Measurement of the Inclusive Charmless Semileptonic Branching Ratio of $B$ Mesons and Determination of $|V_{ub}|$


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We report a measurement of the inclusive charmless semileptonic branching fraction of $B$ mesons in a sample of 89 million $B\bar{B}$ events recorded with the BABAR detector at the $\Upsilon(4S)$ resonance. Events are selected by fully reconstructing the decay of one $B$ meson and identifying a charged lepton from the decay of the other $B$ meson. The number of signal events is extracted from the hadronic mass distribution and is used to determine the ratio of branching fractions

$$\frac{\mathcal{B}(B \to X_u \ell \bar{\nu})}{\mathcal{B}(B \to X\ell \bar{\nu})} = (2.06 \pm 0.25(\text{stat}) \pm 0.23(\text{syst}) \pm 0.36(\text{theo})) \times 10^{-2}.$$
Using the measured branching fraction for inclusive semileptonic $B$ decays, we find \( \mathcal{B}(\overline{B} \to X_u \ell \overline{\nu}) = (2.24 \pm 0.27(stat) \pm 0.26(syst) \pm 0.39(theo)) \times 10^{-3} \) and derive the CKM matrix element \( |V_{ub}| = (4.62 \pm 0.28(stat) \pm 0.27(syst) \pm 0.48(theo)) \times 10^{-3} \).

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The element \( |V_{ub}| \) of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1] plays a critical role in testing the consistency of the Standard Model description of CP violation. In this paper, we present a determination of \( |V_{ub}| \) from a measurement of inclusive charmless semileptonic decays \( \overline{B} \to X_u \ell \overline{\nu} \) [2]. The analysis uses \( \Upsilon(4S) \to B\overline{B} \) events in which one of the \( B \) meson decays hadronically and is fully reconstructed (\( B_{\text{reco}} \)) and the semileptonic decay of the recoiling \( B \) meson is identified by the presence of an electron or muon. While this approach results in a low overall event selection efficiency, it allows for the determination of the momentum, charge, and flavor of the \( B \) mesons. We use the invariant mass \( m_X \) of the hadronic system to separate \( \overline{B} \to X_u \ell \overline{\nu} \) decays from the dominant \( \overline{B} \to X_c \ell \overline{\nu} \) background, which clusters above the \( D \) meson mass [3]. By ensuring a higher signal purity and acceptance than previously achieved [4], and by measuring the fraction of charmless semileptonic decays\( R_u = \mathcal{B}(\overline{B} \to X_u \ell \overline{\nu})/\mathcal{B}(\overline{B} \to X_c \ell \overline{\nu}) \), this analysis leads to substantially smaller systematic uncertainties [5].

The measurement presented here is based on a sample of about 89 million \( B\overline{B} \) pairs collected near the \( \Upsilon(4S) \) resonance by the \( \text{BABAR} \) detector [6] at the \( \text{PEP-II} \) asymmetric-energy \( e^+ e^- \) storage ring operating at SLAC.

We use Monte Carlo (MC) simulations of the \( \text{BABAR} \) detector based on \text{GEANT} [7] to optimize selection criteria and to determine signal efficiencies and background distributions. Charmless semileptonic \( \overline{B} \to X_u \ell \overline{\nu} \) decays are simulated as a combination (see Fig. 1a) of resonant three-body decays \( (X_u = \pi, \eta, \rho, \omega, \ldots) \) [8] and decays to nonresonant hadronic final states \( X_u \) [9], for which the hadronization is performed by \text{Jetset} [10]. The motion of the \( b \) quark inside the \( B \) meson is implemented with the shape function parametrization given in Ref. [9]. The simulation of the \( \overline{B} \to X_u \ell \overline{\nu} \) background uses an HQET parametrization of form factors for \( \overline{B} \to D^* \ell \tau \) [11], and models for \( \overline{B} \to D \pi \ell \tau, D^* \pi \ell \tau \) [12], and \( \overline{B} \to D \ell \tau, D^* \ell \tau \) [8].

To reconstruct a large sample of \( B \) mesons, hadronic decays \( B_{\text{reco}} \to \overline{D^*} \ell \gamma, \overline{D^*} \gamma \gamma \) are selected. Here, the system \( Y^\pm \) consists of hadrons with a total charge of \( \pm 1 \), composed of \( n_1 \pi^\pm + n_2 K^\pm + n_3 K_S^0 + n_4 \rho^0 \), where \( n_1 + n_2 \leq 5, n_3 \leq 2, \) and \( n_4 \leq 2 \). We reconstruct \( D^*^- \to \overline{D^*} \ell \gamma \), \( D^*^0 \to \overline{D^*} \rho^0, \overline{D^*} \rho \), \( D^- \to K^+ \pi^- \pi^- \), \( K^+ \pi^- \pi^- \pi^0, K_S^0 \rho^-, K_S^0 \rho^+ - K^0 \pi^- \pi^- \pi^+ - K^0 \pi^+ \pi^- \); and \( D^0 \to K^+ \pi^- \pi^- \pi^0, K^+ \pi^- \pi^- \pi^+ \), \( K^+ \pi^+ \pi^- \pi^- \), \( K^+ \pi^- \pi^- \pi^+ \). The kinematic consistency of \( B_{\text{reco}} \) candidates is checked with two variables, the beam energy-substituted mass \( m_{\text{ES}} = \sqrt{s/4 - \vec{p}^2_B} \) and the energy difference \( \Delta E = E_B - \sqrt{s}/2 \). Here \( \sqrt{s} \) is the total energy in the \( \Upsilon(4S) \) center of mass frame, and \( \vec{p}_B \) and \( E_B \) denote the momentum and energy of the \( B_{\text{reco}} \) candidate in the same frame. We require \( \Delta E = 0 \) within three standard deviations as measured for each mode.

For each of the 1097 reconstructed \( B \) decay modes, the purity \( \mathcal{P} \) is estimated as the fraction of signal events with \( m_{\text{ES}} > 5.27 \text{GeV} / c^2 \). We only use events for which \( \mathcal{P} \) exceeds a decay mode dependent threshold in the range of 9% to 24%. In events with more than one reconstructed \( B \) decay, we select the decay mode with the highest purity. On average, we reconstruct one \( B \) candidate in 0.3% (0.5%) of the \( B^+ \overline{B}^0, B^+ \overline{B}^+ \) events. The purity for events with a high-momentum lepton is 67% (see Fig. 2a).

Semileptonic decays \( \overline{B} \to X \ell \overline{\nu} \) of the \( \overline{B} \) recoiling against the \( B_{\text{reco}} \) candidate are identified by an electron or muon with a minimum momentum of \( p^* > 1 \text{ GeV} / c \) in the \( \overline{B} \) rest frame. For charged \( B_{\text{reco}} \) candidates, we require the charge of the lepton to be consistent with a prompt semileptonic \( B \) decay. For neutral \( B_{\text{reco}} \) candidates, both charge-flavor combinations are retained and the known average \( B^0, \overline{B}^0 \) mixing rate is used to extract the prompt lepton yield. Electrons are identified [13] with 92% average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1%. Muons are identified [6] with an efficiency ranging between 60% (\( p^* > 1 \text{ GeV} / c \)) and 75% (\( p^* > 2 \text{ GeV} / c \)) and hadron misidentification rate between 1% and 3%. Efficiencies and misidentification rates are estimated from data control samples.

The hadronic system \( X \) in the decay \( \overline{B} \to X \ell \overline{\nu} \) is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the \( B_{\text{reco}} \) candidate or the identified lepton. Care is taken to eliminate fake charged tracks, as well as low-energy beam-generated photons and energy depositions in the calorimeter from charged and neutral hadrons. The neutrino four-momentum \( p_\nu \) is estimated from the missing momentum four-vector \( p_{\text{miss}} = p_Y - p_B - p_X - p_\ell \), where all momenta are measured in the laboratory frame and \( p_Y \) refers to the \( \Upsilon(4S) \) meson. The mass of the hadronic system is determined by a kinematic fit that imposes four-momentum conservation, the equality of the masses of the two \( B \) mesons, and forces \( p_\nu^2 = 0 \). The resulting \( m_X \) resolution is 350 MeV/\( c^2 \) on average.
one charged lepton with $p^* > 1$ GeV/c, charge conservation ($Q_X + Q_l + Q_{B_{reco}} = 0$), and a missing mass consistent with zero ($m^2_{miss} < 0.5$ GeV/c$^2$). These criteria suppress the dominant $B \rightarrow X_c \ell \bar{v}$ decays, many of which contain additional neutrinos or an undetected $K^*_L$ meson. We suppress the $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}$ background by reconstructing the $\pi^+_s$ from the $D^{*+} \rightarrow D^0 \pi^+_s$ decay and the lepton: since the momentum of the $\pi^+_s$ is almost collinear with the $D^{*+}$ momentum in the laboratory frame, we can approximate the energy of the $D^{*+}$ as $E_{D^{*+}} \approx m_{D^{*+}} \cdot E_{\pi^+_s}/145$ MeV/c$^2$ and require for the neutrino $m^2_{\nu} = (p_B - p_{D^{*+}} - p_l)^2 < -3$ GeV$^2$/c$^4$. We veto events with charged or neutral kaons in the recoil $B$ to reduce the background from $B \rightarrow X_c \ell \bar{v}$ decays. Charged kaons are identified [6] with an efficiency varying between 60% at the highest and almost 100% at the lowest momenta. The pion misidentification rate is about 2%. The $K^0 \rightarrow \pi^+ \pi^-$ decays are reconstructed with an efficiency of 80% from pairs of oppositely charged tracks with an invariant mass between 486 and 510 MeV/c$^2$. The impact of the event selection on the $m_X$ distribution is illustrated in Fig. 1b.

We determine $R_u$ from $N_u$, the observed number of $B \rightarrow X_c \ell \bar{v}$ candidates with $m_X < 1.55$ GeV/c$^2$, and $N_{sl}$, the number of events with at least one charged lepton:

$$R_u = \frac{B(B \rightarrow X_c \ell \bar{v})}{B(B \rightarrow X\ell \bar{v})} = \frac{N_u}{N_{sl}} \varepsilon_{m_X}^{sl} \varepsilon_{m_X} \varepsilon_{sl} \varepsilon_{reco} \varepsilon_{reco} \varepsilon_{reco}.$$  

Here $\varepsilon_{m_X} = (34.2 \pm 0.6)\%$ is the efficiency for selecting $B \rightarrow X_c \ell \bar{v}$ decays once a $B \rightarrow X \ell \bar{v}$ candidate has been identified, $\varepsilon_{m_X} = (73.3 \pm 0.9)\%$ is the fraction of signal events with $m_X < 1.55$ GeV/c$^2$, $\varepsilon_{sl}/\varepsilon_{reco} = 0.887 \pm 0.008$ corrects for the difference in the efficiency of the lepton momentum cut for $B \rightarrow X \ell \bar{v}$ and $B \rightarrow X_c \ell \bar{v}$ decays, and $\varepsilon_{reco}/\varepsilon_{reco} = 1.00 \pm 0.03$ accounts for a possible efficiency difference in the $B_{reco}$ reconstruction in events with $B \rightarrow X \ell \bar{v}$ and $B \rightarrow X_c \ell \bar{v}$ decays.

We derive $N_{sl}$ from a fit to the $m_{ES}$ distribution shown in Fig. 2a. The fit uses an empirical description [14] of the combinatorial background from continuum and $B \bar{B}$ events, together with a narrow signal [15] peaked at the $B$ meson mass. The small tail accounts for energy losses in the reconstruction of $\pi^0$ mesons. The residual background in $N_{sl}$ from misidentified leptons and semileptonic charm decays amounts to 6.8% and is subtracted.

We extract $N_u$ from the $m_X$ distribution by a minimum $\chi^2$ fit to the sum of three contributions: the signal, the background $N_c$ from $B \rightarrow X_c \ell \bar{v}$, and a background of $< 1\%$ from other sources (misidentified leptons, secondary $\tau$ and charm decays). In each bin of the $m_X$ distribution, the combinatorial $B_{reco}$ background for $m_{ES} > 5.27$ is subtracted on the basis of a fit to the $m_{ES}$ distribution (Fig. 2b). Fig. 3a shows the fitted $m_X$ distribution. To minimize the model dependence, the first bin is extended to $m_X < 1.55$ GeV/c$^2$. The fit reproduces the data well with $\chi^2/dof = 7.6/6$. Fig. 3b shows the $m_X$ distribution after background subtraction with finer binning. Table I summarizes the results of fits with different requirements on $m_X$, for electrons and muons, for neutral and charged $B_{reco}$ candidates, and for different ranges of the $B_{reco}$ purity $P$. The results are all consistent within the uncorrelated statistical errors.

We have performed extensive studies to determine the systematic uncertainties on $R_u$. To establish that the background from $B \rightarrow X_c \ell \bar{v}$ events is adequately simulated we use previously excluded events with charged or neutral kaons as a control sample. The relative systematic error due to uncertainties in the detection of photons is estimated to be 4.7% by varying the corrections applied to the MC simulation to match the data control samples. An additional error of 1.0% is ascribed to the uncertainty in the simulation of showers generated by $K^0_L$ interactions; it is equal to the shift caused by the removal of the $K^0_L$ energy depositions in the MC simulation.
FIG. 3: The $m_X$ distribution for $\bar{B} \to X \ell \bar{\nu}$ candidates: a) data (points) and fit components, and b) data and signal MC after subtraction of the $b \to c \ell \bar{\nu}$ and the “other” backgrounds.

The uncertainties in the background modeling due to varying the parameters within one standard deviation of the default values. The limited statistics of the simulated event samples adds an uncertainty of 4.5%. The choice of bins for $m_X > 1.55 \text{ GeV}/c^2$ impacts the fit result at a level of 1.2%. All the above mentioned experimental errors add up to 8.7%.

The uncertainties in the background modeling due to branching fraction measurements for $B \to D \ell \nu$, $D^* \ell \nu$, ... and for inclusive and exclusive $D$ meson decays [16] contribute 4.4%.

The error due to the hadronization in the $\bar{B} \to X_u \ell \bar{\nu}$ final state is estimated to be 3.0% by measuring $R_u$, as a function of the charged and neutral particle multiplicities and performing the fit with only the nonresonant part of the signal model. We assign an additional 2.8% error to account for the uncertainties in the inclusive and exclusive branching fractions for charmless semileptonic $B$ decays [16], plus 3.7% for the veto on strange particles. Here, we assume a 100% uncertainty in the $s\bar{s}$ contents for the resonant and 30% for the nonresonant component [17]. These three uncertainties contribute a combined error of 5.5%.

The efficiencies $\varepsilon_{\text{incl}}$ and $\varepsilon_{\text{m}}$ are sensitive to the detailed modeling of the $\bar{B} \to X_u \ell \bar{\nu}$ decays. We assess these uncertainties by varying the nonperturbative parameters in the model [9] within their errors, $\mathcal{A} = 0.48 \pm 0.12 \text{ GeV}$ and $\lambda_1 = -0.30 \pm 0.11 \text{ GeV}^2$, obtained from the results in Ref. [18] by removing terms proportional to $1/m_b^3$ and $\alpha_s^2$ from the relation between the measured observables and $\mathcal{A}$ and $\lambda_1$. Taking into account the correlation of $-0.8$ between $\mathcal{A}$ and $\lambda_1$, we arrive at a theoretical error of 17.5%.

In summary, we obtain

$$R_u = (2.06 \pm 0.25 \pm 0.23 \pm 0.36) \times 10^{-2},$$

where the errors are statistical, systematic (experimental plus signal and background modeling), and theoretical, respectively. Taking into account common errors we compute the double ratio

$$R_u \left( \frac{B(\bar{B} \to X_u \ell \bar{\nu})}{B(\bar{B} \to X \ell \bar{\nu})} \frac{B(B \to X \ell \bar{\nu})}{B(\bar{B} \to X_u \ell \bar{\nu})} \right) = 0.72 \pm 0.18(\text{stat}) \pm 0.19(\text{syst}).$$

Combining the ratio $R_u$ with the measured inclusive semileptonic branching fraction $B(\bar{B} \to X \ell \bar{\nu}) = (10.87 \pm 0.18(\text{stat}) \pm 0.30(\text{syst}))\%$ [13], we have

$$B(\bar{B} \to X_u \ell \bar{\nu}) = (2.24 \pm 0.27 \pm 0.26 \pm 0.39) \times 10^{-3}.$$
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[2] Charge-conjugation is implied throughout the Letter.
[5] The results of this analysis depend on the assumption of parton-hadron duality to which no error is assigned.
[18] D. Cronin-Hennessy et al. [CLEO Collaboration], Phys. Rev. Lett. 87, 251808 (2001). We assume that the shape function parameters in Ref. [9] can be related to the HQET parameters $\tilde{\Gamma}$ and $\tilde{\Lambda}_1$.
[19] The impact of the uncertainty of the fraction of neutral and charged $B$ mesons is negligible.