotation is the product of an atomic (and nuclear) physics term $A$ and a model-dependent gauge theory term $Q_w$. Unfortunately the calculation of $A$ is complicated and somewhat controversial. The value of $A$ calculated by Henley and Wilets for the transition studied by Fortson (in Bismuth) is

$$A_{HW} \approx 2.3 \times 10^{-9}$$

(8.1)

and apparently Sandars calculates a similar number for his own experiment (in Bismuth but for a different transition). Khriplovich finds that similar calculations in another element give answers (which can be tested) that are too high by a factor of about two, and he suggests dividing $A_{HW}$ by two, so

$$A^K \approx 1.1 \times 10^{-9}$$

(8.2)

The calculation of $Q_w$ is straightforward. The dominant term (note that only cross terms can give parity violation) is

$$J^A_{\text{leptonic}} J^V_{\text{hadronic}}$$

(8.3)

In effect this is like electron-nucleus scattering. For the models of Table I, one finds (for appropriate values of $\sin^2 \theta_W$):

<table>
<thead>
<tr>
<th>Model</th>
<th>$Q_w$</th>
<th>$\sin^2 \theta_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-S-GIM</td>
<td>-140</td>
<td>0.3</td>
</tr>
<tr>
<td>Vector</td>
<td>0</td>
<td>all $\theta_W$</td>
</tr>
<tr>
<td>CHYM</td>
<td>+120 for $(N^+_e, E^-)_R$, 0 for $(N^-_e, e^+_R$, -120 for $(E^+, N^-_e, e^-)_R$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

TABLE III
I want to emphasize that if no parity violation is found, it does not mean that the Vector model is indicated. It would probably mean that the electron is vector so that $J_{\text{leptonic}}^A = 0$. However, note that the CHYM model and other models also can have $Q_w = 0$.

Fortson and Sandars have reported a result (an upper limit of $10^{-7}$) which appears to be inconsistent with (smaller than) the W-S-GIM model if one accepts either Eq. (8.1) or (8.2). While experiments may find a small nonzero effect, it might be difficult to rule out systematic errors. Certainly there are few if any "reasonable" or "conservative" models which give a result much smaller than that of W-S-GIM but still nonzero.

Given the difficulty and newness of these experiments and the uncertainty in the calculations of $A$, it is probably wise to wait several months before drawing any conclusions about models from these data.

IX. SUMMARY

The concepts of the naturalness of the GIM mechanism and of CP violation in a model may be philosophical, but they raise interesting questions concerning the origins of these effects. On the basis of present experiments, it is difficult to eliminate the models discussed here at this moment. However, within a year or so, increased statistics in charged-current neutrino scattering, in elastic $\nu p$ scattering, and in elastic $\nu e$ scattering will provide strong restrictions on models. And if, in addition, the problems in theory and experiment for atomic parity-violation are resolved, then our knowledge of weak interactions could in a year's span be greatly enhanced.

X. INTRODUCTION TO PATI-SALAM THEORY

I would like to turn now to an alternative, theoretical point of view. The gauge theory, which was suggested by J. Pati and A. Salam, shares a number of
features in common with the conventional gauge theories of the weak and electromagnetic interactions. However, it departs from fashion in suggesting that quarks, leptons, and gluons have integer charges and are not confined (can exist as free particles).

In the "basic model" there are only the conventional 16 fermions with only left-handed couplings. Their notation and charges are:

\[
\begin{bmatrix}
u_R & u_Y & u_D & \nu_e \\
d_R & d_Y & d_D & e^- \\
s_R & s_Y & s_D & \mu^- \\
c_R & c_Y & c_D & \nu_\mu
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 1 & 0 \\
-1 & 0 & 0 & -1 \\
-1 & 0 & 0 & -1 \\
0 & 1 & 1 & 0
\end{bmatrix}
\]

where R, Y, B are the colors red, yellow, blue, and leptons are the fourth color.

In Ref. 44, Pati and Salam discuss only the "basic model" in summarizing the attributes of their approach (ignoring in that paper the possibility of expanding to include another 16-fold multiplet with right-handed couplings). Since the attractive features of their theory have been discussed widely in the literature, I will concentrate on the phenomenological problems which arise, although limiting myself to the "basic model" (as they did). Some of these problems clearly can be solved in the expanded version.

Pati and Salam consider the color gauge group, SU(4). There are then fifteen color gauge fields as follows:

1. Six "exotics" which couple quarks to leptons (since leptons are just another color). In this gauge theory, the charged exotics mix with \( W^\pm \) (the weak intermediate boson) resulting in the decay of quarks into leptons. From the observed rate of \( K^0 \rightarrow e\mu \), Pati and Salam argue that the mass of exotics \( \geq 10^5 \text{ GeV} \).
2. Eight gluons which change quark colors (therefore some are neutral and some charged). An important point is that, unlike the conventional theory, these unconfined gluons are massive. In this gauge theory the neutral gluon $U^0$ (charged gluons $V^\pm$) mixes with the photon (with $W^\pm$) resulting in phenomena discussed below.\textsuperscript{46}

3. One singlet $S^0$ with mass $> 10^3$ GeV. I have found little discussion of $S^0$ and will ignore it here. It is a neutral current of weak strength.

XI. MIXING $U^0$ AND $\gamma$

It has been observed by a number of authors (Refs. 47-50) that this mixing had significant consequences for some electromagnetic processes. A heuristic discussion of their conclusions follows. For theories with fractional charge quarks, the electromagnetic current can be written as (where I consider only the u, d, and s quarks):

$$J_{em}^F = J_{8,1} = \sum_{c=R,Y,B} \left( \frac{2}{3} \bar{u}_c u_c - \frac{1}{3} \bar{d}_c d_c - \frac{1}{3} \bar{s}_c s_c \right)$$  \hspace{1cm} (11.1)

where 8 refers to flavor octet and 1 to color singlet. However, for the theory with integer-charge quarks, there are both $8,1$ and $1,8$ contributions so that the current can be written as:

$$J_{em}^I = J_{8,1} + J_{1,8} = \bar{u}_R u_R + \bar{u}_B u_B - \bar{d}_Y d_Y - \bar{s}_Y s_Y$$  \hspace{1cm} (11.2)

which then gives

$$J_{1,8} = \sum_{q=u,d,s} - \frac{2}{3} \bar{q} q \frac{2}{3} R q q + \frac{1}{3} \bar{R} q q + \frac{1}{3} B q q$$  \hspace{1cm} (11.3)

While $J_{8,1}$ goes by a photon only (with propagator $(1/q^2)$), $J_{1,8}$ is (in this gauge theory) a mixture of photon and gluon with propagator

$$\frac{1}{q^2} - \frac{1}{q^2 - m_U^2} = \frac{1}{q^2} \left( \frac{m_U^2}{m_U^2 - q^2} \right)$$  \hspace{1cm} (11.4)
As a result \( J_{\text{em}}^{I} \) can be written as

\[
J_{\text{em}}^{I} = J_{8,1}^{I} + \left( \frac{m_{U}^{2}}{m_{U}^{2} - q^{2}} \right) J_{1,8}^{I}
\]  

(11.5)

This result is important when \(|q^{2}| \gg m_{U}^{2}\); then the electromagnetic current for both the integer- and fractional-charge quarks is \( J_{8,1}^{I} \) (Eq. (11.2) does not hold). Therefore, in \( e^{+}e^{-} \) scattering the ratio

\[
R(e^{+}e^{-}) \equiv \frac{\sigma(e^{+}e^{-} \rightarrow \text{hadrons})}{\sigma(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-})}
\]

(11.6)

is the same in the two theories for \( q^{2} \gg m_{U}^{2} \). And in deep-inelastic electroproduction, no differences are expected when color-singlet states are produced at \(|q^{2}| \gg m_{U}^{2} \).

There is another argument (due to H. Lipkin and others) which affects the results naively expected for integer-charge theories. Consider the matrix element

\[
\langle f | J_{8,1}^{I} + J_{1,8}^{I} | i \rangle = \langle f | J_{8,1}^{I} | i \rangle + \langle f | J_{1,8}^{I} | i \rangle
\]

(11.7)

The term \( \langle f | J_{1,8}^{I} | i \rangle \) is zero if \( |f\rangle \) and \( |i\rangle \) contain no color octet terms at the \( q^{2} \) considered (i.e., if no color states are above threshold). In \( e^{+}e^{-} \) annihilation, \( J_{1,8}^{I} \) cannot contribute until \( q^{2} \geq m_{c}^{2} \) (the color continuum threshold). Since quarks also have color, then \( 2m_{q} \geq m_{c} \) where \( m_{q} \) is the mass of free quarks (which are presumed heavier than bound quarks).

Assuming one is above color threshold \( R(e^{+}e^{-}) \) is given by

\[
R(e^{+}e^{-}) = \sum_{\text{color, flavor}} \left| Q_{i}^{f} + \left( \frac{m_{U}^{2}}{m_{U}^{2} - q^{2} + \text{im}_{U}^{I}} \right) Q_{i}^{C} \right|^{2}
\]

(11.8)

where \( Q_{i}^{f} \) and \( Q_{i}^{C} \) are the charges in Eqs. (11.1) and (11.3). Pati and Salam consider in Ref. 44 two possible values of \( m_{U}^{I} \); in regions around 1.5 GeV and
around 4 GeV. Let me show now what I presume is at least one reason why Pati and Salam argue that the color-continuum threshold is above the mass of $U^0$. Consider for now only $u$, $d$, and $s$ quarks; let $\Gamma_U = 1 - 10$ MeV, then for $m_U = 1.5$ GeV and 4 GeV, we find for $R'(e^+e^-)$ (the naive value of $R(e^+e^-)$ if color-continuum threshold were quite low):

TABLE IV

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.1</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'(m_U = 1.5$ GeV)</td>
<td>2.6</td>
<td>2.2</td>
<td>2.1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$R'(m_U = 4$ GeV)</td>
<td>7.4</td>
<td>12</td>
<td>38</td>
<td>735</td>
<td>8.3</td>
<td>3.3</td>
</tr>
</tbody>
</table>

In the fractional-charge quark theory, one expects $R(e^+e^-)$ to be 2 below and 3.3 above charm threshold (or slightly higher due to asymptotic freedom corrections). Given Table IV and the observed low-energy value of $R(e^+e^-)$ which is about 2.3 (and 5.3 above $s = 5$ GeV), one can determine the color-continuum threshold $m_c$ (and set a lower limit on light (free) quark masses, $m_q$). It is clear then that

$$2m_q > m_c > 2.8 \text{ GeV for } m_U = 1.5 \text{ GeV}$$

$$2m_q > m_c > 5.5 \text{ GeV for } m_U = 4 \text{ GeV}$$

(11.9)

The inclusion of the $c$ quark multiplies the values of $R'$ in Table IV by $(3.3/2)$ above charm threshold, but does not affect the conclusions very much.

XII. PHENOMENOLOGY OF A 1.5 GeV GLUON

I will discuss the possibility that $m_U \approx 1.5$ GeV in some detail, and will later discuss other possibilities more briefly. Much of the discussion here is applicable to other masses.
One of the serious problems for the theory without confinement is the prediction of a neutral gluon $U^0$ which is a physical particle. Unfortunately, no $e^+e^-$ annihilation experiment has yet scanned the region around $\sqrt{s} = 1.5$ GeV. But when Frascati, Novosibirsk, or Orsay does scan this region (which may be done in the near future), the $U^0$ should be quite obvious in $e^+e^- \rightarrow$ hadrons. It has a decay width to $e^+e^-$ (or $\mu^+\mu^-$) predicted to be approximately 5 keV and a total width, to be approximately 5 MeV. With these, one can calculate that the area under this narrow resonance should be approximately 50 nb-GeV (the $\psi$, at a higher energy, has an area of 10 ub-GeV).

One can also (see Ref. 52) look for $U^0$ in

$$pp \rightarrow U^0 + X \quad \mu^+\mu^- \quad (or \quad e^+e^-) \quad (12.1)$$

(or in photoproduction). There is an argument due to Bjorken\textsuperscript{53} which indicates that process (12.1) should have been observed; he argues that the relative production of $\mu^+\mu^-$ via $U^0$ and $\gamma$ can be estimated as follows.

Let $\sigma_{U,8}^{U,8} (\gamma,1)$ be the cross section for producing $\mu^+\mu^-$ through the $U^0 (\gamma)$ with the remaining particles being color-octet (color-singlet). Then the cross sections are:

$$\frac{d\sigma_{U,8}}{dm^2} = \sum_{X_8} |X_8 J_8 |^2 \frac{m^4_U}{m^4 ((m^2 - m_U^2)^2 + m^2_{U,8})} A_1$$

$$\frac{d\sigma_{\gamma,1}}{dm^2} = \sum_{X_1} |X_1 J_1 |^2 \frac{1}{m^4} A_2$$

where $A_1 \approx A_2$ = coupling and phase space. Then integrating in $m^2$ over the experimental resolution, $\Delta m$, one obtains
\[ \int \frac{d\sigma_{\gamma \gamma}}{d m^2} \Delta m \approx \sum_{X_8} \frac{|<X_8|J_8|i>|^2}{\sum_{X_1} |<X_1|J_1|i>|^2} \left( \frac{\pi m_U^2}{2 \Gamma_{U} \Delta m} \right) \]  

where \( \left( \frac{\pi m_U^2}{2 \Gamma_{U} \Delta m} \right) = 10^3 - 10^4 \) for \( \Gamma_{U} = 1-10 \text{ MeV} \) and for 20% resolution.

While some dynamical suppression is to be expected in \( |<X_8|J_8|i>|^2 \) relative to \( |<X_1|J_1|i>|^2 \), it is hard for me to imagine a suppression of 3 or 4 orders of magnitude. Pati\(^{54}\) does feel a large suppression is possible; however factors such as \( Q^2/2m_c^2 \), discussed in their papers, can give only a factor of 10 (I believe an upper limit on \( m_c \) is \( 2m_U \)). Since the above calculation shows whether the \( U^0 \) should stand above the background in pp → µ⁺µ⁻X, it would appear that with reasonable assumptions it is difficult to understand how it could have been missed.

Since in this "basic model" there is no heavy lepton, one needs another explanation of the \( e^+e^- \rightarrow \mu^+\mu^-X \) (neutral energy) events seen at SPEAR. Pati and Salam argue that the threshold for producing free quarks (which are heavier than bound quarks) might coincide with the threshold for \( \mu e \) events (i.e., \( m_q \approx 1.8 \text{ GeV} \)); the process is

\[ e^+e^- \rightarrow d_R^- + \bar{d}_R^+ \]

\[ \nu_e \bar{\nu}_e \quad \nu_\mu \bar{\nu}_\mu \]

The only quarks which can be expected to decay to \( \mu \) are \( d_R \) and \( s_R \). However, this decay is dominant only if red quarks are less than a pion mass heavier than yellow and blue quarks (which is plausible) and if \( d_R \) and \( s_R \) are heavier than charged gluons (which was found for neutral gluons, Eq. (11.9)). Following Eq. (11.8), \( d_R \) and \( s_R \) contribute about 1/9 each to \( R(e^+e^-) \); in addition, two charged
gluons contribute 1/16 each. From this I calculate the branching ratio of $d_R$, $s_R$, or the gluons, which would explain the observed $\mu e$ rate, to be about 30% (which is consistent with the expected branching ratios of these particles).

An important question is whether this theory with $m_U \approx 1.5$ GeV can provide an understanding of $R(e^+ e^-)$. It is assumed that $\psi, \psi'$, etc., are charmonium states, and the rise of $R(e^+ e^-)$ at $\sqrt{s} \approx 4$ GeV is due to reaching the threshold for production of charmed mesons. In the region above $\sqrt{s} = 3$ GeV, the cross section calculated (see Table IV) is essentially identical to that of the fraction-charge quark theory (though gluon production adds 1/8 to $R$). Therefore above, say, $\sqrt{s} = 5$ GeV, $R(e^+ e^-)$ should be equal to 3.5 plus asymptotic freedom corrections. However, it has been argued in Refs. 55 and 56 that asymptotic freedom corrections cannot account for the discrepancy between 3.5 and the experimental $R(e^+ e^-) \approx 5.3$. It would seem necessary to have a heavy lepton and thereby abandon the "basic model" (as Pati and Salam have considered long ago). Pati informs me that Elias, Salam, Strathdee, and he are considering presently an alternative explanation of $R(e^+ e^-)$ within the "basic model".

Turning to neutrino scattering, we consider the effects of the $V^\pm$ gluons mixing with $W^\pm$. It will be assumed that the $V^\pm$ (like the $U^0$) have a mass of approximately 1.5 GeV. As for the electromagnetic case, there is a damping factor associated with the color-octet term, so that asymptotically there are no new quark contributions over that expected in the theory with confined quarks (although there are asymptotic gluon contributions). However, for appropriate $m_U$ and $m_q$, one expects temporary effects. One must reach color threshold before new contributions arise, but the damping factor soon overcomes those quark contributions.
Sidhu, Mohapatra, and Pati (SMP) have studied\textsuperscript{57} the consequences of color production on charged-current neutrino scattering. There are several problems to be faced in calculating $R_c$ and $\langle y \rangle_{\nu}$ (defined earlier). The bound quarks (unlike free quarks) are quite light, so although the threshold for producing color is high, $\xi$ scaling\textsuperscript{24} is considered inappropriate. Lacking $\xi$ scaling as a suppression mechanism, SMP use a threshold factor which is chosen to show the expected approach to scaling. Their fits to $R_c$ and $\langle y \rangle_{\nu}$ are shown in Figs. 7 and 8 (see Fig. 1 for references). The dotted (dash-dotted) curves are for the Pati-Salam model with $m_U = 2$ GeV ($m_U = 4$ GeV), but without asymptotic freedom corrections. For comparison, the solid (dashed) curves are for the W-S-GIM (CHYM) model with the corrections. For light gluons, $m_U \approx 2$ GeV, the results of SMP show that $R_c$ has little rise with energy, and $\langle y \rangle_{\nu}$ rises too quickly. The problem for $\langle y \rangle$ might be eased by raising $m_c$ (color threshold) but then I expect $\langle y \rangle$ will not be high enough at large energies and $R_c$ will show no significant rise. It is argued by Pati and Salam\textsuperscript{44} that asymptotic freedom is applicable to their theory, but I do not feel it will help the case with light gluons. It should be
acknowledged that the W-S-GIM model also has difficulty explaining these data.

In this approach, dimuons in neutrino scattering can result from the processes:

\[ \nu + N \rightarrow \mu + C + x \]  \hspace{1cm} (12.5)

where

\[ C = D \rightarrow \mu \nu + K + x \]  \hspace{1cm} (12.6)

or

\[ C = V_{\text{color}} \rightarrow \mu \nu \]  \hspace{1cm} (12.7)

Unlike charm production, \(V^\pm\) production has no accompanying K (usually) and is dominantly a two-body decay.

For the deep-inelastic, neutral-current neutrino scattering, a rise in \(\sigma/E\) with \(E\) is predicted after color threshold, unlike conventional theories where little energy dependence is expected for \(\sigma/E\).

No predictions have been published yet for \(\nu e\) elastic scattering or for atomic physics parity violation experiments, although work is in progress.

In summary, for \(m_{\nu} \approx 1.5\) GeV the following problems arise. The observed absence of a peak in \(e^+e^-\) or \(\mu^+\mu^-\) invariant mass plots in pp (or pN) collisions is evidence against the existence of a \(U^0\) gluon of this mass. There will soon be scans in \(e^+e^-\) annihilation in this energy region which will completely resolve this issue. Since the area under \(U^0\) should be about 50 nb - GeV, and since limits that are at least one order of magnitude lower will be set, it will be difficult to escape this problem (unless it is found). There is no obvious explanation of \(R(e^+e^-)\) without a heavy lepton (or other contributions) so one must go beyond the "basic model". With \(m_V \approx 2\), the \(<\gamma>_{\nu}\) in charged-current \(\bar{\nu}\) scattering rises much too quickly.
XIII. OTHER POSSIBILITIES AND CONCLUSIONS ABOUT THE PS THEORY

Are other masses for $U^0$ possible and helpful in solving the above problems? The lack of lepton pairs in pp scattering is observed at higher masses. A heavy lepton (or other contributions) is still needed to explain $R(e^+e^-)$. In addition it was shown earlier that the higher masses of $U^0$ require higher masses for free quarks, so that the explanation of $e^+e^- \to \mu eX^0$ as quark decays fails and here too a heavy lepton is needed (requiring at least that one go beyond the "basic model" with only 16 fermions). The fits to charged-current data (see Figs. 7 and 8) are somewhat better with $m_U = 4 \text{ GeV}$.

However, the crucial point is that the entire region from 1.9 to 7.6 GeV has now been scanned (with no gaps) in $e^+e^-$ annihilation at a level sufficient to see the $U^0$, and the $U^0$ has not been found there. The areas under the $U^0$ resonance, calculated from $\Gamma_{ee}$ given by Pati and Salam, are considerably above the area limits quoted by the SPEAR and Frascati collaborations. So it is difficult to understand how the $U^0$ could have a mass between 1.9 and 7.6 GeV.

Let us consider four cases for the masses of gluons and of quarks.

1. Both quarks and gluons are light. This case (advocated by Pati and Salam) is discussed in Sec. XII.

2. Gluons are light but quarks heavy. The problems of case 1 remain. In particular the $U^0$ must be found.

3. Quarks are light but gluons heavy. $R(e^+e^-)$ would equal 4 at low energies (instead of the experimental value of 2.3). There are other similar problems.

4. Both quarks and gluons are heavy. Then color is frozen out, and all phenomenology is identical to that for the confined, fractional-charge quark theory, and there is no point for discussion.
There are a number of problems for the Pati-Salam gauge theory for unconfined, integer-charge gluons and quarks. I have not considered the advantages of extending the "basic model" to include other fermions and right-handed currents. Clearly, some problems can be solved that way. However, sooner or later one must see the U° or see free quarks.

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