A SIMPLE DESCRIPTION OF THE $J^P = 1^+ K^\pm \pi^\mp$ SYSTEM

IN THE REACTIONS $K^\pm p \rightarrow K^\pm_\pi \pi^- p$ *


Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A model in which $Q_1$ and $Q_2$ resonance contributions add coherently to a gaussian background is shown to reproduce the mass dependence of the $J^P = 1^+ K^*\pi$ and $\rho K$ partial waves in $K^\pm p \rightarrow K^\pm_\pi \pi^- p$ at 13 GeV/c. Through a fit to the data, the mass and total width for $Q_1$ are found to be $m = 1289 \pm 3(25) \text{ MeV}$, $\Gamma = 150 \pm 9(\pm 70) \text{ MeV}$ and for $Q_2$:

$m = 1404 \pm 3(\pm 10) \text{ MeV}$, $\Gamma = 142 \pm 4(\pm 15) \text{ MeV}$, where estimated systematic errors are given in parentheses. While a significant background is required for the $1^+ K^*\pi$ system, none is needed for the $1^+ \rho K$ system.

*Work supported by the Energy Research and Development Administration.
†Now at the Physics Department, Carleton University, Ottawa, Ontario, Canada K15 5B6.
‡Now at the Physics Department, Oxford University, Oxford, England.
§Now at Laboratoire de l'Accélérateur Linéaire, Orsay, France.
In previous letters [1, 2] we have presented results for the $J^P = 1^+ K^* \pi$ and $\rho K$ partial waves obtained from an analysis of the $K\pi\pi$ system observed in the reactions

$$K^+ p \rightarrow K^+ \pi^- \pi^- p$$

(1)
at 13 GeV/c. The structure observed in the mass dependence of the cross section and relative phase of these $1^+$ partial waves was qualitatively interpreted as due to the presence of two axial vector $Q$ mesons and a low mass "Deck" background. In this paper we demonstrate that a straightforward model which coherently adds background and two resonance contributions does indeed quantitatively reproduce all the mass dependent features of these partial waves. From fits of this model to the data, we obtain values for the mass, width, and decay couplings of the $Q_1$ and $Q_2$ mesons. A good knowledge of these parameter values for the $Q$ mesons provides an important clue in predicting the properties of the missing axial vector mesons such as the $A_1$.

We first review those features of the data motivating a description in terms of two resonances and a background contribution. The $1^+ K^* \pi$ and $\rho K$ partial waves are shown as a function of $K\pi\pi$ mass in figs. 1 and 2. The wave notation $J^P M^\gamma$ denotes the $K\pi\pi$ spin, parity, magnetic substate, and the exchange naturality. The $Q_1$ meson is associated with the peak $\sim 200$ MeV wide in the $1^+ \rho K$ partial waves centered at $\sim 1300$ MeV (fig. 2). If $Q_1$ has a small $K^* \pi$ coupling, the large forward phase variation ($\sim 70^\circ$ relative to $1^+ 0^+ K^* \pi$) which these waves exhibit in this region would substantiate a resonance interpretation. The $Q_2$ meson is associated with the structure observed in the cross section for the $1^+ 0^+ K^* \pi$ waves at $\sim 1400$ MeV with a width of $\sim 160$ MeV (fig. 1a, b). Assuming $Q_2$ has a small coupling to $\rho K$, the backward relative phase motion of the $1^+ \rho K$ waves (fig. 2) in the 1400 MeV region is further evidence for the second
resonance. In addition, the $2^+ 1^+ K^*\pi$ wave describing the $K^*(1420)$ meson shows little phase variation relative to the $1^+ 0^+ K^*\pi$ wave (fig. 1e,f). Finally the large peak in the $1^+ K^*\pi$ waves at $\sim 1200$ MeV is attributed to a "Deck" background contribution.

We now introduce a simple model which describes the experimental features of the data† summarized above. We write each partial wave amplitude, $N_{ij}$, as the sum of a background or "Deck" component, $D_{ij}$, and two resonance contributions, $R_{ij}$,

$$N_{ij} = D_{ij} + R_{ij}^{(1)} + R_{ij}^{(2)} .$$

The subscript $i$ denotes $J^P M^\eta$ and $j$, the isobar channel ($K^*\pi$ or $\rho K$). We assume that each resonance contribution factorizes into a "production" part depending only on p-p momentum transfer ($t'$) and a Breit-Wigner part depending only on $K\pi\pi$ mass $m$:

$$R_{ij}^{(n)} = a_{ij}^{(n)} e^{i\phi_{ij}^{(n)}} C_j \gamma_j^{(n)} \frac{q_j^{2L_j+1}}{m_n^2 - m^2 - i m_n \Gamma_n} , \quad n=1,2 .$$

The total width, $\Gamma_n$, is related to the reduced partial width couplings, $\gamma_j^{(n)}$, by

$$m_n \Gamma_n = \sum_j \left( \gamma_j^{(n)} \right)^2 \frac{q_j^{2L_j+1}}{m} .$$

In eqs. (3) and (4), $\langle q_j \rangle$ denotes the momentum of the Q meson decay products averaged over a Breit-Wigner intensity for the isobar, and $L_j$ is the orbital angular momentum between the decay products. The $C_j$ are isospin coefficients for the $K^*\pi$ (-2/3) and $\rho K$ (1/$\sqrt{3}$) channels. The parameters $a_{ij}^{(n)}$, $\phi_{ij}^{(n)}$ correspond

†Similar features have also recently been observed in an analysis combining K-bubble chamber data at 10, 14, and 16 GeV/c (ref. [3]).
to the production part of the amplitude averaged over the momentum transfer interval \(-t' < 0.3 \text{ GeV}^2\). We emphasize that the features of the K\(\pi\pi\) partial waves when studied as a function of \(t'\) differ significantly with beam particle [2] and isobar channel [2,3]. Consequently the only constraint we place on the \(a^{(n)}_1, \phi^{(n)}_1\) is that resonance production be independent of decay channel. The parametrization of eq. (3) was chosen so that the different production properties [2,3] of the \(1^+K^*\pi\) and \(1^+\rho K\) systems may be accommodated by further parametrizing \(a^{(n)}_1, \phi^{(n)}_1\) as functions of \(t'\). For the background contribution we write

\[
D_{ij} = A_{ij} e^{i\phi_{ij}} q_j^{1/2} \exp \left[ -\frac{(m-m_{0ij})^2}{2\sigma_{ij}^2} \right] .
\]

Here we assume a gaussian mass dependence for the "Deck" contribution and specify the averaged \(t'\) dependence by means of \(A_{ij}\) and \(\phi_{ij}\).

For each partial wave the cross section is given by

\[
\frac{d^2\sigma}{d\Omega dt'} \bigg|_{ij} = |N_{ij}|^2 / C_{ij, kl}^2 .
\]

and the relative phase between partial waves by

\[
\phi_{rel} = \text{arg} \left( N_{ij}^* N_{kl} \right) .
\]

In eq. (6) \(C_{ij, kl}\) is the coherence parameter between two waves defined by

\[
C_{ij, kl} = \frac{|\rho_{ij, kl}|}{|\rho_{ij, ij}|^{1/2} |\rho_{kl, kl}|^{1/2}} ,
\]

where \(\rho_{ij, kl}\) is the density matrix element between waves \(ij\) and \(kl\). The observation [5] that the \(1^+0^+, 1^+1^+\) and \(2^+1^+K^*\pi\) waves are to a good approximation coherently produced relative to one another leads us to set the coherence
parameter to the value one for these waves. For the $1^+0^+$ and $1^+1^+\rho K$ waves, we use values of the coherence parameter consistent with our measurements.†

The values of the parameters are determined by least squares fits to the $K^-$ and $K^+$ data separately, with the resonance parameters $\gamma^{(n)}_j$ and $m_n$ constrained to be the same in both cases. The fits were performed using the data over the full mass range for the $1^+K^*\pi$ partial waves, the region $1.21 \leq m \leq 1.46$ GeV for the $1^+\rho K$ waves, and $1.33 \leq m \leq 1.50$ GeV for the $2^+1^+K^*\pi$ waves. Phase data for $1^+1^+K^*\pi$ were not used for $m \leq 1.10$ GeV. To use eqs. (2)-(5) in fits to the data, one phase parameter for both $K^+$ and $K^-$ must be fixed. We chose to make $D_{ij}$ purely real for the $1^+0^+K^*\pi$ amplitude ($\theta(1^+0^+K^*\pi) = 0$). Thus all phases are measured relative to this background component. The $K^*(1420)$ mass and width were fixed at the nominal values [4].

The curves resulting from our fit are shown in figs. 1 and 2. The overall $\chi^2$ is 772 for 252 data points. The errors are statistical only; systematic uncertainties due to the neglect of small partial waves and isospin 3/2 contributions, for example, have not been included. The overall agreement of the curves with the data is satisfactory, the most noticeable systematic discrepancy being in the 1.2 GeV region for the $1^+0^+K^*\pi$ waves. A somewhat more flexible background than eq. (5) does do better in describing the low mass peaks. In addition we recall that there are ambiguous solutions [1] in the $K^-$ case for $1.14 \leq m \leq 1.25$ GeV; these are indicated by crosses in figs. 1 and 2. The contributions of

†The $1^+\rho K$ system is found [1, 3] not to be coherently produced relative to $1^+0^+K^*\pi$. The coherence parameters (with $k\ell=1^+0^+K^*\pi$) used for the $1^+\rho K$ waves are for $K^+$, 0.75 ($1^+0^+\rho K$), 0.85 ($1^+1^+\rho K$) and for $K^-$, 0.76 ($1^+0^+\rho K$), 0.87 ($1^+1^+\rho K$). The measured $t$-channel coherence parameters [5] are essentially independent of $K\pi\pi$ mass. We nevertheless identify the $1^+\rho K$ relative phase data with our model through eq. (7). This is justified provided the magnitude of the true [5, 6] $1^+0^+K^*\pi$ nucleon helicity nonflip amplitude is large compared to that of the helicity flip amplitude.
background and $Q_2$ ($Q_1$ is negligible) to $1^+0^+K^*\pi$ are shown in figs. 1a and b by dotted and dashed curves, respectively. Our background is considerably narrower than that expected for "Deck" mechanisms. However, it has been noted by several authors [7,8] that a "Deck" amplitude should in principle be unitarized in the presence of $K^*\pi$ or $\rho K$ resonance effects; such a procedure could modify a broad input background to agree with that found here. The model successfully describes the different $K^+$ and $K^-$ mass dependence of both $1^+0^+$ and $1^+1^+K^*\pi$ waves near 1400 MeV. We note that the incoherently combined background and resonance contributions account for no more than $\sim60\%$ of the observed $1^+0^+K^*\pi$ cross sections, indicating sizeable interference contributions.

The $1^+\rho K$ partial cross sections in fig. 2 are reproduced essentially by a single Breit-Wigner corresponding to $Q_1$. Any attempt to include a background of the form eq. (5) is rejected by the fit, essentially because of the large relative phase motion between the $1^+0^+K^*\pi$ and $1^+\rho K$ waves. The absolute phase of the model amplitude for the $1^+0^+K^*\pi$ wave does not start to move until about 1.3 GeV, where it then increases according to the Breit-Wigner description of $Q_2$ to about 110° at 1.5 GeV. Thus the relative phase motion shown in fig. 2 corresponds essentially to the difference of two Breit-Wigner phase curves for resonances of similar widths but separated by $\sim100$ MeV in mass.

The parameters of our fit are presented in tables 1-2. The partial widths of $Q_1$ and $Q_2$ (table 2) were calculated as indicated in eq. (4) using $<q_{K^*}> = 306$ MeV, $<q_{\rho}> = 141$ MeV for $Q_1$ and $<q_{K^*}> = 395$ MeV, $<q_{\rho}> = 279$ MeV for $Q_2$. The errors in parentheses for the resonance parameters (table 2) are estimates of systematic uncertainties in our parametrization. They are based on the spread of values we have observed from fits with different width parametrizations, different background shapes, and different choices of $K^-$ solutions in the ambiguous...
region [1]. The large systematic uncertainty in $\Gamma_{\rho K} (Q_1)$ is due principally to the fact that removing the factor of m in eq. (4) results in a larger value for this width; this sensitivity to width parametrization for $Q_1$ is not surprising in view of its proximity to the nominal $\rho K$ threshold. In addition we note that there is some coupling [5] of $Q_1$ to $\kappa \pi$ and, possibly $\epsilon K$; the inclusion of these smaller modes in our fit would decrease $\Gamma_{\rho K} (Q_1)$ by $\sim 25\%$, while leaving the total width unchanged. Similarly, $\Gamma_{K^*\pi} (Q_2)$ would decrease by $\sim 20\%$ were the $\epsilon K$ channel [5] included in the fit. The most striking feature of the present results is the nearly complete decoupling of $Q_1$ from the $K^*\pi$ mode and of $Q_2$ from the $\rho K$ mode.

In a conventional mixing scheme for $Q_A$ and $Q_B$, this decoupling pattern of $Q_1$ and $Q_2$ suggests a mixing angle near $45^\circ$ (modulo $\pi$) and is not unexpected from quark model estimates [9].

In conclusion we have presented a simple model which quantitatively reproduces the features in the mass dependence of the $1^+ K^*\pi$ and $\rho K$ partial waves. We emphasize that the structure of the model allows for the different production features observed in the data for the $1^+ K^*\pi$ and $\rho K$ systems [2, 3] as well as the difference between the $K^+$ and $K^-$ reactions [2]. We find that the $1^+ K^*\pi$ system is described by a coherent sum of contributions from a low mass ($\sim 1.2$ GeV) background and, essentially, a single resonance ($Q_2$) with a mass of 1404 MeV and a width of 142 MeV. In contrast, the $1^+ \rho K$ system is almost entirely described in terms of a single resonance ($Q_1$) of mass 1289 MeV and width 150 MeV. While the model we have presented is admittedly not unique [7, 8], it nevertheless provides a simple description of the data. In addition it demonstrates that the inclusion of interference effects between the resonance production and "Deck" background amplitudes is probably essential to a quantitative understanding of the experimental measurements.
REFERENCES


FIGURE CAPTIONS

1. Comparison of the $1^+ 0^+$, $1^+ 1^+$, and $2^+ 1^+ K^*\pi$ cross sections and relative phases with the results (solid curves) of the fit described in the text. The reference wave is $1^+ 0^+ K^*\pi$. The crosses denote the $K^-$ ambiguous solutions (lower likelihood). The dotted and dashed curves in (a) and (b) correspond to the background and $Q_2$ contributions, respectively.

2. Comparison of the $1^+ 0^+$ and $1^+ 1^+ \rho K$ cross sections and relative phases with the results of our fit. The reference wave is $1^+ 0^+ K^*\pi$. The crosses denote the $K^-$ ambiguous solutions (lower likelihood).
Table 1
Background parameters.

<table>
<thead>
<tr>
<th>J^P M</th>
<th>Beam</th>
<th>A(\mu b^{1/2}/GeV^2)</th>
<th>\Phi (deg.)</th>
<th>m_0 (MeV)</th>
<th>\sigma (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1^+0^+</td>
<td>K^+</td>
<td>60.0\pm0.9</td>
<td>0</td>
<td>1135\pm4</td>
<td>134\pm4</td>
</tr>
<tr>
<td></td>
<td>K^-</td>
<td>68.5\pm1.3</td>
<td>0</td>
<td>1136\pm3</td>
<td>131\pm4</td>
</tr>
<tr>
<td>1^+1^+</td>
<td>K^+</td>
<td>18.7\pm0.5</td>
<td>-154\pm2</td>
<td>1154\pm5</td>
<td>98\pm4</td>
</tr>
<tr>
<td></td>
<td>K^-</td>
<td>17.5\pm0.5</td>
<td>-151\pm3</td>
<td>1190\pm9</td>
<td>147\pm11</td>
</tr>
<tr>
<td>$I^G$</td>
<td>$\Lambda$</td>
<td>10$^4$ MeV</td>
<td>1420</td>
<td>$K^*$</td>
<td>$\frac{\Lambda}{\Lambda}$</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td>-----------</td>
<td>------</td>
<td>------</td>
<td>----------------</td>
</tr>
<tr>
<td>1.420</td>
<td>$\ell$</td>
<td>3,7 ± 0.2</td>
<td>3,6 ± 0.1</td>
<td>158 ± 6</td>
<td>85 ± 4</td>
</tr>
<tr>
<td>1,986</td>
<td>$\ell$</td>
<td>8,7 ± 0.2</td>
<td>9,1 ± 0.3</td>
<td>118 ± 6</td>
<td>100 ± 6</td>
</tr>
<tr>
<td>1,986</td>
<td>$\ell$</td>
<td>8,7 ± 0.2</td>
<td>9,1 ± 0.3</td>
<td>118 ± 6</td>
<td>100 ± 6</td>
</tr>
<tr>
<td>1,986</td>
<td>$\ell$</td>
<td>8,7 ± 0.2</td>
<td>9,1 ± 0.3</td>
<td>118 ± 6</td>
<td>100 ± 6</td>
</tr>
</tbody>
</table>

Table 2

Systematic errors for masses and widths are given in parentheses.

Resonance parameters. The total width for $1^+$ is 150 ± 9 (170) MeV and for $2^+$ 142 ± 4 (165) MeV.
$K^+ p \rightarrow K^+ \pi^+ \pi^- p \quad 13 \text{ GeV}$

$-t' < 0.3 \text{ GeV}^2$

$K^+_T$

Fig. 1
$K^+\pi^-\pi^+\pi^-\rho$ 13 GeV
$-t' < 0.3$ GeV$^2$

$K^+$

(a) $l^+O^+\rho K$

(b) $l^+O^+\rho K$

(c) $l^+l^+\rho K$

(d) $l^+l^+\rho K$

Fig. 2